CONGESTION CONTROL FOR REAL-TIME TRAFFIC IN HIGH-SPEED NETWORKS

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Congestion Control for Real-Time Traffic in High-Speed Networks[1]

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Abstract

Traditional feedback congestion control is not suitable for high-speed networks due to their high bandwidth-to-delay ratio. In this paper, we investigate the possibility of locally controlling short-term congestion for loss-tolerant, but delay-sensitive traffic such as packet voice through selective discarding of packets based on the virtual work found by a packet on arrival to a queue (local deadline). Five models, $M/D/1/K$, $M/M/1/K$, bounded system time, bounded waiting time and a discrete-time finite-buffer bulk-arrival system, are considered. For the last three, it is shown by analysis and simulation of a multi-stage virtual circuit that this approach can cut voice-tolerable loss rates in half for high loads. Further, we show that the simple case of using the same local deadline throughout the network performs nearly as well as taking reduced interior traffic into consideration and optimizing loss performance over a set of heterogeneous control parameters. By example, the issue of establishing control parameters at call setup time is also considered. Simulation results suggests that queue scheduling based on time remaining to deadline can reduce losses by up to two orders of magnitude.

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Chapter 1

Introduction

1.1 Real-Time Traffic and Packet Loss

The higher bandwidths promised by broadband integrated services digital networks (BISDN) have made applications with real-time constraints, such as control, command, and interactive voice and video communications, feasible.

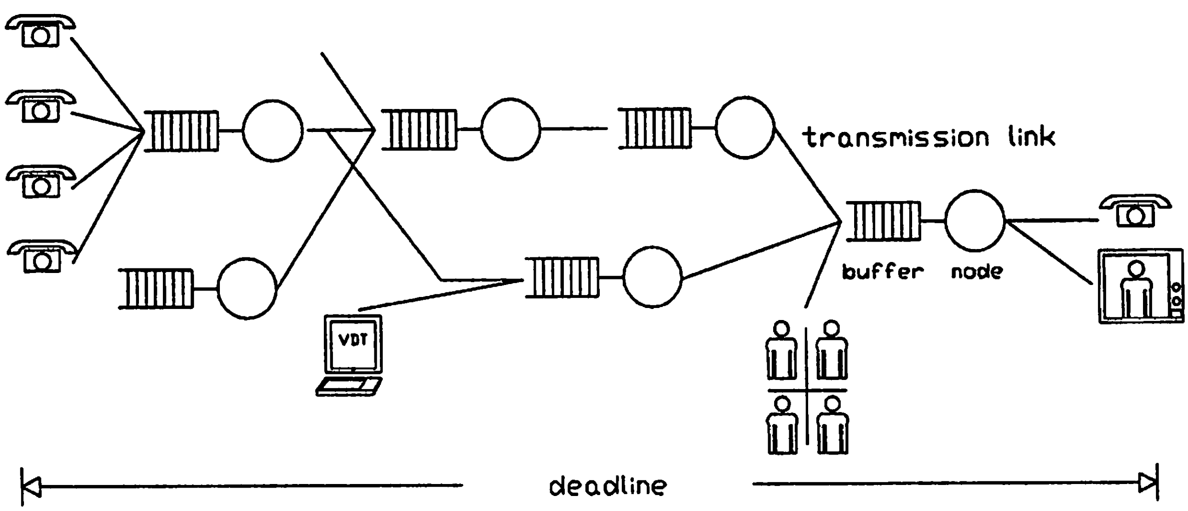

Figure 1.1: Example of high-speed integrated services digital network

In the past, design and performance evaluation have concentrated almost exclusively on averages, emphasizing minimization of mean delays and queue lengths as design goals. For real-time applications, however, performance measures related to the tail of distributions, for example the probability of loss by exceeding a specified end-to-end or global delay, are more meaningful. Figure 1.2 shows that two distributions with the same mean and variance may have distinctly different areas under the tail of the distribution.

