RESEQUENCING DELAY AND BUFFER OCCUPANCY, IN SELECTIVE REPEAT ARQ WITH MULTIPLE RECEIVERS

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ABSTRACT

Of all the Automatic Repeat Request (ARQ) protocols, Selective-Repeat (SR) is the most efficient, both for single and multiple receivers. However, under this protocol, a receiver may accept packets from the channel out of their original order. To forward the packets in proper order, a receiver must first queue those that arrive out of sequence. In this paper we consider an SR ARQ protocol with one source and multiple receivers; each receiver acknowledges all packets and handles its resequencing buffer based only on the packets that it receives error-free. An analysis of the resequencing delay and buffer occupancy at a receiver is presented. We construct a model that enables us to derive steady-state results, taking into consideration such system parameters as number of receivers, propagation delay, packet error probabilities, and acknowledgments. We focus on two measures of occupancy. The first measure corresponds to the number of packets awaiting to be resequenced whereas the second measure includes, in addition, the buffer space reserved for packets that cause resequencing delays. The main results are the distribution of the resequencing delay, and the distribution of the number of packets occupying the receiver's buffer. We also present an expression for the mean of the second occupancy measure, as well as simple expressions for the mean of the first buffer occupancy in the limit as the packet error probability tends to one. Finally, we provide numerical results to illustrate the effects of system parameters on the resequencing delay and buffer occupancy.

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1 INTRODUCTION

Data integrity in communication networks is usually maintained using automatic repeat request (ARQ) protocols. ARQ is established by adding error-detecting code (for instance, cyclic redundancy check (CRC)) to the information bits, and requiring that packets be retransmitted when errors are reported via a feedback channel. Of the three basic ARQ protocols—Stop-and-Wait, Go Back(N), and Selective Repeat (SR)[4]—the last, SR, uses channels most efficiently, since packets are sent continuously, and only packets containing errors are retransmitted. ARQ protocols have also been applied to a *multicast* environment, in which a single transmitter sends packets reliably to a set of receivers. It has been shown that SR ARQ is the most efficient protocol for this environment as well [14,5].

Packets queue at the transmitter under all ARQ protocols, because of the random process by which packets arrive and the occurrence of errors. Under the SR protocol, packets may be accepted by the receiver not in the order in which they were first transmitted. Many applications, however, require the receiver to deliver packets to the next processing entity in their original order. Consequently, the receiver must temporarily hold in its buffers packet that arrive out of order, thereby causing *resequencing delay.* To simplify buffer design, the receiver allocates storage not only for packets waiting to be resequenced but also for packets that have not yet arrived but are causing resequencing delays.

Previous analyses of SR ARQ queueing have focused on the transmitter's buffer for a single source-destination environment [10,1]. An analysis has been made of resequencing delays in a configuration consisting of a single transmitter-receiver pair communicating over a single channel [13]. This model was subsequently extended to include multiple parallel channels with differing error probabilities [12]. To the authors' best knowledge, queueing at the multicast SR ARQ transmitter has not yet been analyzed, while variations of this protocol have been evaluated only for their throughput.

The objective of this paper is the quantitative evaluation of the resequencing phenomenon for an SR ARQ protocol operating in a multicast environment. Although resequencing represents only a part of the total packet delay, by concentrating on this component of the overall packet queueing we are able to analyze the receiver's buffer at a level of detail that would be hard to achieve if we attempted an overall model construction. The results so generated can provide us with guidelines for receiver design, and enable us to compare protocols that yield the same throughput but differ in their resequencing performance.

We will consider the basic transport mechanism of an SR ARQ multicast protocol, which includes packet (re)transmissions, and positive/negative acknowledgments sent for every packet. In a "real-life" system, an SR multicast protocol must include additional mechanisms to keep track of the active receivers, and to set up and terminate sessions. In this paper, however, we assume a stable receiver population and ignore the additional traffic required for session management, so that we may focus on the transmission mechanism and the effect of important system parameters on the protocol's performance.

The model we use to analyze the multicast protocol is a generalization of a model developed for a single transmitter-receiver pair, operating over a channel with noiseless feedback [13]. That model was used to obtain the statistics of the resequencing delay and the statistics of the number of packets waiting in the receiver's buffer for resequencing (excluding the storage reserved for packets not yet received). The generalized model used here, in addition to taking multiple receivers into consideration, also accounts for noisy feedback channel that results in lost acknowledgments. Our concerns in this paper are the analysis of: 1. Resequencing delay. 2. The number of packets awaiting resequencing (which we refer to as *buffer occupancy*), and 3. the number of packets waiting to be resequenced, plus the number of packets responsible for the queueing (*buffer occupancy with reservations*). Specifically, we will determine the distribution of a packet's resequencing delay, the distribution of receiver's buffer occupancy, and the mean buffer occupancy with reservations. We also will derive simple expressions for the mean buffer occupancy in the limit as the packet error probability tends to one.

We note that a growing literature focuses on resequencing in queueing systems. Kleinrock et al. considered the problem in the context of database updates [9]: in their model, customers (packets) incur delay from exponential distribution, independently of each other. After this delay, customers are stored until they can be delivered in order. Harrus and Plateau generalized this model, by allowing the delays to come from a general distribution [7]. This model was further generalized, to include additional processing of the packets after delivery [2].

Kumar and Kermani studied resequencing at the output of a multiserver exponential queue; they analyzed the resequencing delay under the assumption of equal service rates to all servers [3]. Yum and Ngai extended that analysis, to include servers with different service rates [15]. More recently, Illiadis and Lien considered a two-server exponential queue with threshold service policies, in which the slow server is fed the kth message in the queue [8]. Resequencing at the output of queues and servers in parallel has been treated by Jean-Marie [11]. Although all of these studies have been concerned with resequencing delays, and some of them have also dealt with the buffer occupancy, none of them have covered either buffer occupancy with reservations or retransmissions.

This paper is organized as follows. In Section 2 we describe the model in detail, and the assumptions that underlie it. In Section 3 we analyze resequencing delay, and determine the distribution of that delay. We also discuss some special cases of the model, which give some insight into the effect of the system parameters, such as the number of receivers and the error rate of the feedback channel. We also derive simple expressions for the limiting values of the packet delay, as the packet loss probability tends to 1. The analysis contained in Section 4 yields the distribution of the number of packets in the receiver's buffer, and also a simple expression for the limiting value of the mean buffer occupancy, as the packet loss probability tends to one. The section concludes with a derivation of the mean buffer occupancy with reservations. Section 5 contains numerical results illustrating the behavior of the multicast SR ARQ protocol, for different parameter values.

