RESEQUENCING DELAY AND BUFFER OCCUPANCY IN SELECTIVE REPEAT ARQ WITH MULTIPLE RECEIVERS

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RESEQUENCING DELAY AND BUFFER OCCUPANCY IN SELECTIVE REPEAT ARQ WITH MULTIPLE RECEIVERS

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ABSTRACT

Of all of the Automatic Repeat Request (ARQ) protocols, the Selective-Repeat (SR) protocol achieves the highest efficiency, both for a single receiver and multiple receivers. However, under this protocol a receiver may accept packets out of order from the channel. If a receiver is required to transfer the packets to the user in order, it must queue packets that arrive out of sequence before they can be delivered. We consider an SR ARQ protocol with one source and multiple receivers, where each receiver acknowledges all packets and handles its resequencing buffer based only on the packets that it receives error-free. An analysis of the resequencing delay and buffer occupancy at a receiver is presented. We construct a model that allows us to derive steady-state results and that takes into consideration system parameters such as number of receivers, propagation delay, and error probabilities of packets and acknowledgments. The main results of the analysis are the distribution of the resequencing delay and the distribution of the number of packets occupying the receiver's buffer. We also derive simple expressions for the mean buffer occupancy at the limit as the packet error probability tends to one. Numerical results are provided to illustrate the effects of system parameters on the behavior of the expected resequencing delay and the expected buffer occupancy.

1 INTRODUCTION

Data integrity in communication networks is usually preserved by means of automatic repeat request (ARQ) protocols. ARQ is accomplished by adding error-detecting code (e.g., CRC) to the information bits, and retransmitting packets when errors are reported via a feedback channel. Of the three basic ARQ protocols—Stop-and-Wait, Go Back(N), and Selective Repeat (SR)[4]—the last, SR, achieves the highest channel efficiency, since packets are sent continuously and only packets that arrive in error at the receiver are retransmitted. ARQ protocols have also been applied to a *multicast* environment in which a single transmitter sends packets reliably to a set of receivers. It has been shown that SR ARQ is the most efficient for this environment as well [13,5].

Packets queue at the transmitter under all ARQ protocols because of the random nature of the packet arrival process and the error process. The SR protocol, however, has an additional side-effect; namely, it causes packets to be accepted by the receiver not necessarily according to the order in which they were first transmitted. Many applications require the receiver to deliver packets in their original order to the next protocol layer or to the next node on the route. Consequently, packets that arrive out of order have to be temporarily held at the receiver, thereby resulting in additional delay in the packets' path and requiring the receiver to allocate storage hardware.

Previous queueing analyses of SR ARQ protocols have focused on the first of the aforementioned delays for a single transmitter-receiver pair [10,1]. An analysis of the resequencing delay has been reported for a configuration consisting of a single transmitter-receiver pair communicating over a single channel [12]. This model was subsequently extended to include multiple parallel channels that differ in their error probabilities [11]. To the authors' best knowledge, no analytic models for the queueing at the multicast SR ARQ transmitter have been developed so far, and existing analyses of variations of this protocol are restricted to evaluating link throughput.

The objective of this paper is the quantitative study of the resequencing phenomenon under an SR ARQ operating in a multicast environment. Although it provides only a part of the total packet delay, concentrating on the second component of the overall packet queueing allows us to analyze the receiver's buffer at a level of detail that would be hard to achieve if an overall model construction was attempted. The results so generated can provide guidelines to the receiver design, and permit us to compare various protocols, that yield the same throughput but differ in the resequencing performance.

We consider here the basic mechanism of a SR ARQ multicast protocol, which includes packet (re)transmissions and positive/negative acknowledgments sent for every packet. In a real-life system, a SR multicast protocol must include additional mechanisms to keep track of the set of active receivers and to set up and terminate sessions. In

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this paper, however, we assume a stable receiver population and ignore the additional traffic required for session management so that we may focus on the transmission mechanism and the effect of the important system parameters on the protocol's performance.

The model used to analyze the multicast protocols is a generalization of the model developed in [12] for a single transmitter-receiver pair. The model used here also relaxes the noiseless-feedback assumption [12] and allows us to account for lost acknowledgments. The main results of this paper are the derivation of the distribution of a packet's resequencing delay and the distribution of receiver's buffer occupancy. We also derive simple expressions for the mean buffer occupancy in the limit as the packet error probability tends to one.

We note that there is a growing literature that focuses on the area of resequencing in queueing systems. Kleinrock et. al. considered the problem in the context of database updates [9]. In their model, customers (packets) incur delays from an exponential distribution independent of each other. Following this delay, customers are stored until they can be delivered in order. Plateau generalized this model by allowing the delays to come from a general distribution [7]. This model was further generalized to include additional processing of the packets after delivery [2].

Resequencing at the output of a multiserver exponential queue was considered by Kumar and Kermani, who analyzed the resequencing delay under the assumption of equal service rates to all servers [3]. Yum and Ngai extended that analysis to include servers with different rates [14]. More recently, Illiadis and Lien considered a two-server exponential queue with threshold service policies, in which the slow server is fed the kth message in the queue [8].

The paper is organized as follows. In Section 2 we describe the model in detail and the assumptions that underlie it. Section 3 contains the analysis of resequencing delay and provides the distribution of that delay. We also discuss some special cases of the model which provide insight into the effect of the system parameters, such as the number of receivers and the error rate of the feedback channel. Simple expressions for the limiting values of the packet delay as the packet loss probability tends to 1 are also derived. Section 4 contains the analysis that yields the distribution for the number of packets in the receiver's buffer and a simple expression for the limiting value of the mean buffer occupancy as the packet loss probability tends to one. Section 5 contains numerical results that illustrate the behavior of the multicast SR ARQ protocol for different parameter values.