Many types of real-time traffic are characterized by "perishable", but redundant messages. In other words, excessive delay renders them useless, but a certain loss can be tolerated without serious

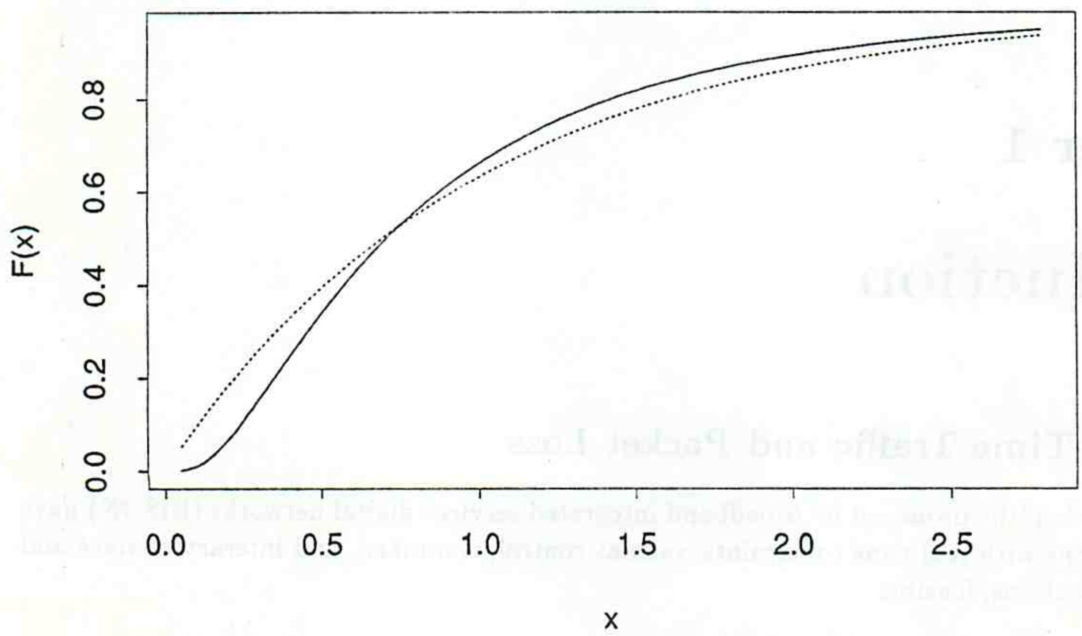

Figure 1.2: Log-normal and exponential distribution with mean 1 and variance 1

degradation in the grade of service [1]. Real-time packets are lost for several reasons [2]:

late loss The packet arrives at the receiver after the end-to-end deadline has expired. For virtual circuits, this implies that the packet suffered excessive waiting times in the intermediate nodes.

error loss The packet content or header may be corrupted by the transmission medium or, usually less likely, by the switching nodes. For headers of 2 to 8 bytes length, the header error rate in ATM networks is estimated at 10^{-9} on average and 10^{-5} at worst [3]. This work ignores error loss as it is not materially affected by control policies; for the systems studied, its contribution to overall packet loss is also rather small.

drop loss Nodes in the network may be dropped intentionally in order to limit congestion[1] or unintentionally because of buffer overflow.

The tolerance for packet loss varies with the type of traffic carried and the measures the receiver is prepared to take to recover or reconstruct lost packets. In general, higher compression ratios and longer packets incur greater sensitivity to losses. For speech coded with PCM and adaptive DPCM, [4] showed that losses from 2 to 5% are tolerable without interpolation, while odd-even interpolation raises the threshold of objectionable loss to between 5% and 10%. For the CCITT 32-KBits/s ADPCM algorithm and a cell size of 32 octets, reconstruction based on pitch estimation allows for loss rates of 3%, while for 64 Kbits/s PCM 8% are acceptable [5]. Another reconstruction scheme

[1]Closed-loop flow control mechanisms such as back pressure or sliding windows are not suited for high-speed networks due to their large delay-bandwidth product.

[2] classifies 8-ms speech packets into background noise, voiced speech, fricatives and "other", with different encoding and reconstruction mechanism for each class. Losses of 47%, 5%, 8% and 4% are not perceptible, respectively, except for voiced speech, where degradation is noticeable even at the loss rate of 5%. Following Coviello [6], many of our examples will use a loss rate of 5% as noticeable, but acceptable.

For high-quality audio, the compact disc data format and error correction and compensation mechanism suggests itself. Here, lost packets would have similar effects as surface scratches.

Compressed video data is far less tolerant of lost packets. A variable-rate DPCM coder operating at around 30 to 34 Mbits/s [7] requires a loss rate of below 10^{-11} without interpolation. However, recent developments in packet video coders reported in [8, 9, 10] and elsewhere have raised the error threshold considerably.

Congestion can be controlled during call setup (*admission control*) or during the data transfer phase of a call[2]. We are concerned here with the latter aspect of the problem.