2 THE MODEL

2.1 Model Assumptions

Consider a system that consists of a transmitter delivering packets to M receivers, reliably, over a broadcast channel on which a single transmission reaches all the receivers. Each receiver returns an acknowledgment (ACK) to the transmitter for every packet received correctly. The transmitter records the identities of the receivers that have acknowledged correct receipt of each packet. If the transmitter lacks acknowledgments from one or more receivers after a time-out period following packet transmission, the transmitter retransmits that packet; otherwise it discards the packet. Notice that only the first ACK from each receiver for a specific packet need be recorded by the source; the other ACKs are ignored. Each receiver manages its own resequencing buffer, which contains all the correctly-received packets for which there is a packet with a lower sequence number that is still missing. On receipt of the packet with the lowest expected sequence number, the receiver releases the largest set of packets with contiguous sequence numbers, beginning with the packet whose sequence number is the lowest. Observe that errors are not reported to the transmitter by negative acknowledgments, but rather are detected by time-out, and that the transmitter discards any ACK received in error. For a more detailed description of a multicast SR ARQ based on positive acknowledgments, see Chandran and Lin [5]. Other multicast protocols designed for multiple sources and destinations have also been reported [6].

Notice that the contents of the various receivers' buffers are not necessarily the same, because of independent resequencing by each receiver and because packets do not arrive successfully at all receivers at the same time. In this paper, we analyze the resequencing buffer of one such receiver, taking into consideration the effects of multicast transmission.

The model we use is based on the following assumptions:

- All packets are of the same length.
- The time axis is partitioned into slots, each of which can accommodate exactly one packet.
- A transmitted packet is received correctly by receiver i ($i = 1, 2, \ldots, M$) with probability p_i.
- An ACK sent from receiver i is received correctly at the source with probability q_i.
- Successes/failures of packets/ACKs independently, both from receiver to receiver and among successive transmissions.
- Acknowledgments of a packet are received by the source $S - 1$ slots, after the packet is transmitted.
- A receiver's resequencing buffer never overflows. Note that this assumption is not very strong, because, as we show later, buffer occupancy is unlikely to grow to a very large size, even under very noisy channel conditions.

The performance measures of interest to us are

- The packet resequencing delay at the i-th receiver, W_i,
- The buffer occupancy at the i-th receiver, N_i, that is, the number of packets held in the receiver's buffer.
- The buffer occupancy with reservations at the i-th receiver, N_i', which consists of, in addition to N_i, the packets in the gaps of the sequence held at the receiver. This measure reflects the buffer size needed by the receiver.

Figure 1 illustrates the differences between N and N'. The receiver buffer in this example contains packets 4 and 6. Packets 1 and 2 have been released and packets 3, 5, 7, and 8 will arrive in that sequence. Here $N = 2$ (packets 4 and 6 are in the buffer) and $N' = 4$ (storage is allocated for packets 3 and 5). Notice that upon arrival of the missing packet with the lowest sequence number (packet 3), all the packets with contiguous numbers (at least packets 3 and 4) are instantly released.

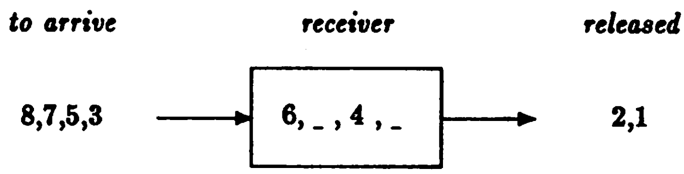

Figure 1: The Two Buffer Occupancies, $N_i = 2$, $N_i^! = 4$.

columns

frames		1	2	...	$S-1$	S
	1	$i+1$	$i+2$	...	$i+S-1$	$i+S$
	2	$i+S+1$				
	3	$i+2S+1$				
	$\vdots$			$\vdots$		
	n	$i+(n-1)S+1$				$i+nS$

Figure 2: Slots, Frames and Columns

To clarify our analysis, we show the sequence of slots partitioned into frames, each of which is S slots long, as illustrated in Figure 2. A sequence of slots read vertically, such as $\{i+1, i+S+1, i+2S+1, \ldots\}$, is called a *column*. The constant acknowledgment delay assumption causes of a packet's (re)transmitted to take place every Sth slot, that is on contiguous slots of a single column. A new packet is transmitted on a column only after all of the previous packet's acknowledgments arrive. Since the error events are independent, and since columns carry different packets, the transmission processes on the various columns are independent. Therefore, we will study the properties of the transmission process on a single column and then consider the effect of the other columns' processes on the overall delay and buffer occupancy.

2.2 Analysis of the Transmission Process on a Column's

We now consider the transmission process on the k-th column ($k = 1, 2, \ldots, S$), that is, the sequence of slots $\{k + nS,\ n = \ldots, 0, 1, 2, \ldots\}$. Because a packet is transmitted on consecutive slots of a column, the transmission process on a single column is equivalent to a multicast Stop-and-Wait ARQ protocol, in which a node waits for a timeout period before making its next transmission. The transmitter sees that process as a packet transmission followed by M immediate responses, each of which is either recorded or ignored. A new packet is taken from the transmitter's pool (which we assume to be an infinite supply); only after receipt of the last ACK of the packet previously transmitted on that column.

Receiver i's active participation in the packet transmission process on a column is divided into two phases, forward and backward:

The *Forward* (fwd) phase is defined as the sequence of consecutive slots on the column, occupied by a packet, from the first slot on which the packet is transmitted, until the slot in which it is received by receiver i. Let X_i be a random variable representing the length of this phase in column-slots, or, in other words, the number of transmissions a packet undergoes until it is received correctly by i. The distribution of X_i is given by

$$P(X_i = k) = p_i \overline{p}_i^{k-1} \quad k = 1, 2, \ldots \tag{1}$$

The *backward* (bwd) phase is defined as the sequence of slots on a column following the reception of a new packet by receiver i, until ACKs from all M receivers arrive at the source, see Figure 3.

Observe that a packet is in a given mode *with respect to a specific receiver* and, at a specific slot, may be at different modes for different receivers.

We use the notation $\overline{p}_i = 1 - p_i$ and $\overline{q}_i = 1 - q_i$, to denote the probabilities of packet loss on the forward and backward channel, respectively, with respect to receiver i. We define Y_i^f to be the number of frames measured from the first slot, following the arrival of a new packet at receiver i, until the acknowledgment arrives at the source. The distribution of Y_i^f is

$$P(Y_i^f = k) = q_i \overline{q}_i^{k} \quad k = 0, 1, 2, \ldots \tag{2}$$

Note that this distribution assumes that in every slot each receiver sends an ACK for the last packet it received on that column. This assumption can be easily relaxed by modifying the distribution in Eq (2). Observe that $Y_i^f = 0$, if the ACK is received correctly on its first transmission. The random variables $\{X_i, Y_i^f\}$ are independent by the model's assumptions.