2 THE MODEL

2.1 Model assumptions

Consider a system that consists of a transmitter delivering packets to M receivers reliably over a broadcast channel in which a single transmission reaches all the receivers. Each receiver is required to respond to each packet that it receives correctly by returning an acknowledgment (ACK) to the transmitter. The transmitter records the identities of the receivers that have acknowledged correct receipt of each packet. If the transmitter lacks acknowledgments from one or more receivers after a time-out period following packet transmission, the transmitter retransmits that packet; otherwise it discards the packet. Only the first ACK from each receiver for a specific packet is recorded by the source; the other ACKs are ignored. Each receiver manages its own resequencing buffer and clears from the buffer the maximum set of packets with contiguous sequence numbers upon receipt of the packet with lowest sequence number it has been waiting for. Observe that errors are not reported to the transmitter by negative acknowledgments; they are detected by the transmitter through the time-out mechanism. Also observe that an ACK received in error is detected by the time-out mechanism and is treated as if the corresponding packet had been received in error by the receiver. For a more detailed specification of a multicast SR ARQ which is based on positive acknowledgments, see Chandran and Lin [5]. Other multicast protocols that rely on negative acknowledgments have also been reported in the literature [6]; these protocols however, require a fully connected network, which is not assumed here.

The reader should observe that the contents of the various receivers' buffers are not necessarily the same because of independent resequencing by each receiver and because packets do not arrive successfully at all receivers at the same time. In this paper we analyze the resequencing buffer of one such receiver, taking into consideration the effect of the multicast transmission.

The model we use is based on the following assumptions:

- All packets are of the same length.
- The time axis is partitioned into slots, each of which can accommodate exactly one packet.
- A transmitted packet is received correctly by receiver i ($i = 1, 2, \ldots, M$) with probability p_i.
- An ACK sent from receiver i is received correctly at the source with probability q_i.
- Successes/failures of packets/ACKs are independent, both from user to user and among successive transmissions.
- Acknowledgments of a packet are received by the source following a round-trip propagation delay of $S-1$ slots after the packet is transmitted (unless they are in error).
- A receiver's resequencing buffer never overflows. Note that this assumption is not very strong, because, as we show later, buffer occupancy is unlikely to grow to a very large size, even under very noisy channel conditions.

		columns				
		1	2	...	$S-1$	S
frames	1	$i+1$	$i+2$	...	$i+S-1$	$i+S$
	2					
	⋮			⋮		
	n					$i+nS$

Figure 1: Slots, frames and columns

The analysis is facilitated by partitioning the sequence of time slots into frames, each of which is S slots long, as illustrated in Figure 1. The sequence of slots, vertically separated S slots from one another, which appears in the matrix in Figure 1, is called a *column*. The constant round-trip acknowledgment delay, which is typical of satellite channels, has the effect that a packet is transmitted every Sth slot until it is acknowledged by all of its receivers. That is, all the transmissions of a given packet are carried on contiguous slots of a single column, and a new packet is transmitted on a column only after the previous packet's acknowledgment is received. Since the error events are independent, and since columns carry different packets, the transmission processes on the various columns are independent. Our approach is thus to to study the properties of the transmission process of a single column and then to consider the effect of the other columns' processes on the overall delay and buffer occupancy.

Notice that under this model a receiver can send the sequence number of the last packet it has received correctly at the end of each slot on that column. Only the first ACK from each receiver for a specific packet is recorded by the source; the other ACKs are ignored.

2.2 Analysis of a column's transmission process

We consider now the transmission process on the k-th column ($k = 1,2,\ldots,S$), i.e., the sequence of slots $\{k + nS,\ n = \ldots,0,1,2,\ldots\}$. Since a packet is transmitted on consecutive column slots, this process is similar to a multicast ARQ protocol over a link with zero acknowledgment delay. The transmitter sees that process as a packet transmission followed by M immediate responses, and as mentioned above, each response is either recorded or ignored. A new packet is taken from the transmitter's pool (which we assume to have of infinite supply) only after the last ACK is received for the packet already in transmission on that column.

Receiver i's active participation in the packet transmission process on a column is divided into two phases, forward and backward:

Forward (fwd) This is the sequence of consecutive slots on the column, occupied by a packet, from the first time a packet is transmitted until it arrives error-free to receiver i. Let X_i be a random variable representing the length of this phase in column-slots. Its distribution is given by

$$P(X_i = k) = p_i \bar{p}_i^{k-1} \quad k = 1,2,\ldots \tag{1}$$

Notice that during this phase, receiver i send ACKs for the previous packet transmitted on that column, except for the last slot, at the end of which an ACK for the packet just received correctly is sent.

Backward (bwd) This is the sequence of slots from the first slot after a new packet is correctly received until ACKs from all M receivers arrive at the source. We denote the length of this interval as Y_i.

In Eqs. (1) and (2), $\bar{p}_i = 1 - p_i$ and $\bar{q}_i = 1 - q_i$ are probabilities of transmission error on the forward and backward channel, respectively. We define Y_i' to be the number of frames measured from the first slot following the arrival of a new packet at receiver i until the acknowledgment arrives at the source. The distribution of Y_i' is:

$$P(Y_i' = k) = q_i \bar{q}_i^k \quad k = 0,1,2,\ldots \tag{2}$$

Observe that $Y_i' = 0$ if the ACK is received correctly on its first transmission. The random variables $\{X_i,\ Y_i'\}$ are independent by the model's assumptions.