This report endeavors to investigate congestion control policies at individual nodes with the goal of decreasing end-to-end packet loss. addresses congestion The policies investigated strive to improve end-to-end loss by selectively discarding messages at intermediate nodes, applying the principle of "Think globally, act locally". Messages that stand little chance of making their end-to-end deadline should be discarded as early in the virtual circuit as possible. Discarding, judiciously applied, has two beneficial effects. First, *instantaneous* congestion at the discarding node is relieved. Note that this type of congestion is due to the statistical fluctuations in the arrival process and can occur even in relatively lightly loaded systems. Secondly, downstream nodes are not burdened with traffic that has little chance of meeting its deadline at the receiving end, speeding up service for the rest. Reduced downstream traffic counteracts *long-term* congestion, expressed as high average load.

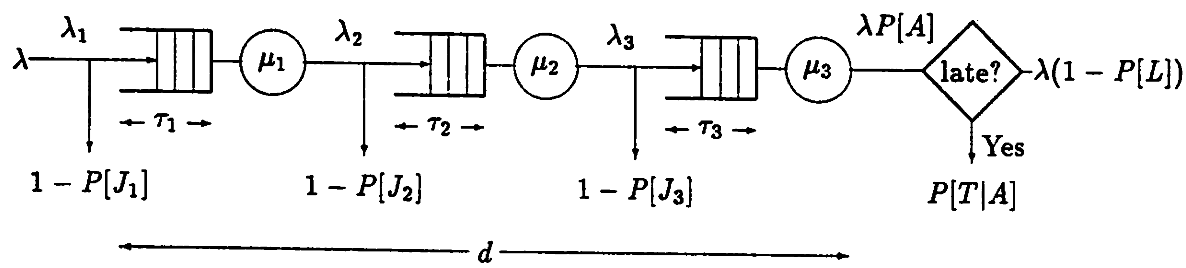

Figure 1.3: Sample virtual circuit

Barring clairvoyance, we have three choices in discarding packets, depending on our degree of caution and amount of available information (see Fig. 1.3):

1. discard a packet if the time spent at the node under consideration exceeds the end-to-end deadline d (also refered to as the time to extinction);

[2] A rarely exercised and analyzed option is the termination of existing connections which might be necessary if available capacity decreases suddenly or high-priority traffic needs to be accomodated.

2. discard a packet whose time spent in the virtual circuit so far, including time in current node, exceeds the end-to-end deadline d
3. discard a packet if the time to be spent in current node i exceeds a fixed *local* deadline τ_i

When we refer to *system time*, we include service, i.e., transmission, time. *Waiting time* is meant to refer to the delay between arrival and start of service.

The first two schemes above are conservative in the sense that no packet will ever be dropped voluntarily at the intermediate nodes that would not have been late at the destination. The third approach risks that packets that would have made the end-to-end deadline are discarded. The parameter space investigated will cover the first and third possibility, as the second requires "travel history" to be carried with each packet.

We note that in a high-speed environment, control measures such as backpressure or choke packets [11, 12] lose effectiveness since the high delay-to-bandwidth ratio makes the feedback information obsolete by the time it reaches the controlling agent. Because of this, local, distributed control mechanisms are preferable to centralized algorithms in WANs as long as they can be used to meet global performance objectives.

1.2 Outline of Report

After outlining the system model underlying this report in Chapter 2 and the general equations governing packet loss in Chapter 3, we investigate the following mechanisms for discarding packets, expressed as queueing models:

M/D/1/K Fixed-length packets arrive with exponentially interarrival time. An arriving packet is discarded if the queue length exceeds a specified local deadline K.

M/M/1/K Service and interarrival times are exponentially distributed. An arrival is admitted if and only if the number of packets in the system is below a specified threshold K.

FIFO-BW Service and interarrival times are again exponentially distributed. However, In this queue, the virtual wait and not the queue length is bounded by the parameter τ.

FIFO-BS Service and interarrival times are exponentially distributed. Here, the system time for an arriving packet is bounded by τ.

$D^{[M]}$/D/1/K Unlike in the continuous-time models above, here time is slotted, with each arrival requiring one time slot of service. Arrivals to the finite queue occur in Poisson-distributed bulks.

Note that in all queue models, admitted customers are served in order of arrival (FIFO). Rejected customers are lost and do not return. For each model, the expressions for the conditional distribution of the system or waiting time available in the literature will be reviewed and extended where necessary. For reasons of analytic tractability, a complete analysis will be restricted to the last three models mentioned above. Through transform methods, expressions for the end-to-end delay distributions will be derived for FIFO-BS and FIFO-BW, evaluated numerically and compared to simulation results. Node rejection probability and the tail of the delay distribution establish the end-to-end loss probability under the various congestion control schemes. A simple scheme of using the same value τ for all local deadlines τ_i in the VC performs will be shown to perform almost as well as the more difficult scheme of selecting optimal τ_i for each node. We attempt to demonstrate that the policy can be implemented in high-speed networks and compares favorably to random discarding [13] as a congestion countermeasure.