We denote by *cycle* the sequence of slots over which a packet is transmitted until the last of the M ACKs is received. Let Z be the length of a cycle in slots measured along a column, that is, the number of slots a packet occupies. For example, in Figure 3 the cycle length on column 6 is 6. From the description above it is clear that Z is given by

$$Z = \max_{1 \le i \le M} \{X_i + Y_i'\} \quad . \tag{3}$$

The distribution of Z is given by

$$P(Z \le z) = \prod_{i=1}^{M} P(X_i + Y_i' \le z) = \prod_{i=1}^{M} \frac{p_i q_i}{\bar{q}_i - \bar{p}_i} \left[\frac{\bar{q}_i}{q_i}(1 - \bar{q}_i^z) - \frac{\bar{p}_i}{p_i}(1 - \bar{p}_i^z) \right], \; z = 1, 2, \ldots \tag{4}$$

and its expectation by

$$E(Z) = \sum_{z=0}^{\infty} [1 - P(Z \le z)] \quad . \tag{5}$$

The transmission process on a column can thus be modeled by a renewal process, with the beginning of the first transmission of a packet as a renewal point; the times between renewals are i.i.d. and distributed as Z. Notice that $1/E(Z)$ is the rate of packet departure from the transmitter which equals the arrival rate of packets to the receiver. $1/E(Z)$ is also the overall rate of packet arrival, since the transmission processes on the columns are statistically the same.

Let F_i denote the probability that at a randomly selected slot a packet transmitted to receiver i is found in the fwd mode. By simple arguments, one can easily show that F_i is given by

$$F_i = \frac{E(X_i)}{E(Z)} \quad . \tag{6}$$

Let $\overline{F}_i = 1 - F_i$.

Let us momentarily digress to examine two special cases which can shed some light on the system behavior.

$M = 1$: Dropping the subscript for the probabilities of transmission success for the single receiver case we get $E(Z) = (p\bar{q} + q)/(pq)$.

$q = 1$: Assuming that for all i, $p_i = p$ and $q_i = q = 1$ we can readily obtain

$$E(Z) = \sum_{z=0}^{\infty} \left[1 - (1 - \bar{p}^z)^M \right] = \sum_{k=1}^{M} (-1)^{k+1} \binom{M}{k} \frac{1}{1 - \bar{p}^k} \quad .$$

In both cases $E(X) = 1/p$. Notice that $F = 1$ for $M = 1$, $q = 1$ and that F decreases to 0 as q decreases to 0 and as M increases to ∞. We can also observe similar behavior in more general cases, as shown below.

3 DELAY ANALYSIS

In this section we study the delay, W_i, that a packet incurs from the time it is accepted at receiver i's buffer until the time it is released from that buffer. We derive an expression for the distribution of the resequencing delay and study the behavior of the mean resequencing delay as the packet loss probability tends to one.

3.1 The Delay Distribution

Consider a test packet and assume, without loss of generality, that it is transmitted on column S and that its first transmission occurs on the k-th frame. Figure 3 depicts the transmissions of such a packet, ($S = 6$), and those of packets on the other columns. From the protocol and model description, it is clear that the only packets that can hold the test packet in the resequencing buffer are those that are in the fwd mode during the k-th frame. The test packet is released from the buffer when the last of the following two events occurs:

1. The test packet is accepted by receiver i.
2. The last of the packets that were initially in the fwd mode for receiver i at the k-th frame (in columns $1, 2, \ldots, S-1$) ends its fwd mode by arriving error-free to receiver i.

We now define and use several variables, all of which have the subscript i since they refer to receiver i. Recall that X_i denotes the number of times the test packet is transmitted in its fwd mode. Consider now the slots $1, 2, \ldots, S-1$ of the k-th frame. Denote by V_i the number of frames, starting from the kth frame, until the last of the packets of those $S-1$ slots ends its fwd mode. If all those $S-1$ packets are in the bwd mode in the k-th frame, $V_i = 0$. Also, whenever $V_i > 0$, let U_i denote the number of the column on which the test packet is released from the buffer. For example, in Figure 3 $X_i = 3$, $V_i = 7$ and $U_i = 4$.

The probability that $V_i = 0$ is

$$P(V_i = 0) = \overline{F}_i^{S-1}. \tag{7}$$

Since the column processes are independent, the joint distribution of V_i and U_i is, for $v > 0$ and $1 \leq u \leq S-1$,

$$P(V_i = v, U_i = u) = F_i p_i \overline{p}_i^{v-1} \left(1 - F_i \overline{p}_i^{v-1}\right)^{S-u-1} (1 - F_i \overline{p}_i^{v})^{u-1}. \tag{8}$$

We observe that if $X_i \geq V_i$, then $W_i = 0$, since the test packet is released immediately upon its error-free reception. In general, the distribution of W_i is

$$P(W_i = w) = \begin{cases} P(V_i \leq X_i) \\ P(V_i = X_i + n,\ U_i = u)\,, \end{cases} \tag{9}$$

where the first equality holds for $w = 0$ and the second for $w = S(n-1) + u$, $1 \leq u \leq S-1$; $n \geq 1$. Since

$$P(V_i \leq n) = [1 - F_i \bar{p}_i^n]^{S-1} \quad n \geq 0 \ , \tag{10}$$

the term $P(V_i \leq X_i)$ can be expressed as

$$P(V_i \leq X_i) = \sum_{n=1}^{\infty} P(V_i \leq n) P(X_i = n) = \sum_{l=0}^{S-1} \frac{(-1)^{S-l-1}\binom{S-1}{l} F_i^l p_i}{1 - \bar{p}_i^{l+1}} \quad . \tag{11}$$

The term $P(V_i = X_i + n,\ U_i = u)$ can be expressed as

$$P(V_i = X_i + n,\ U_i = u) = \sum_{\ell=1}^{\infty} P(X_i = \ell) P(V_i = \ell + n, U_i = u)$$

$$= F_i p_i^2 \sum_{\ell=1}^{\infty} \bar{p}_i^{2\ell+n-2} \left(1 - F_i \bar{p}_i^{\ell+n-1}\right)^{S-u-1} \left(1 - F_i \bar{p}_i^{\ell+n}\right)^{u-1}$$

$$= F_i \bar{p}_i^n p_i^2 \sum_{m=0}^{S-u-1} \sum_{j=0}^{u-1} \binom{S-u-1}{m} \binom{u-1}{m} (-1)^{S-(m+j)} F_i^{m+j} \bar{p}_i^{n(m+j)+j} \frac{1}{1 - \bar{p}_i^{m+j+2}} \ . \tag{12}$$

The moments of the waiting time can now be computed. For example, the mean $E(W_i)$, is given by

$$E(W_i) = \sum_{u=1}^{S-1} \sum_{n=0}^{\infty} [nS + u] P(W = nS + u)$$

$$= \sum_{u=1}^{S-1} \sum_{n=0}^{\infty} [nS + u] P(V_i = X_i + n + 1, U_i = u) \ . \tag{13}$$

Higher moments can be computed in a similar fashion. Substituting Eq. (12) into Eq. (13) along with some tedious algebraic manipulations yields a closed-form solution for the first moment of the packet delay. However, it is computationally more efficient to use Eqs. (12) and (13) and truncate the summation. Using Little's result, the expected buffer occupancy at receiver i is given by

$$E(N_i) = \frac{E(W_i)}{E(Z)}. \tag{14}$$

3.2 Asymptotic Behavior of the Mean Resequencing Delay

Observe that when $p_i = q_i = 1, i = 1, \ldots, M$, or when $p_i = 0$, the resequencing delay is zero. In the first case all packets are received and acknowledged correctly on the first transmission, whereas in the second case, no packets arrive at all at the receiver. On the other hand, for a given M, the average resequencing delay at receiver i is a decreasing function of p_i whenever $0 < q_j \leq 1$, $j = 1, \ldots, M$, $0 < p_j \leq 1$, $j \neq i$. Thus, there is a discontinuity at $p_i = 0$, and as $p_i \downarrow 0$ the mean packet resequencing delay reaches its maximum. We proceed now to obtain upper and lower bounds for the expected delay of small values of p_i.