We denote by *cycle* the number of times a packet is transmitted until the last of the M ACKs is received. Let Z be the length of a cycle. Then, from the description above it is clear that Z is given by

$$Z = \max_{1\le i\le M} \{X_i + Y_i'\} \ . \tag{3}$$

The distribution of Z is given by

$$P(Z \le z) = \prod_{i=1}^{M} P(X_i + Y_i' \le z)$$
$$= \prod_{i=1}^{M} \frac{p_i q_i}{\bar{q}_i - \bar{p}_i} \left[\frac{\bar{q}_i}{q_i}\left(1 - \bar{q}_i^z\right) - \frac{\bar{p}_i}{p_i}\left(1 - \bar{p}_i^z\right) \right],\ z = 1,2,\ldots \tag{4}$$

and its expectation by

$$E(Z) = \sum_{z=0}^{\infty} [1 - P(Z \le z)] \ . \tag{5}$$

Last, observe that according to the definition of Y_i,

$$Y_i = Z - X_i \ . \tag{6}$$

The transmission process in each column can thus be modeled by a renewal process, with the beginning of the first transmission of a packet as a renewal point; the times between renewals are i.i.d. and distributed as Z. Notice that $1/E(Z)$ is the rate of packet departure from the transmitter which equals the arrival rate of packets to the receiver. $1/E(Z)$ is also the overall rate of packet arrival, since the transmission processes on the columns are statistically the same.

Let F_i denote the probability that at a randomly selected slot a packet transmitted to receiver i is found in the fwd mode. By simple arguments one can easily show that F_i is given by

$$F_i = \frac{E(X_i)}{E(Z)} \quad . \tag{7}$$

Let $\overline{F}_i = 1 - F_i$.

Some understanding of the effects of the system parameters can be gained by considering two special cases

$M = 1$: Dropping the subscript for the probabilities of transmission success for the single receiver case we get: $E(Z) = (p\overline{q} + q)/(pq)$.

$q = 1$: Assuming that for all i, $p_i = p$ and $q_i = q = 1$ we can readily obtain

$$E(Z) = \sum_{s=0}^{\infty} \left[1 - (1 - \overline{p}^s)^M\right] = \sum_{k=1}^{M} (-1)^{k+1} \binom{M}{k} \frac{1}{1 - \overline{p}^k}$$

In both cases $E(X) = 1/p$. Notice that $F = 1$ for $M = 1$, $q = 1$ and/or that F decreases to 0 as q decreases to 0 and as M increases to ∞.

3 DELAY ANALYSIS

In this section we study the delay a packet incurs from the time it is accepted at receiver i's buffer until the time it is released from that buffer. We derive expressions for both the delay distribution and the limiting value of the mean delay as the probability of packet loss tends to one.

3.1 The delay distribution

Consider a test packet and assume, without loss of generality, that it is transmitted on column S and that its first transmission occurs on the k-th frame. Figure 2 depicts the transmissions of one such packet, and those of packets on the other columns. From the protocol and model description it is clear that the only packets that can hold the test packet in the resequencing buffer are those that are in the fwd mode during the k-th frame. The test packet is released from the buffer at the time the last of the following two events occurs:

a. The test packet is accepted by receiver i.

b. The last of the packets that were initially in the fwd mode for receiver i at the k-th frame (in columns $1, 2, \ldots, S-1$) ends its fwd mode by arriving error-free to receiver i.

We now define several variables, all of which have the subscript i since they refer to receiver i. Let X_i denote the number of times the test packet is transmitted in its fwd mode. Consider now the slots $1, 2, \ldots, S-1$ of the k-th frame. Denote by V_i the number of frames, starting from the k-th slot, until the last of the packets of those $S-1$ slots ends its fwd mode. If all those $S-1$ packets are in the bwd mode in the k-th frame, $V_i = 0$. Also, whenever $V_i > 0$, let U_i denote the number of the column on which the test packet is released from the buffer. For example, in Figure 2 $X_i = 3$, $V_i = 7$ and $U_i = 4$.

The probability that $V_i = 0$ is

$$P(V_i = 0) = F_i^{S-1}. \tag{8}$$

Since the column processes are independent, the joint distribution of V_i and U_i is, for $v > 0$ and $1 \leq u \leq S-1$,

$$P(V_i = v, U_i = u) = F_i p_i \overline{p}_i^{v-1} \left(1 - F_i \overline{p}_i^{v-1}\right)^{S-u-1} (1 - F_i \overline{p}_i^{v})^{u-1}. \tag{9}$$

Last, let W_i denote the number of slots in which the test packet is stored in receiver i's buffer before it is released. Notice that if $X_i \geq V_i$ then $W_i = 0$ since the test packet is released immediately upon its error-free reception. In general, the distribution of W_i is

$$P(W_i = w) = \begin{cases} P(V_i \leq X_i) \\ P(V_i = X_i + k,\ U_i = u)\ , \end{cases} \tag{10}$$

where the first equality holds for $w = 0$ and the second for $w = S(k-1) + u$, $1 \leq u \leq S-1$; $k \geq 1$. Since $P(V_i \leq k)$ is

$$P(V_i \leq k) = \left[1 - F_i \overline{p}_i^k\right]^{S-1} \quad k \geq 0\ , \tag{11}$$

the term $P(V_i \leq X_i)$ can be expressed as

$$\begin{aligned} P(V_i \ \leq \ X_i) &= \sum_{k=1}^{\infty} P(V_i \leq k) P(X_i = k) \\ &= \sum_{l0}^{S-1} \frac{(-1)^{S-l-1} \binom{S-1}{l} F_i^l p_i}{1 - \overline{p}_i^{l+1}}. \end{aligned} \tag{12}$$