Finally, a simulation study in Chapter 9 addresses the combination of deadline-based scheduling and discarding. The policies discussed include:

- minimum-laxity scheduling, where laxity is defined as the time to expiration of the global deadline;
- minimum-laxity scheduling, where laxity is defined as the time to expiration of the global deadline divided by the number of hops left to travel;
- history-based admission, which admits a packet only if the time spent waiting upstream plus the virtual wait fall below a threshold which may depend on the number of nodes left to traverse;
- other service-orderings such as LIFO

If the queues at each node are served in first-come, first-serve order, the queueing delay at the node can be determined on entering the queue since the service time of preceding packets can reasonably be assumed to be known even in case of general service time distribution. If the queue is not served in the order of arrival, it may be more appropriate to queue packets and then preempt them as they age beyond their per-node allowance.

1.3 Delays for Packet Voice and Video

1.3.1 Delay Components

The delay experienced in a packet-switched network consists of a fixed component, known at circuit setup, and a variable, stochastic component that changes with each packet. Note that the fixed component may depend on the path selected and thus change even for the same source-destination pair.

The fixed delay component can be further subdivided, listed below in rough order of (worst-case) magnitude [23, 24, 6].

Propagation delay: The speed of light imposes a minimum delay of 3.3 μs/km in vaccuum and approximately 5 μs/km in optical fiber (refractive index of $\eta = 1.5$). Thus, the longest connection in the continental United States (Fairbanks, Alaska to Miami, Florida) of 6900 km would incur a one-way propagation delay of 35 ms. One hop to a geosynchronous satellite parked at an altitude of 36,000 km, on the other hand, adds 360 ms to the delay budget.

Cell assembly and coding delay: The maximum delay caused by coding and packetization depends on the source coding method chosen. For most schemes proposed, both packet size and coding rate are fixed, resulting in a fixed assembly delay. It is given by the number of user bits per packet divided by the user bit rate plus. For block coders, such as linear predictive coding and vector quantization, packetization and coding are integrated, while for most waveform coders, the quantization and source coding time has to be added. In the common case of 64 kbits/second PCM, source coding (essentially digitization) adds 125 μs of delay. The packetization or cell assembly time naturally depends on the packet length. For the 48-byte packets proposed for ATM, packetization and coding adds 6.1 ms if standard PCM is used, 12 ms for 32 kbit ADPCM. For odd-even sample interpolation [25] and similar schemes spreading sample information over several cells, the cell assembly delays double.

Packet arrival delay: Unless cut-through switching is used, each switching node awaits the arrival of a full packet before commencing re-transmission to the next link, even if no other packets are contending for the output link. This delay is given by the total packet length, including header, divided by the trunk transmission rate. For a 150 Mbps link and 53 byte ATM cells, each switch adds 2.8 μs of delay, for a total of 14 μs for the assumed five-hop connection.

Decoding delay: The delay inserted by the decoder naturally depends on the coding algorithm and the speed of the access line since the hardware will most likely be handing complete packets to the voice decoder. The cell assembly delay may serve as a conservative upper bound.

Processing delays are typically ignored in the analysis [6] due to their small contribution.

Variable delays are mainly due to resource contention, i.e., queueing, occurring in multiplexers [26, 27] and switching nodes. The latter source of delay is the focus of this technical report. For a heavily loaded T1 (1.544 Mbps) voice multiplexer carrying 120 conversations, about 60 buffers, incurring a maximum delay of 20 ms, are needed to limit packet losses to 0.25% [27, Table 5][3] Due to the higher trunk speeds, similar queue lengths at switching nodes incur less delay. Exact figures depend very much on the trunk configuration, the traffic burstiness, priority schemes and so on. For a dynamic time division multiplexing structure, Singh *et al.*[24] report maximum queueing delays of 1 ms, while DePrycker *et al.*[28] estimate it at 40 μs per switch for their switch design. Simulation results in Chapter 8 indicate that constraining queueing delays (in a five-hop network) to twenty transmission time units yields negligible losses.

In summary, packetized real-time traffic experiences additional delay compared to circuit-switching the same traffic. This delay is due to packetization and queueing, with exact numbers depending heavily on the system architecture and data rates. For voice, a delay penalty of about 20 to 30 ms can be expected, which, as discussed below, may force widespread use of echo cancellation.