Consider the test packet at the frame in which it is first transmitted and assume, as before, that it is transmitted on column S. Let $R_i = \max\{X_i, V_i\}$ be the number of *frames* during which the packet is either transmitted or resides in the receiver's buffer. The cumulative distribution for R_i is

$$\begin{aligned} P(R_i \leq r) &= (1 - F_i\bar{p}_i^r)^{S-1}(1 - \bar{p}_i^r) \\ &= 1 - \bar{p}_i^r + \sum_{j=1}^{S-1} (-1)^j \binom{S-1}{j} F_i^j \left[\bar{p}_i^{jr} - \bar{p}_i^{(j+1)r}\right], \end{aligned} \tag{15}$$

and the average number of frames, $E(R_i)$ is

$$\begin{aligned} E(R_i) &= \sum_{r=0}^{\infty} \left[\sum_{j=1}^{S-1} (-1)^{j+1} \binom{S-1}{j} F_i^j \left[\bar{p}_i^{jr} - \bar{p}_i^{(j+1)r}\right] + \bar{p}_i^r\right] \\ &= \frac{1}{p_i} + \sum_{j=1}^{S-1} (-1)^{j+1} \binom{S-1}{j} F_i^j \left[\frac{1}{1-\bar{p}_i^j} - \frac{1}{1-\bar{p}_i^{j+1}}\right]. \end{aligned} \tag{16}$$

For small p_i, $E(R_i)$ can be written

$$E(R_i) = \frac{1}{p_i} + \frac{1}{p_i} \sum_{j=1}^{S-1} (-1)^{j+1} \binom{S-1}{j} F_i^j \frac{1}{j(j+1)} + O(1) \quad . \tag{17}$$

Denoting the summation of the right-hand side as f_S,

$$\begin{aligned} f_S &= \sum_{j=1}^{S-1}(-1)^{j+1}\left[\binom{S-2}{j}+\binom{S-2}{j-1}\right]F_i^j\frac{1}{j(j+1)} \\ &= f_{S-1}+\frac{1}{(S-1)SF_i}\sum_{j=2}^{S}(-1)^j\binom{S}{j}F_i^j \end{aligned}$$

Thus, we can express f_S recursively as

$$f_S = f_{S-1}+\frac{1}{F_iS(S-1)}\left[(1-F_i)^S-1+SF_i\right] \quad (18)$$

where $f_2 = F_i/2$.

The first term on the right-hand side of Eq. (17) represents the expected number of frames the packet spends in its fwd phase, and f_S/p_i is the expected number of frames spent in the resequencing buffer. Thus, for small p_i

$$S\left(\frac{f_S}{p_i}-1\right) \le E(W_i) \le \frac{Sf_S}{p_i} \quad (19)$$

As p_i decreases, so does the relative difference between these two bounds.

4 RECEIVER'S BUFFER OCCUPANCY

In this section we analyze N_i and N_i^r which, as we recall, are the ith receiver's buffer occupancy without and with reservations, respectively. We derive the distribution of N_i; a limiting expression (as $p \to 1$) for its mean, $E(N_i)$; and the average buffer occupancy with reservations, $E(N_i^r)$.

4.1 The Buffer Occupancy Distribution

We obtain the distribution of N_i as observed immediately prior to the end of a randomly selected slot (the *observation instant*). Without loss of generality, we assume that that slot is in the S-th column and in the t-th frame, $-\infty < t < \infty$. The receiver's buffer, observed at that instant, either is empty, or contains packets that have been received error-free and that are ordered according to their sequence numbers. The series of numbers contains at least one gap: a nonempty buffer means that the receiver is still waiting for a packet with a sequence number lower than those residing in the buffer. The packet with the lowest sequence number among the packets not yet delivered by the receiver to the user is called the *oldest packet*. In addition to the gap due to the oldest packet, there may be other gaps in the packet sequence, each such gap representing one or more packet that have been transmitted unsuccessfully so far, followed by a successfully-received packet.

If a packet is in the fwd mode at the observation instant, we denote by its *age* the number of times it has been transmitted (unsuccessfully). The age of packets in the bwd mode is, by convention, zero. One can readily show that the oldest packet has the highest age, and that of all the packets observed with that age, the oldest packet is the one transmitted on the smallest numbered column. The approach for deriving the buffer occupancy is to condition it first on the age of the oldest packet. In the remainder of this section we drop the explicit reference to receiver i, to simplify the ensuing notation.

Let A_j denote the age of the packet currently under transmission in column j during the frame containing the randomly-chosen slot. The distribution of A_j is

$$P(A_j = n) = \begin{cases} \overline{F} + Fp & n = 0 \\ Fp\overline{p}^n & n > 0 \end{cases} \quad . \tag{20}$$

Let A denote the age of the oldest packet, and C the column in which that oldest packet is transmitted. Then we have the following expressions:

$$P(A = 0) = [\overline{F} + Fp]^S \quad , \tag{21}$$

$$P(A = n, C = k) = Fp\overline{p}^n \left(1 - F\overline{p}^n\right)^{k-1} \left(1 - F\overline{p}^{n+1}\right)^{S-k} \quad . \tag{22}$$

Let $N_{c,n}$ denote the number of packets that arrive at the receiver within column c during the preceding n frames. We want to compute the probability distribution for that number conditioned on the event that the age of the packet currently under transmission in that column is less than n, i.e., $R(k,n) = P(N_{c,n} = k | A_c \le n)$, $0 \le k \le n$. Normally, if we observe column c at a randomly chosen instant, the number of slots remaining to the end of the cycle at that column, denoted here by Z^{re}, has the distribution of the residual life in a renewal process. The distribution of Z^{re} is

$$P(Z^{re} = k) = \left(1 - \sum_{j=1}^{k-1} P(Z = j)\right) / E(Z). \tag{23}$$

However, we observe the column c at frame $t-n+1$, and we know that since that column does not contain the oldest packet, the cycle ended at or before the observation instant. Thus, we are interested in the statistics of Z^{re} conditioned on the event that the packet under transmission in column c at frame $t-n+1$, if one exists, will not require more than n additional transmissions. Denoting this event by E, we are interested in $P(Z^{re}=k|E)=P(Z^{re}=k,\ E)/P(E)$, where

$$P(E)=1-F_i\bar{p}^n \quad . \tag{24}$$

For $(0 \leq k \leq n)$ the joint probability $P(Z^{re}=k,\ E)$ is

$$P(Z^{re}=k,\ E)=P(Z^{re}=k) \quad . \tag{25}$$

The value of this probability when $k>n$ is harder to calculate. Fortunately we do not use these values in our remaining calculations.