The term $P(V_i = X_i + k,\ U_i = u)$ can be expressed as

$$\begin{aligned} &P(V_i \ = \ X_i + k,\ U_i = u) \\ &= \sum_{\ell=1}^{\infty} P(X_i = \ell) P(V_i = \ell + k, U_i = u) \\ &= F_i p_i^2 \sum_{\ell=1}^{\infty} \overline{p}_i^{2\ell+k-2} \left(1 - F_i \overline{p}_i^{\ell+k-1}\right)^{S-u-1} \left(1 - F_i \overline{p}_i^{\ell+k}\right)^{u-1} \quad (13) \\ &= F_i \overline{p}_i^k p_i^2 \sum_{m=0}^{S-u-1} \sum_{j=0}^{u-1} \binom{S-u-1}{m} \binom{u-1}{m} (-1)^{S-(m+j)} \\ &\quad \cdot F_i^{m+j} \overline{p}_i^{k(m+j)+j} \frac{1}{1 - \overline{p}_i^{m+j+2}}. \quad (14) \end{aligned}$$

The moments of the waiting time can now be computed. For example, the mean $E(W_i)$, is given by

$$\begin{aligned} E(W_i) &= \sum_{u=1}^{S-1} \sum_{k=0}^{\infty} [kS + u] P(W = kS + u) \\ &= \sum_{u=1}^{S-1} \sum_{k=0}^{\infty} [kS + u] P(V_i = X_i + k + 1, U_i = u)\ . \end{aligned} \tag{15}$$

Higher moments can be computed in a similar fashion. Substituting Eq. (14) into Eq. (15) along with some tedious algebraic manipulations yields a closed-form solution for the first moment of the packet delay. However, it is computationally more efficient to use Eqs. (13) and (15) and truncate the summation. Using Little's result, the expected buffer occupancy at receiver i is given by

$$E(N_i) = \frac{E(W_i)}{E(Z)}. \tag{16}$$

3.2 Limiting value of the mean packet delay

When $p_i = q_i = 1, i = 1, \ldots, M$, or when $p_i = 0$, the resequencing delay is zero. In the first case all packets are received and acknowledged correctly on the first transmission, whereas in the second case, no packets arrive at all at the receiver. For a given M and $0 < q_i \le 1$, $i = 1, \ldots, M$, and $0 < p_j$, $j \neq i$, the delay is a decreasing function of p_i. Thus, there is a discontinuity at $p_i = 0$, and as $p_i \downarrow 0$ the mean packet resequencing delay reaches its maximum. We proceed now to obtain upper and lower bounds on the expected delay for small values of p_i.

Consider the test packet at the frame in which it is first transmitted and assume, as before, that it is transmitted on column S. Let $R_i = \max\{X_i, V_i\}$ be the number of *frames* during which the packet is either transmitted or resides in the receiver's buffer. The cumulative distribution for R_i is

$$\begin{aligned} P(R_i \le r) &= (1 - F_i\bar{p}_i^r)^{S-1}(1 - \bar{p}_i^r) \\ &= 1 - \bar{p}_i^r + \sum_{j=1}^{S-1}(-1)^j \binom{S-1}{j} F_i^j \left[\bar{p}_i^{jr} - \bar{p}_i^{(j+1)r}\right] \end{aligned} \tag{17}$$

and the average number of frames, $E[R_i]$ is

$$\begin{aligned} E(R_i) &= \sum_{r=0}^{\infty}\left[\sum_{j=1}^{S-1}(-1)^{j+1}\binom{S-1}{j}F_i^j\left[\bar{p}_i^{jr} - \bar{p}_i^{(j+1)r}\right] + \bar{p}_i^r\right] \\ &= \frac{1}{p_i} + \sum_{j=1}^{S-1}(-1)^{j+1}\binom{S-1}{j}F_i^j\left[\frac{1}{1-\bar{p}_i^j} - \frac{1}{1-\bar{p}_i^{j+1}}\right] \end{aligned} \tag{18}$$

As $p_i \downarrow 0$, $E(R_i)$ becomes

$$\lim_{p_i \downarrow 0} E(R_i) = \frac{1}{p_i} + \frac{1}{p_i}\sum_{j=1}^{S-1}(-1)^{j+1}\binom{S-1}{j}F_i^j\frac{1}{j(j+1)}. \tag{19}$$

Denoting the summation of the right-hand side as f_S,

$$\begin{aligned} f_S &= \sum_{j=1}^{S-1}(-1)^{j+1}\left[\binom{S-2}{j} + \binom{S-2}{j-1}\right]F_i^j\frac{1}{j(j+1)} \\ &= f_{S-1} + \frac{1}{(S-1)SF_i}\sum_{j=2}^{S}(-1)^j\binom{S}{j}F_i^j. \end{aligned}$$

Thus, we can express f_S recursively as

$$f_S = f_{S-1} + \frac{1}{F_iS(S-1)}\left[(1-F_i)^S \quad , -1 + SF_i\right] \tag{20}$$

where $f_2 = F_i/2$.