1.3.2 Delay Bounds

The nature of voice communication imposes two delay constraints on any voice communication system, largely independent of the underlying transport mechanism. The first, lower one, is caused by echo effects, the second by the psychological effects of delay by itself. Echo effects are only noticeable for roundtrip delays[4] exceeding a few tens of milliseconds; shorter echos are perceived as reverberation [29]. Speech reflected once at the far end is referred to as *talker echo* and occurs

[3]The references cited here use 64-octet packets instead of the 53-octet cells assumed previously, but the results should be within 10% of the true values.

[4]In this section, delays are understood to be round-trip unless noted otherwise.

after one round-trip delay, while another reflection at the talker's end back to the listener causes *listener echo.*

Echos in telephone networks are of acoustic and electrical origin. The acoustical coupling (with an attenuation of 30–32 dB for regular handsets [24, p. 0129] and unknown, but lower values for speaker phones) of the handset microphone and speaker has the talker hear his or her voice with one roundtrip delay. However, the major source of echo coupling in today's phone networks is the impedance mismatch in the hybrid (transformer) that connects the two-wire subscriber loop to the four-wire backbone plant. Talker echo path losses (TEPL) as low as 20 dB have been measured [30, p. 989]. Gruber [30] provides a graph showing the fraction of connections judged good or better (% GOB) as a function of TEPL and talker echo path delay. Currently, connections with delays of more than 45 ms are equipped with echo cancellers, resulting in a total TEPL of 50 dB. At that level, the degradation through echo is not noticeable. This predominant source of echo would disappear once all subscriber loops have been converted to ISDN, however, even with only acoustical echo, echo cancellers would be required on connections experiencing a delay of more than about 40 ms. Tolerable connections can be supported without echo cancellation with delays of up to 90 – 100 ms with the (acoustic) echo return loss (ERL) of 30 dB [24, Fig. 3]. While single-chip echo cancellers are now feasible, higher delays generally complicate the design of these adaptive filters as they require longer tap-delay lines with correspondingly higher demands on the processor that updates the filter coefficients. Echo suppressors, as used on satellite channels, are generally considered undesirable since they turn the voice channel into a half-duplex connection.

At much higher delays, and without echo effects, the conversational behavior changes, even though the participants in the conversation may not be aware of the nature of the impairment [31]. Delays of this magnitude affect the "signaling" component of the audio channel, allowing for orderly talker alternation or interruption, particularly important in the absence of visual cues. Delays of 600 and 1200 ms, as encountered in single-hop and double-hop satellite connections, were investigated by Brady [32] and others [33, 29], who found that confusions and involuntary interruptions increased markedly.[5] No studies known to the authors address the effect of pure delays below 600 ms, although Ross *et al.*[34, p. 1294] claim that pure delays below about 200 to 250 ms have no ill effect.

The effect of variable, uncompensated delays, i.e., changes in the spacing between talkspurts, are investigated by Gruber [35]. He finds that for hangovers of less than 100 ms, even small average delays of less than 100 ms were perceptible. The range of values reported, however, is of little use for the variable delays encountered in high-speed networks.

No information on the effect of delay on interactive video connections could be located. The issue was of no apparent concern when video telephones were first introduced on a larger scale around 1970. It is surmised that delays below human reaction time (150 to 200 ms for visual stimuli [36, p. 499]) are not noticed, as long as synchronization between voice and video is maintained.

1.4 Timestamps and Playout

Since speech packets suffer variable delays within the network, a playout buffer at the packet voice receiver has to compensate by adding a controlled variable delay before playing out each packet,

[5] The situation is analogous to the effect of propagation delay on multiple access systems with collision detection, such as CSMA/CD.

assuring that the decoder sees the interpacket intervals generated by the encoder. For all playout mechanisms, a packet is considered lost if it is not present when scheduled for playout. A sequence number within the packet header allows for the detection of missing packets. Note that the sequence numbers need to cover a range that is large enough that it is highly unlikely that a whole range worth of packets is lost. Reusing sequence numbers for each talkspurt seems reasonable due to their separation in time.

The mechanism used to ensure synchronous playout vary depending on the coding scheme and the use of silence suppression (TASI). In the *null timing information* scheme [14, 15, 16] packets do not bear a timestamp. Rather, the receiver adds a fixed delay corresponding to the maximum anticipated queueing delay variation to the first packet in each talkspurt. The remaining packets in a talkspurt are played out at fixed intervals corresponding to the packet generation rate. This scheme is intended for fixed-rate coders with or without silence suppression. With silence suppression, synchronization problems may occur. Since the delay added to the first packet in a talkspurt has no causal connection to the actual delay suffered, silence periods between talkspurts are not reconstructed accurately. In addition, special measures have to be taken to deal with lost first packets.