We consider two subcases:

1. $N_{c,n}=0$. In order for column c to add at least one packet, the following two conditions must be met:
   (a) The cycle present in column c at frame $t-n+1$ must complete in less than n slots.
   (b) The subsequent fwd phase at the receiver must also complete before the observation instant.

   Based on these events, the probability that column c does not contribute to the buffer occupancy as seen at the observation instant is expressed as

$$R(0,n)=1-\sum_{j=1}^{n}P(Z^{re}=k|E)(1-\bar{p}^{n-j}) \quad . \tag{26}$$

2. $N_{c,n}>0$. In this case, column c can be partitioned into the following segments between $t-n+1$ and t:
   (a) The last part of a cycle. During this segment, column c does not contribute any packet to the buffer.
   (b) m complete cycles, whose lengths are i.i.d., each distributed as Z. Each cycle contributed exactly one packet to the buffer.
   (c) The first part of a cycle. A packet is accepted to the buffer from this segment only if the column has completed its fwd mode.

Each of these segments may be of zero length during the aforementioned period. Hence $N_{c,n} = m$ either if the second segment contains m cycles and all of the third segment is in the fwd mode, or the second segment contains $m-1$ cycles and the column ends its fwd mode before the end of the third segment. The distribution for the number of packet arrivals given over the interval $t-n+1$ and t is

$$R(k,n) = \sum_{j=1}^{n-m}\sum_{l=m}^{n-j} P(Z^{re}=j|E)P^{(m)}(Z=l)\bar{p}^{n-j-l}$$

$$+\sum_{j=1}^{n-m-1}\sum_{l=m-1}^{n-j} P(Z^{re}=j|E)P^{(m-1)}(Z=l)\left[P(Z>n-j-l)-\bar{p}^{n-j-l}\right] \tag{27}$$

where $P^{(m)}(Z)$ is the m-th convolution of the distribution of Z.

Let N denote the number of packets in the receiver's buffer at the observation slot. The distribution of N, conditioned on the oldest packet residing in column c and having age n, is

$$\begin{aligned} P(N=m|A=n,\ C=c) &= P(\sum_{j<c} N_{j,n-1} + \sum_{j>c} N_{j,n} = m|A=n,\ C=c) \\ &= \sum_{m_1+m_2=m} R^{(c-1)}(m_1,n-1)R^{(S-c)}(m_2,n) \quad , \end{aligned} \tag{28}$$

where $R^{(l)}(m,n)$ is the lth convolution of $R(m,n)$ and $1 \le c \le S; m = 0,1,\cdots$. Removing the conditioning on the age and location of the oldest packet yields

$$P(N=m) = \begin{cases} P(A=0)+\sum_{c,n} P(A=n,C=c) & \\ \quad \cdot R^{(c-1)}(0,n-1)R^{(S-c)}(0,n), & m=0 \\ & \\ \sum_{c,n} P(A=n,C=c)\sum_{m_1+m_2=m} & \\ R^{(c-1)}(m_1,n-1) \quad R^{(S-c)}(m_2,n) & m>0. \end{cases} \tag{29}$$

4.2 Limiting Value of the Mean Buffer Occupancy

In this subsection, we consider a system where $p_i = p$, $1 \le i \le M$. We derive an expression for the mean buffer occupancy as $p \downarrow 0$. Applying Little's result to Eq. (19) yields

$$\lim_{p\downarrow 0} S(f_S/p-1)/E(Z) \le \lim_{p\downarrow 0} E(N) \le \lim_{p\downarrow 0} Sf_S/(pE(Z)). \tag{30}$$

One can show that $\lim_{p\downarrow 0}(pE(Z))^{-1} = 1/H_M$, where $H_M = \sum_{i=1}^{M} 1/i$.
In addition, $\lim_{p\downarrow 0} 1/E(Z) = 0$. Consequently,

$$\lim_{p\downarrow 0} E(N) = Sf_S/H_M \quad . \tag{31}$$

Finally, the recurrence relation for f_S (Eq. (18)) requires the value of F_i as $p \downarrow 0$. One can show that $\lim_{p\downarrow 0} F_i = 1/H_M$.

4.3 The Mean Buffer Occupancy with Reservations

In this subsection, we obtain the mean buffer occupancy with reservations, $E(N_i')$. Let W_i'' denote the time that a buffer space is allocated to a packet until it is accepted by the receiver. We derive the distribution of W_i'' along with its mean. Little's result allows us to obtain $E(N_i')$,

$$E(N_i') = (E(W_i'') + E(W_i))/E(Z) \tag{32}$$

where $E(W_i)$ is given in Eq. (13).

Consider a randomly chosen packet P. For convenience, we assume that P begins its transmission in the ith frame of column S. We first determine the circumstance under which the receiver allocates storage to P before it accepts P. This occurs if a packet younger than P is accepted before P. We are interested in the delay between when the ***first*** packet younger than P is accepted and when P is accepted by the receiver. If P is accepted before all younger packets, then $W_i'' = 0$.

Let l be the frame at which P is first transmitted and $l + V_i'$ the frame during which the first packet younger than P is accepted. Let U_i' be the first column in the $l + V_i'$-th frame during which this occurs. The distribution of W_i'' is

$$P(W_i'' = w) = \begin{cases} P(X_i \leq V_i'), & w = 0, \\ \sum_{v=1}^{\infty} \bar{p}_i^{v+k-1} p_i P(V_i' = v, U_i' = u), & w = kS - u, k = 1, 2, \cdots. \end{cases} \tag{33}$$

The number of frames required for the first packet younger than P to be received correctly in a column, consists of one less the residual cycle time for the packet that was under transmission in the i-th frame and the transmission time for the subsequent packet, $Z^{re} - 1 + X_i$. Hence we can write the following expressions for $P(X_i \leq V_i^l)$ and $P(V_i^l = v, U_i^l = u)$,

$$P(X_i \leq V_i^l) = \sum_{x=1}^{\infty} \bar{p}_i^{x-1} p_i P(Z^{re} + X_i > x)^{S-1}, \tag{34}$$

$$P(V_i^l = v, U_i^l = u) = P(Z^{re} + X_i > v+1)^{S-u-1} P(Z^{re} + X_i = v+1) P(Z^{re} + X_i > v)^{u-1}.$$
$$u = 1, \cdots, S-1;\ v = 1, 2, \cdots. \tag{35}$$

The expected value of W_i'' can be computed by substituting equation (33) into

$$E(W_i'') = \sum_{w=1}^{\infty} w P(W_i'' = w).$$

5 NUMERICAL RESULTS

Numerical results of the model are shown in Figures 4 through 9. Figures 4 and 6 illustrate the means of the two measures of buffer occupancy for different system parameters. Figure 5 illustrates the limiting behavior of the mean buffer occupancy, $E(N_i)$. The remaining figures illustrate the behavior of the tail of the distribution for the number of packets in the buffer. In all cases, we consider a system in which the error processes are the same for all receivers, $p_i = p$ and $q_i = q$ for all i, and, except for Figure 5, $S = 20$.