The first term on the right-hand side of Eq. (19) represents the expected number of frames the packet spends in its fwd phase; thus f_s/p_i is the expected number of frames spent in the resequencing buffer. Thus,

$$S\left(\frac{f_S}{p_i} - 1\right) \le \lim_{p_i \downarrow 0} E(W_i) \le \frac{Sf_S}{p_i}. \tag{21}$$

As p_i decreases, so does the relative difference between these two bounds.

4 RECEIVER'S BUFFER OCCUPANCY

In this section we provide a similar analysis of the buffer occupancy at a receiver. We derive expressions for the distribution of the buffer occupancy, the number of packets in a receiver's buffer, and a limiting expression for the mean buffer occupancy as the packet loss probability tends to one.

4.1 The buffer occupancy distribution

We now proceed to obtain the distribution of the number of packets in the buffer of receiver i as observed immediately prior to the end of a randomly selected slot (the *observation instant*). Without loss of generality, we assume that that slot is in the S-th column, in the t-th frame, $-\infty < t < \infty$. The receiver's buffer, observed at that instant, contains packets that have been received error-free and that are ordered according to their sequence numbers. The series of numbers contains at least one gap: a nonempty buffer means that the receiver is still waiting for a packet with a sequence number lower than those already in the buffer. The packet with the lowest sequence number among the packets not yet delivered by the receiver to the user is called the *oldest packet.* In addition there may be other gaps in the packet sequence, each such gap representing one or more packet that have been transmitted unsuccessfully so far.

We denote by the *age* of an observed packet the number of times that packet has been transmitted unsuccessfully before the observation instant. A packet, of course, must be in the fwd mode to have an age greater than zero. One can readily show that the oldest packet has the highest age at the observation instant, and that among all the packets observed with that age, the oldest packet is the one transmitted on the smallest numbered column. The approach for obtaining the buffer occupancy is to condition it first on the age of the oldest packet. In the remainder of this section we drop the explicit reference to receiver i to simplify the ensuing notation.

Let A_j denote the age of the packet currently under transmission in column j of the receiver during the frame containing the randomly chosen slot. If the packet in column j has been received successfully (i.e., the column is in the bwd mode), then $A_j = 0$. The distribution of A_j is

$$P(A_j = n) = \begin{cases} \overline{F} + Fp & ,n = 0 \\ Fp\overline{p}^n & ,n > 0. \end{cases} \quad (22)$$

Let A denote the age of the oldest packet in the buffer over all columns, and C the column in which that oldest packet is transmitted. Then we have the following expressions:

$$P(A = 0) = [\overline{F} + Fp]^S, \quad (23)$$

$$P(A = n, C = k) = Fp\overline{p}^n \left(1 - F\overline{p}^n\right)^{k-1} \left(1 - \overline{p}^{n+1}\right)^{S-k} \quad (24)$$

Let $N_{c,n}$ denote the number of packets that arrive at the receiver within column c during the preceding n frames. We are interested in computing the probability distribution for that number *conditioned* on the event that the age of the packet currently under transmission in that column is less than n, i.e., $R(k,n) = P(N_{c,n} = k | A_c \le n)$, $0 \le k \le n$. Normally, if we observe column c at a randomly chosen instant, the remaining number of slots to the end of the cycle at that column, denoted here by Z^{re}, has the distribution of the residual life in a renewal process. The distribution of Z^{re} is

$$P(Z^{re} = k) = \left(1 - \sum_{j=1}^{k-1} P(Z = j)\right) / E(Z). \quad (25)$$

However, we observe the column c at frame $t - n + 1$ and we know that since that column does not contain the oldest packet, the cycle ended at or before the observation instant. Thus, we are interested in the statistics of Z^{re} conditioned on the event that the packet under transmission in column c at frame $t - n + 1$, if one exists, will not require more than n additional transmissions. If we let E denote this event, then we are interested in $P(Z^{re} = k|E) = P(Z^{re} = k,\ E)/P(E)$ where

$$P(E) = 1 - F_i\overline{p}^n. \quad (26)$$

For $(0 \le k \le n)$ the joint probability $P(Z^{re} = k,\ E)$ is

$$P(Z^{re} = k,\ E) = P(Z^{re} = k). \quad (27)$$

The value of this probability when $k > n$ is more difficult to calculate. Fortunately these values are not required in our remaining calculations.

We consider two subcases:

1. $N_{c,n} = 0$. In order for column c to add at least one packet, the following two conditions must hold:
   (a) The cycle present in column c at frame $t - n + 1$ must complete in less than n slots and,
   (b) The subsequent fwd phase at the receiver must also complete before the observation instant.

   Based on these events, the probability that column c does not contribute to the buffer occupancy as seen at the observation instant is given by

$$R(0,n) = 1 - \sum_{j=1}^{n} P(Z^{re} = k|E)(1 - \overline{p}^{n-j}) \quad . \quad (28)$$

2. $N_{c,n} > 0$. In this case column c can be partitioned into the following segments between $t - n + 1$ and t:
   (a) The last part of a cycle. During this segment column c does not contribute any packet to the buffer.
   (b) m complete cycles, whose lengths are i.i.d., each distributed as Z. Each cycle contributed exactly one packet to the buffer.
   (c) The first part of a cycle. A packet is accepted to the buffer from this segment only if the column has completed its fwd mode.