In the *complete timing information* approach [15], packets are marked with timestamps. The content of the timestamp field leads to tradeoffs between processing, reliability and field size. Since speech packets have to be small for packetization delay and reconstruction reasons, the length of the header becomes a major consideration.

A timestamp field containing a global absolute clock is unsuitable because of the difficulty of synchronizing nodes across the network; also, field lengths would tend to be long. (However, [14, p. 221] analyze just such an arrangement.) A second choice would put the relative distance between packets, measured in units of speech sample periods, into the timestamp field. However, lost packets and the large maximum value of the timestamp, equal to the potentially unbounded silence period measured in speech samples, create problems.

Other approaches to timestamping require that each node inspects and modifies the timestamp. A *delay-stamp* [17] is initialized to zero by the transmitter and incremented by each node by the amount of time the packet has spent in that node, possibly including any propagation delays. Initializing the delay stamp to the end-to-end deadline and decrementing it along the virtual circuit has the advantage that the receiving node immediately obtains the delay to be added. Also, packets with negative time to extinction can be thrown out on inspection. The time stamp in the CCITT draft recommendation G.PVNP for a packetized voice networking protocol [18] accumulates queueing delays at switching nodes with a resolution of one millisecond, with a maximum of 200 ms.

For speech coding without silence suppression and with fixed packet generation intervals, only sequence numbers are necessary, as pointed out in [19, p. 647]. Contrary to the assertion of the authors, we can do without explicit timestamps even when using silence suppression if we restrict silence periods to be a multiple of the packet size. During silence periods, the sequence counter would still be incremented. The final packet in a talkspurt could be tagged to avoid engaging the reconstruction algorithm at the receiver[6]. It remains to be investigated whether transmitting "silence" at the beginning of the first packet in a talkspurt outweighs the header bits saved by not needing timestamps.

[6]The tagging operation incurs an additional, possibly unacceptable, packetization delay, however.

For all timing recovery methods, the maximum delay can be made to vary with the network load [14], either quasi-statically at call setup or dynamically based on delay estimates. Adjustments of the playout delay would have to take place in a talkspurt. The implementation cost and potential rise in the number of lost packets would probably only be justified if the maximum variable delay is indeed a dominant factor in the overall end-to-end delay.

The receiver has to compensate also for slight differences in clock speeds. For example, if the receiver clock was slower than the sender clock, the receiver would fall further and further behind the sender, with eventual buffer overflow. Clock synchronization schemes for this application, combined with the compensation of the stochastic network delay component, are described in [20, 21, 16]. The methods measure the long-term filling level and adjust the output clock frequency correspondingly, implementing a "digital phase locked loop" (see also [22]. If silence suppression is used, it may be sufficient, however, to ensure that the receive (playout) clock is never slower than the send clock. Also, for packet voice with silence suppression, the silence period could be shortened by a small fraction to compensate for any clock speed differential. For video, a whole frame could be duplicated or omitted. This is referred to as "phase slips" by Cochennec *et al.*[21].

As an aside it should be note that the receiver cannot simply replace silence periods by complete silence as this would result in very unnatural changes in the background noise level [16]. Petr *et al.*[19] avoid this problem by transmitting background noise, but as low-priority packets to be discarded on overload.

1.5 Notation

In general, lower case roman function names denote probability density functions, while upper-case functions stand for the corresponding distribution function. Upper-case roman letters also represent events and random variables. This listing contains only symbols that are used beyond a single section. If a symbol is not listed, it may assume different meanings depending on context. The context also determines whether a variable refers to the end-to-end value or a single-node value.

$(\cdot)_n$	Pochhammer's symbol: $(\cdot)_{n+1} = (\cdot)_n(\cdot + n)$
$\cdot^*(s)$	Laplace transform
$\mathcal{L}(\cdot)$	Laplace transform
$\mathcal{L}^{-1}(\cdot)$	inverse Laplace transform
$\cdot_i$	node index
$\cdot(\cdot\|J)$	conditional pdf or cdf for joining packets
$P[\cdot]$	probability of an event
$\Re$	the set of real numbers
$\Re(\cdot)$	the real part of a number
$\Im(\cdot)$	the complex part of a number
a	FIFO-BS: $\equiv b^{1-\rho}e^{\rho(b-1)}$
b	FIFO-BS: $\equiv e^{-\mu\tau}$
b_n	FIFO-BS: service time of nth packet
d	end-to-end deadline
i	node index