Figure 4 shows the expected buffer occupancy, $E(N_i)$ as a function of the error probability in the forward channel with the number of receivers, M, as a parameter. We can see that, for a fixed value of p, the expected buffer occupancy decreases as M increases. Because the expected length of the fwd mode is determined by p, whereas $E(Z)$ increases with M, a packet that begins its transmission is likely to find fewer packets in the fwd mode on the other columns. Since these are the packets that may hold that new packet in the resequencing buffer, the fewer there are, the shorter is the time the packets must wait before the last of them is successfully transmitted. Figure 4 also illustrates the expected buffer occupancy with reservations $E(N_i^l)$. This measure of buffer occupancy shows the same behavior as $E(N_i)$. Note that ignoring reservations results in an estimate that is typically 5% to 20% too low.

We also observe from Figure 4 that the two measures of expected occupancy decrease, as the probability of correct reception increases in the range $0 < p < 1$. From the model description, one can see that the resequencing buffer is empty for the two extreme values of p. When $p = 1$, all the packets are received without errors, whereas when $p = 0$ none of the packets is received correctly. A special point of operation is when $p \downarrow 0$, where the mean buffer occupancy achieves its largest value. Since for $p = 0$ the occupancy is 0, as mentioned above, this is a point of discontinuity in the graph. Such discontinuity has also been observed in the single-channel case [13].

Figure 5 depicts the values of the expected buffer occupancy at $p \downarrow 0$ as a function of S with M as parameter. As expected, the occupancy increases with S and decreases as M increases. Notice, however, that the *total* expected buffer occupancy in the system, i.e., the expected node buffer occupancy times M, increases with M.

The effect of the feedback channel's error probability is shown in Figure 6 in which $S = 20$ and $M = 4$. In general, the higher the ACK loss probability (lower q), the lower the expected buffer occupancy with or without reservations. This can be explained in a manner similar to the effect of increasing M, since both lower the value of F. Notice, however, that the value of q has the lowest effect at the extreme values of p, where the buffer occupancy is determined mainly by p. Also note that the difference between buffer occupancies with and without reservations decreases as q decreases.

It is interesting to study the tail of the buffer-occupancy distribution, as this provides insight into the buffer size required by the receiver. We conjecture that the probability $P(N > B)$ obtained from our model is an upper bound on the probability that a receiver's buffer of size B will overflow. Figure 7 illustrates the probability that the buffer occupancy exceeds B where $B = 2, 4, 16, 32$ when there are no feedback errors as a function of the forward error probability p for $M = 16$ receivers. We observe that the buffer occupancy rarely exceeds 32 for the range of error probabilities depicted here. Consequently the receiver's buffer size can be chosen to be less than 33. Figure 8 illustrates $P(N > B)$ for the case where the feedback error probability is 0.7. In this case, we observe that the receivers require even less buffer capacity than in the case $q = 1$. For example, if one is willing to tolerate a probability of exceeding B of .05, then $B = 16$ is adequate for this system.

Last, we illustrate $P(N > B)$ for $p = .9$ and $q = 1$, as a function of the number of receivers. We observe that $P(N > B)$ decreases quickly as a function of the number of receivers when that number is small, and decreases slowly for $M > 40$. When the number of receivers is 150, then $B = 9$ is sufficient so that the probability that the buffer occupancy exceeds B is less than .05.

Before we end this section, we briefly consider another interesting case where one of the receivers is a bottleneck, e.g., $p_1 \downarrow 0$ and $p_j > 0$ for all $j \neq i$. In this case, $F_i \to 1$ and $F_j \to 0$, $j \neq i$, $1 \leq j \leq M$. Let us focus first on node i. Substituting of $F_i = 1$ into Eq. (18) and solving the resulting recurrence yields

$$f_S = H_S. \tag{36}$$

This can be substituted into Eq. (19) to yield tight bounds on the average resequencing delay at the ith receiver. When $M = 1$, the bounds correspond to those obtained in [13]. In the case of receiver j, $j \neq i$, one can show that $E(W_j) \to 0$.

6 CONCLUSION

In this paper, we have presented a model for evaluating the resequencing phenomenon in multicast SR ARQ protocol, in terms of a receiver's buffer occupancy and the delay a packet encounters in the resequencing buffer. We have derived the distribution of both occupancy and delay and have obtained a simple formula for the expected delay as the packet loss probability tends to 0. We also introduced a new measure of buffer occupancy, that includes packets that have not been received, but that cause other packets to await resequencing. By means of numerical results, we have shown that the average buffer occupancy increases as the acknowledgment delay or the loss proability of ACKs increases, or as the number of receivers or the packets' loss probabilities decrease. We have also presented curves for probabilities of buffer occupancies above some thresholds. These curves can be useful in designing receivers for proper operation under SR ARQ protocol.

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Figure 3: The test packet's transmissions and waiting with respect to receiver i.

Figure 4: The expected buffer occupancy as a function of p with M as the parameter

Figure 5: The limiting value of the expected occupancy as $p \downarrow 0$

Figure 6: The expected buffer occupancy as a function of p with q as the parameter

Figure 7: The tail of the buffer occupancy distribution as a function of p, $(q = 1)$.

Figure 8: The tail of the buffer occupancy distribution as a function of p, $(q = 0.7)$.

Figure 9: The tail of the buffer occupancy distribution as a function of M.