Each of these segments may be of zero length during the aforementioned period. Hence $N_{c,n} = m$ if either the second segment contains m cycles and all of the third segment is in the fwd mode, or the second segment contains $m - 1$ cycles and the column ends its fwd mode before the end of the third segment. The distribution for the number of packet arrivals given over the interval $t - n + 1$ and t is

$$\begin{aligned} R(k,n) &= \sum_{j=1}^{n-m} \sum_{l=m}^{n-j} P(Z^{re} = j|E) P^{(m)}(Z = l) \overline{p}^{n-j-l} \\ &+ \sum_{j=1}^{n-m-1} \sum_{l=m-1}^{n-j} P(Z^{re} = j|E) P^{(m-1)}(Z = l) \\ &\cdot \left[P(Z > n - j - l) - \overline{p}^{n-j-l}\right] \end{aligned} \quad (29)$$

where $P^{(m)}(Z)$ is the m-th convolution of the distribution of Z.

Let N denote the number of packets in the receiver's buffer at the observation slot. The distribution of N conditioned on the oldest packet residing in column c and having age n is

$$\begin{aligned} &P(N = m|A = n,\ C = c) \\ &= P\Big(\sum_{j<c} N_{j,n-1} + \sum_{j>c} N_{j,n} = m|A = n,\ C = c\Big) \quad (30) \\ &= \sum_{m_1+m_2=m} R^{(c-1)}(m_1, n-1) R^{(S-c)}(m_2, n), \end{aligned}$$

where $R^{(l)}(m,n)$ is the lth convolution of $R(m,n)$ and $1 \le c \le S; m = 0, 1, \cdots$. Removal of the conditioning on the age and location of the oldest packet yields

$$P(N=m)=\begin{cases} P(A=0)+\sum_{c,n}P(A=n,C=c) \\ R^{(c-1)}(0,n-1)R^{(S-c)}(0,n), \\ \\ \sum_{c,n}P(A=n,C=c)\sum_{m_1+m_2=m} \\ R^{(c-1)}(m_1,n-1)R^{(S-c)}(m_2,n), \end{cases} \tag{31}$$

4.2 Limiting value of the mean buffer occupancy

In this section we consider a system where $p_i = p$, $1 \le i \le M$. We derive an expression for the mean buffer occupancy as $p \downarrow 0$. An application of Little's result to Eq. (21) yields

$$\lim_{p \downarrow 0} S(f_S/p - 1)/E(Z) \le \lim_{p \downarrow 0} E(N) \le \lim_{p \downarrow 0} S f_S/(pE(Z)). \tag{32}$$

One can show that $\lim_{p \downarrow 0}(pE(Z))^{-1} = 1/H_M$ where $H_M = \sum_{i=1}^{M} 1/i$.
In addition, $\lim_{p \downarrow 0} 1/E(Z) = 0$. Consequently,

$$\lim_{p \downarrow 0} E(N) = S f_S / H_M. \tag{33}$$

Finally, the recurrence relation for f_S (Eq. (20)) requires the value of F_i as $p \downarrow 0$. One can show that $\lim_{p \downarrow 0} F_i = 1/H_M$.

5 RESULTS

Numerical results of the model are presented in Figures 3 to 8. In the Figures 3 through 5, we have chosen to demonstrate the resequencing performance by the mean buffer occupancy. Later, we study the tail of the distribution for the number of packets in the buffer. In all cases, we consider a system in which the error processes are the same for all receivers, $p_i = p$ and $q_i = q$ for all i, and, except for Figure 4, $S = 20$.

Figure 3 shows the expected buffer occupancy as a function of the error probability in the forward channel with the number of receivers, M as a parameter. We can see that, for a fixed value of p, the expected buffer occupancy decreases as M increases. Because the expected length of the fwd mode is determined by p, whereas $E(Z)$ increases with M. Thus, a packet that begins its transmission is likely to find fewer packets in the fwd mode on the other columns. Since these are the packets that may hold that new packet in the resequencing buffer, the fewer there are, the shorter is the time the packets must wait before the last of them is successfully transmitted.

We also observe from Figure 3 that the expected occupancy decreases as the probability of correct reception increases in the range $0 < p < 1$. From the model description, one can immediately see that the resequencing buffer is empty for the two extreme values of p. When $p = 1$ all the packets are received without errors, whereas in $p = 0$ none of the packets is received correctly by the receiver. A special point of operation is when $p \downarrow 0$, where the mean buffer occupancy accepts its largest value. Since for $p = 0$ the occupancy is 0, as mentioned above, this is a point of discontinuity in the graph. Such discontinuity has been also observed in the single-channel case [12].

Figure 4 depicts the values of the expected buffer occupancy at $p \downarrow 0$ as a function of S with M as parameter. As expected, the occupancy increases with S and decreases as M increases. Notice, however, that the *total* expected buffer occupancy in the system, i.e., the expected node buffer occupancy times M, increases with M.

The effect of the feedback channel's error probability is depicted in Figure 5 in which $S = 20$ and $M = 4$. In general, the higher the ACK loss probability (lower q), the lower the expected resequencing buffer occupancy. This can be explained in a manner similar to that of increasing M, since both lower the value of F. Notice, however, that the value of q has the lowest effect at the extreme values of p, where the buffer occupancy is determined mainly by p.