K	maximum number of packets in system
M	number of hops, length of VC
$P[A]$	probability that a random packet traverses the whole virtual circuit
$P[D]$	probability that packet is discarded en route (dropped)
$P_i[J]$	probability that a packet joins the ith queue, i.e., is not rejected, assuming it has cleared the previous nodes
$P[J]$	probability that packet joins system (is not rejected)
$P[L]$	probability that packet is discarded or misses end-to-end deadline (lost)
$P[T\|A]$	probability that an arriving packet is late (tardy)
Q	normalization factor
$s(t)$	pdf of system time
$S(t)$	cdf of system time
$u(t)$	unit step: 0 for $t < 0$, 1 otherwise
v_n	amount of virtual work on arrival of nth packet
$w(t)$	pdf of waiting time
$W(t)$	cdf of waiting time
$w^*(s)$	Laplace transform of waiting time
z	$\equiv be^{\mu K}$
α_i	FIFO-BW: normalization constant
$\delta(t)$	Kronecker delta: 1 for $t = 0$, 0 otherwise
λ_i	traffic intensity arriving at queue controller i (before dropping)
λ_{VC}	traffic intensity entering the virtual circuit
μ_i	service rate at node i
$\hat{\Pi}_\nu$	probability of ν packets queued in corresponding infinite-capacity system
$\Pi(z)$	generating function of number of packets in system
ρ	load $\equiv \lambda/\mu$
r	load $\equiv 1 - \rho$
τ	local deadline
τ^*	optimal local deadline
ν_i	traffic indicator $\equiv \mu_i - \lambda_i$

1.6 Abbreviations

BISDN	broadband integrated services digital network
BS	bounded system time
BW	bounded waiting time
cdf, CDF	cumulative distribution function
DSI	digital speech interpolation
FIFO	first-in, first-out (≡ FCFS)
GOS	grade of service
LIFO	first-in, last-out (≡ LCFS)
mgf	moment generating function
mux	(voice) multiplexer
pmf	probability mass function
pdf	probability density function
RT	real time
RV	random variable
SAD	speech activity detection (≡ TASI)
TASI	time-assigned speech interpolation
TO	time out
VC	virtual circuit
vwt	virtual waiting time

Chapter 2

Modeling Assumptions

In general, our assumptions for modeling and simulation should be guided by their justification in high-speed networks suitable for real-time traffic. As mentioned above, real-time traffic is characterized by the limited useful life (perishability) of packets on one hand and a tolerance for some loss (redundancy) on the other. Since the work is motivated by packet voice and video traffic, we assume that losses on the order of one percent are not objectionable (see introduction, Chapter 1).

Although it may be a practical concern, we will gloss over the distribution of the time between packet losses and simply assume that the probability of loss is a sufficient indicator of the quality of service. The approaches we study are also applicable to other real-time traffic, such as sensor data and control commands, assuming that it shares a similar tolerance for loss, similar traffic characteristics and performance metrics. Since packet voice and video will very likely constitute a substantial fraction of the traffic in a high-speed network, with well-known characteristics (at least for voice), our terminology and examples will emphasize this particular type of traffic.

The tight bounds on end-to-end delay rule out error correction by request for retransmission (ARQ). Forward error correction may be used, but is presumably not very effective for mitigating lost (vs. corrupted) packets unless the redundancy bits are spread across packets. Without loss of generality, the channel is assumed to be free of bit errors. For one, channel errors are not of interest in the evaluation of these scheduling protocols. Also, for the media of interest, the packet loss due to channel errors is several orders of magnitude below that considered here.

All real-time (RT) connections are assumed to be virtual circuits, i.e., the number of hops and the end-to-end propagation delay (given by the type of channel and the physical separation of source and destination) are constant and known after the VC has been established. Thus, any such fixed and known delays are subtracted *a priori* from the total allowable end-to-end delays. Henceforth, allowable delay is meant to refer only to the maximum delay inflicted upon a packet due to the statistical multiplexing, i.e., queueing delay. The virtual circuit approach is favored for real time traffic since it allows resource allocation and keeps the receiver from having to reorder packets having traversed the network on different paths.

The length M of the VC will be varied over a range from two to ten. Ten nodes were used by Takahashi *et al.* [3] and by Boyer *et al.* in [23]; between four and ten by [37]. In a simulation study of network delay for packetized voice, Gopal *et al.* assumed circuit lengths of seven, while De Prycker [38] suggested four to five as typical for anticipated ATM networks. The current North American long-distance telephone network routes call through between two and eleven switching nodes (tandems) [39, p. 630]. Following [38], many of the examples presented will have five nodes.