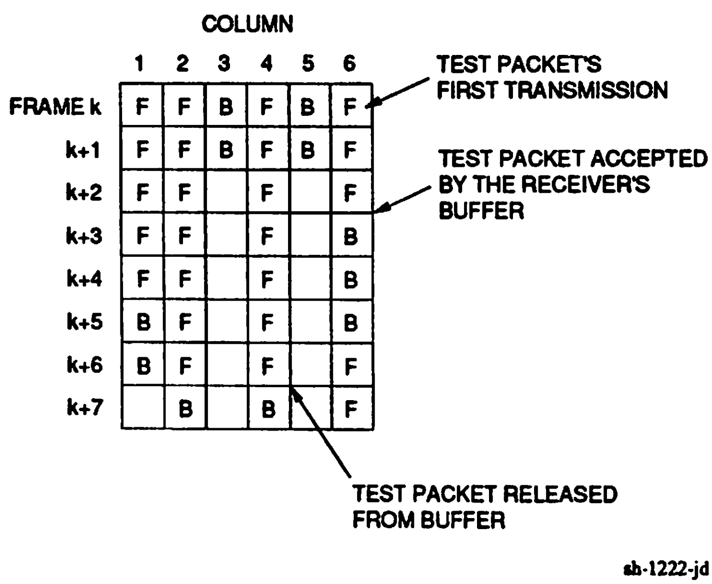

FIGURE 3 TRANSMISSION PROCESS OF THE TEST PACKET

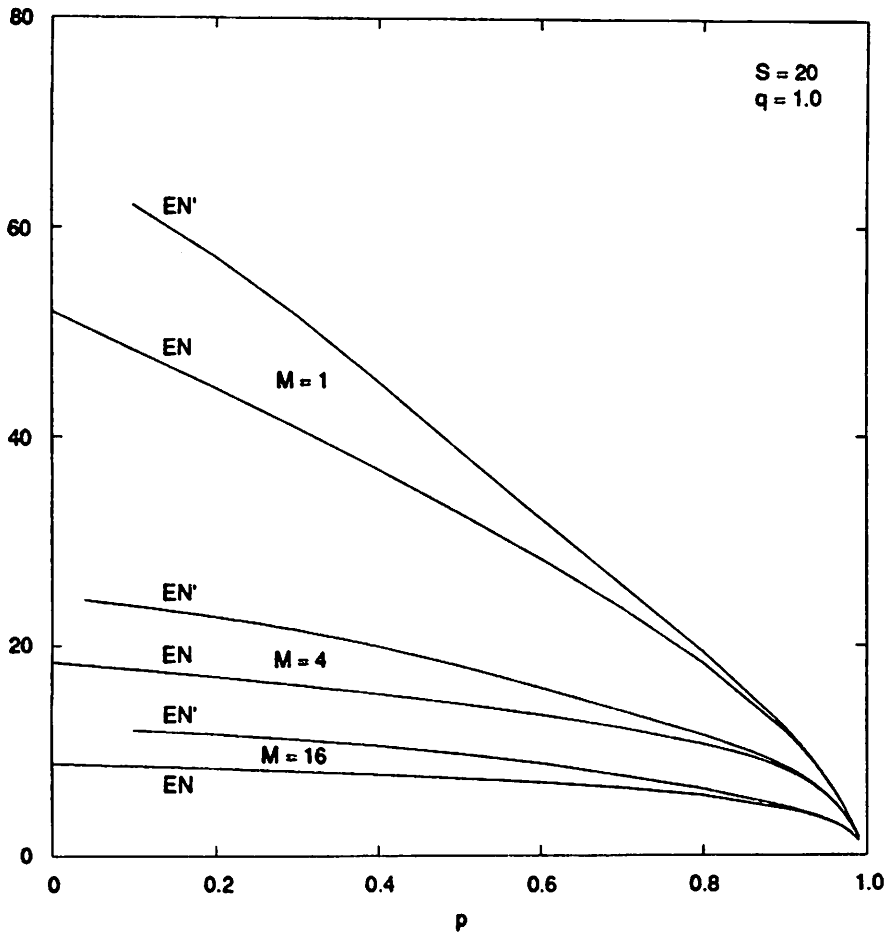

FIGURE 4 EXPECTED BUFFER OCCUPANCY AS A FUNCTION OF *p* WITH *M* AS THE PARAMETER

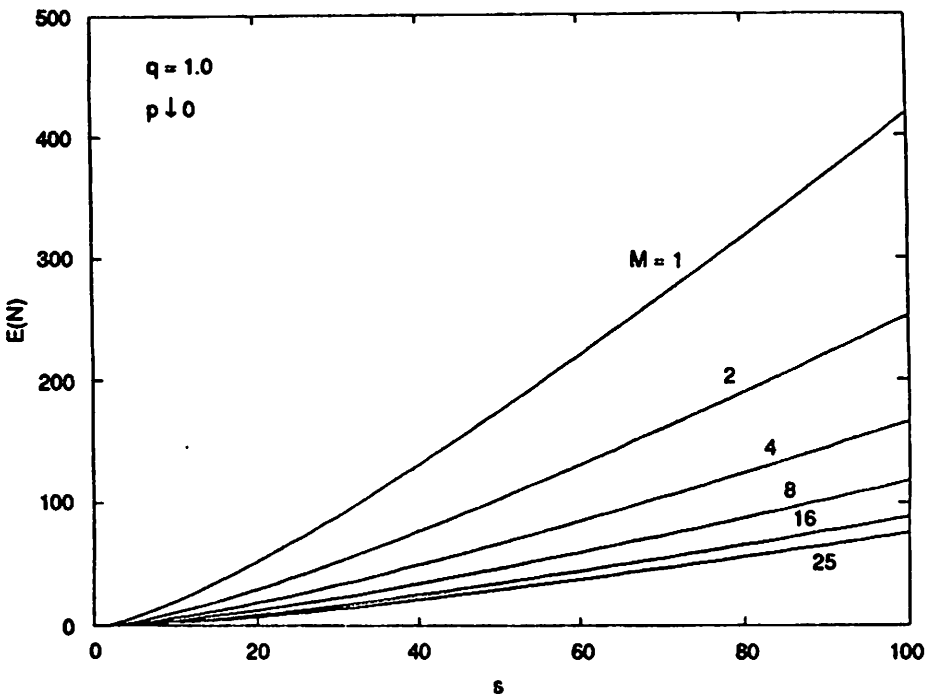

FIGURE 5 THE LIMITING VALUE OF THE EXPECTED OCCUPANCY AS p ↓ 0

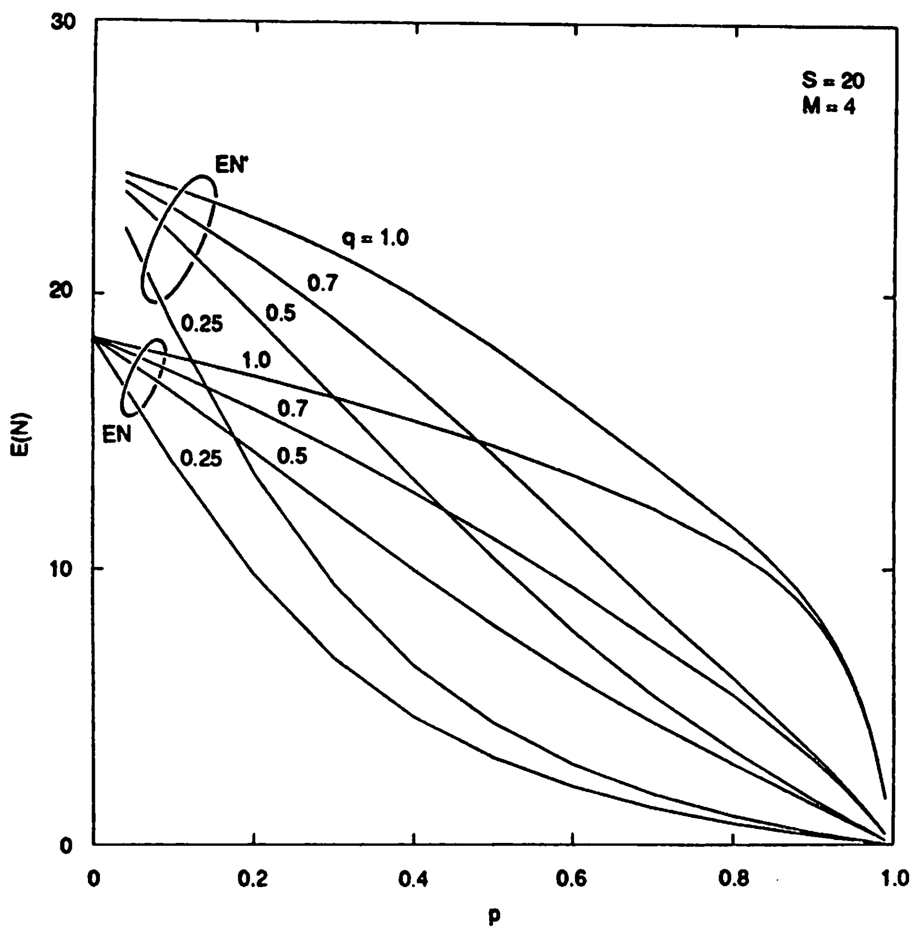