It is interesting to study the tail of the buffer occupancy distribution, as this provides insight regarding the size of buffer required by the receiver. We conjecture that the probability $P(N > B)$ obtained from our model is an upper bound on the probability that a receiver's buffer of size B will overflow. Figure 6 illustrates the probability that the buffer occupancy exceeds B where $B = 2, 4, 16, 32$ when there are no feedback errors as a function of the forward error probability p for $M = 16$ receivers. We observe that the buffer occupancy rarely exceeds 32 for the range of error probabilities depicted here. Consequently the buffer size at the receiver can be chosen to be less than 33. Figure 7 illustrates $P(N > B)$ for the case where the feedback error probability is 0.7. In this case we observe that the receivers require even less buffer capacity than in the case $q = 1$. For example, if one is willing to tolerate a probability of exceeding B of .05, then $B = 16$ is adequate for this system.

Last, we illustrate $P(N > B)$ for $p = .9$ and $q = 1$ as a function of the number of receivers. We observe that $P[N > B]$ decreases quickly as a function of the number of receivers when that number is small, and slowly for $M > 40$. When the number of receivers is 150, then $B = 9$ is sufficient so that the probability of that the buffer occupancy exceeds B is less than .05.

Before we end this section, we briefly consider another interesting case where one of the receivers is a bottleneck, e.g., $p_1 \downarrow 0$ and $p_j > 0$ for all $j \ne i$. In this case, $F_i \to 1$ and $F_j \to 0$, $j \ne i$, $1 \le j \le M$. Let us focus first on node i. Substitution of $F_i = 1$ into Eq. (20) and solving the resulting recurrence yields

$$f_S = H_S. \tag{34}$$

This can be substituted into Eq. (21) to yield tight bounds on the average resequencing delay at the ith receiver. When

$M = 1$, the bounds correspond to those obtained by Shacham [12]. In the case of receiver j, $j \neq i$, one can show that $E(W_j) \to 0$.

6 CONCLUSION

In this paper we have presented a model for evaluating the resequencing phenomenon in multicast an SR ARQ protocol in terms of a receiver's buffer occupancy and the delay a packet encounters in the resequencing buffer. We have derived the distribution of both occupancy and delay and have obtained simple formulation for the expected delay as the packet loss probability tends to 0. By means of numerical results, we have shown that the average buffer occupancy increases as the acknowledgment delay or the loss proability of ACKs increases, or as the number of receivers or the packets loss probabilities decrease. Curves for probabilities of buffer occupancies above some thresholds that are also presented can aid in designing the receivers for a proper operation under SR ARQ protocol.

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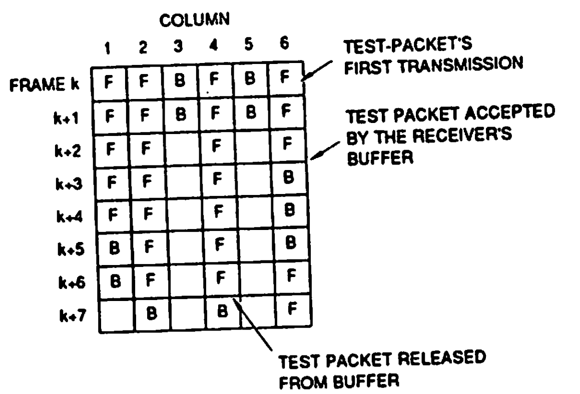

FIGURE 2 TRANSMISSION PROCESS OF THE TEST PACKET

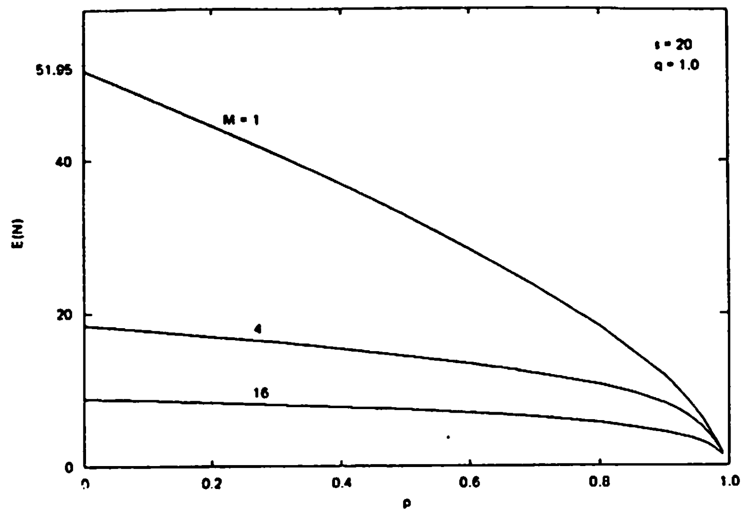

FIGURE 3 EXPECTED BUFFER OCCUPANCY AS A FUNCTION OF p
(M is parameter)