FIGURE 6 EXPECTED BUFFER OCCUPANCY AND BUFFER OCCUPANCY WITH RESERVATIONS

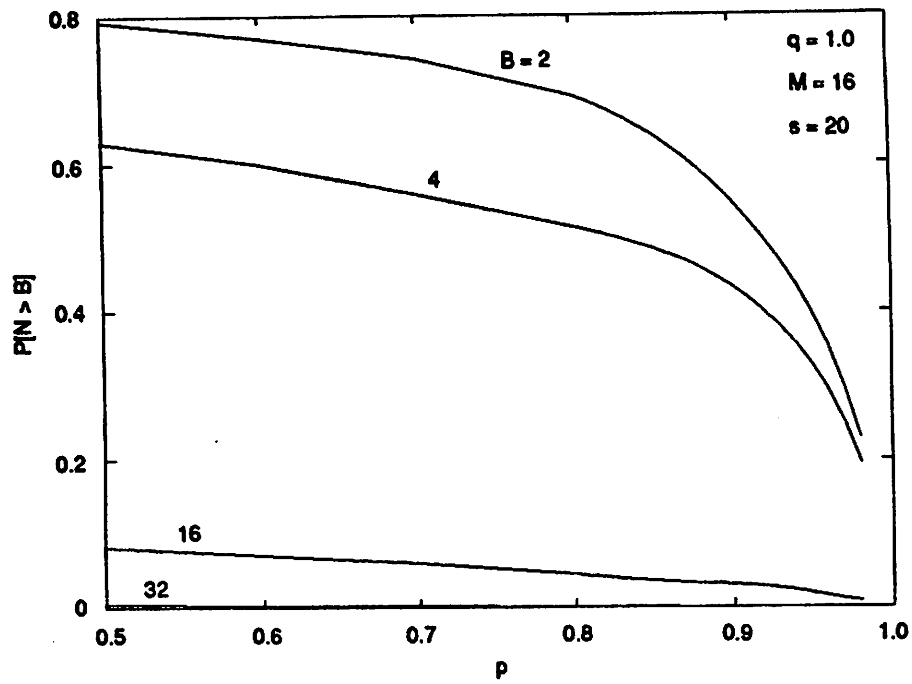

FIGURE 7 P[N > B] AS A FUNCTION OF *p*, q = 1.0

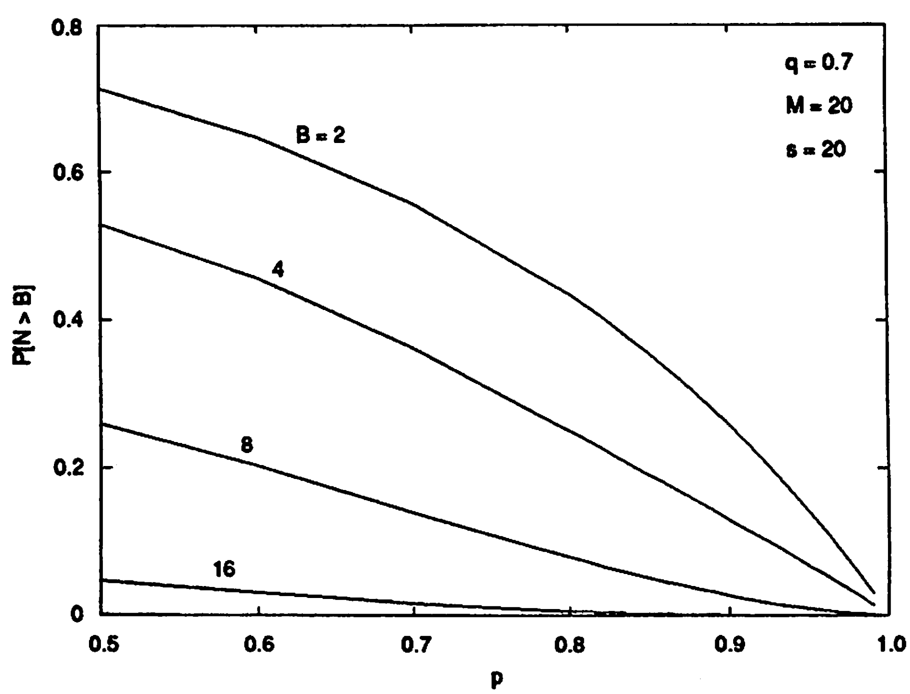

FIGURE 8 P[N > B] AS A FUNCTION OF *p*, q = 0.7

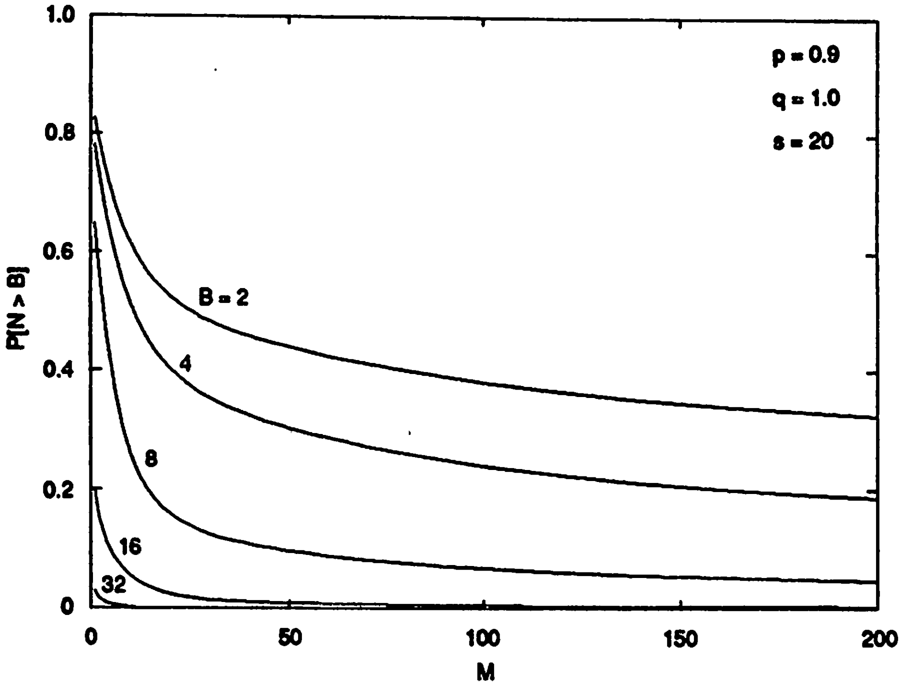

FIGURE 9 P[N > B] AS A FUNCTION OF *M*