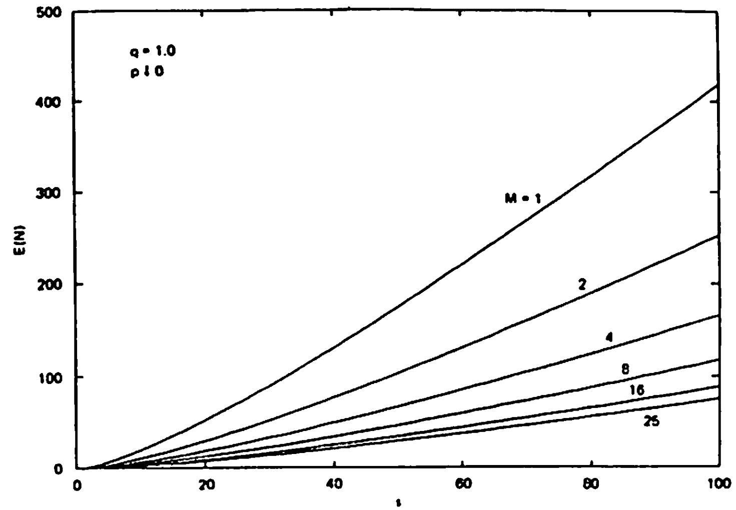

FIGURE 4 THE LIMITING VALUE OF THE EXPECTED OCCUPANCY AS p ↓ 0

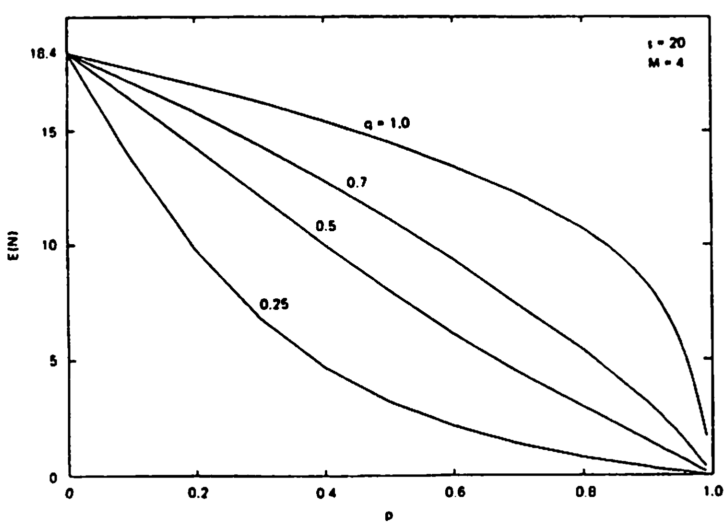

FIGURE 5 EXPECTED BUFFER OCCUPANCY AS A FUNCTION p
(q is parameter)

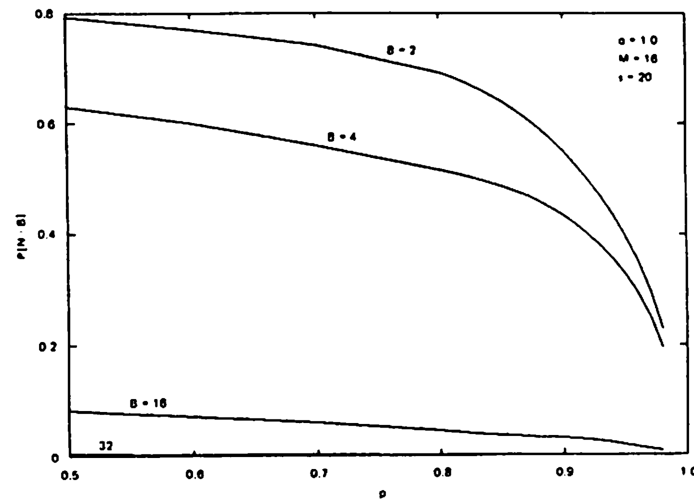

FIGURE 6 P(N > B) AS A FUNCTION OF p, q = 1.0

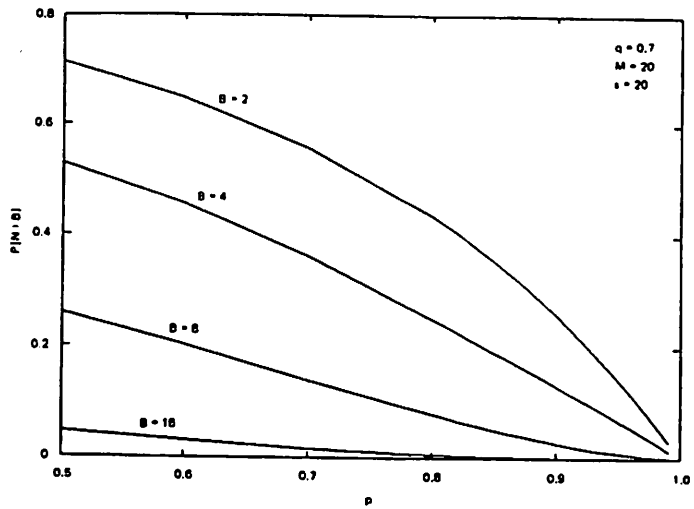

FIGURE 7 P[N > B] AS A FUNCTION OF p, q = 0.7

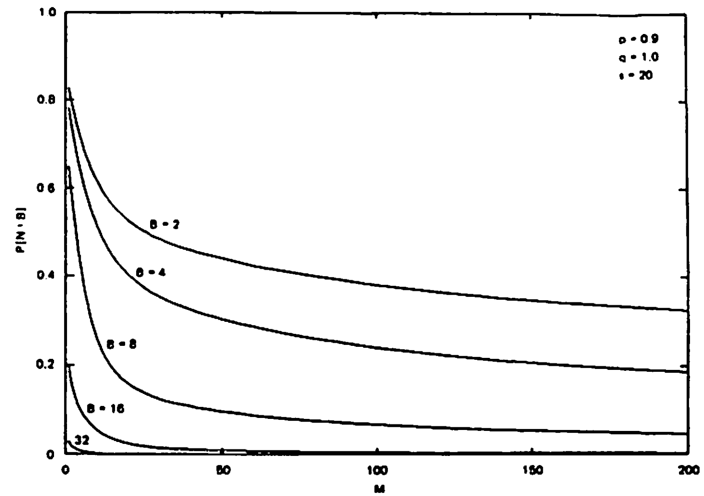

FIGURE 8 P[N > B] AS A FUNCTION OF M