As in [37], both the situation of VCs sharing a node buffer pool and separate buffers for each VC can be considered. To limit the number of model parameters, we will restrict our attention to the case of all VCs sharing a single buffer. The influence of interfering traffic is accounted for by reducing the service rate μ [40, 41, 37, 42]. Fig. 1.3 summarizes the notation used.

Following convention, packets are either of fixed (deterministic) length, as found in ATD-type systems [43], or of exponentially distributed length. Comparison of the $M/D/1$ and $M/M/1$ queue as well as more complicated systems [44] indicate, that the exponential assumption leads to loss results that bound the results for deterministic service from above, with similar general shape of the loss vs. traffic curve.

Without loss of generality, all times are scaled relative to the average service time, i.e., $\mu = 1$. Non-uniform channel capacities or service rates could be accounted for under the penalty of a slightly more cumbersome notation.

Packet arrivals to all nodes are approximated by Poisson processes with arrival rate λ_i. For a voice multiplexer, modeling a superposition of voice sources subject to speech activity detection (SAD), also known as time-assigned speech interpolation (TASI), as a Poisson source leads to overly optimistic estimates [45]. However, for the case of voice trunks considered here, the number of voice sources is possibly two orders of magnitude higher than that found at the T1 rate (1.544 Mbps) multiplexer studied by [26] and others. A 150 Mbit channel can support 4380 64 KBit/sec PCM sources of speech activity factor 0.42 with a utilization of 0.8. Thus, following the arguments of [46], we can conclude that for short time spans, the superposition process is adequately modeled by a Poisson process. The discarding mechanism further limits the interaction period of packets in the queue, improving the approximation

The analysis, but not the simulation, takes the input stream of each interior node i to be a Poisson process with rate equal to the arrival rate $\lambda_i P[J_i]$, which is strictly true only for the $M/M/m$ case and FIFO service discipline (*Burke's theorem*, [47]). As a consequence, when arrivals in an open queueing network are no longer Poisson, they do not see time averages. This raises the question whether job-based estimates of performance parameters are still valid. As an alternative, time-based estimates would not suffer these disadvantages and may also offer lower variance [48, p. 140]. However, since the number of packets dropped at each node is small ($\ll 1\%$), the deviation from the Poisson distribution should be negligible and is henceforth ignored. This assertion is supported by simulation [49, 50].

Some care has to be taken to ensure independence of the individual nodes, with differing requirements depending on whether the loss measurements are based on system or waiting times. A subtle difference between the traditional tandem queue and a store-and-forward communication network should be pointed out. In a tandem queue, each node sees the same sequence of packets, albeit with redrawn service times. It can be shown [51, p. 77] that even in this case, system times of a given job at different nodes are independent. However, the waiting times of a given job are not independent [52].

The independence assumption can be justified on other grounds. In a communication network, virtual circuits between a single source-destination pair will typically occupy only a very small fraction of the channel capacity at each intermediate node. Since the interfering traffic from other virtual circuits can reasonably be assumed to be independent of the VC under study, the waiting times at each node then become independent as a packet from the VC under study is a random observer of the next queue. (This assumes that none of the packets preceding our observer packet at queue i appear at queue $i+1$ before the observer packet.) Since the waiting time depends only on the prior arrivals, whether or not service times are redrawn at each node has no influence on the waiting times and their independence. Thus, for the case of bounded waiting times, the Kleinrock assumption is unneccessary.

In summary, both the system and waiting time of an individual packet are assumed to be independent from node to node, with nodes linked only by the reduction in input traffic rates for the circuit under study afforded by packet dropping. The issue of independence will resurface in the design of simulation experiments, as discussed in the Section 7.4.

For the communication systems of interest, it is reasonable to presume that the virtual work on arrival is proportional to the number of bits in the buffer. Also, the service requirements of each arriving packet are determined by its length in bits and are thus known on entering the queue. (The results for FIFO-BW do not change if the packet leaves the queue if it has not started service within the local deadline; the service time of an individual packet does not enter into consideration. For FIFO-BS, we have to assume that the service time is known on joining the queue.) Therefore, we can implement virtual-work based policies very easily by restricting the buffer size, measured in bits. The traditional $M/M/c/K$ systems are less suitable for modeling typical communication systems with finite buffer since the buffer is restricted in the number of bits it can hold, not the number of customers.

A number of researchers have investigated the effect of packet dropping on a single packet voice multiplexer (e.g., [53, 54], and that of bit dropping on a virtual circuit [55], but the performance of packet dropping in a VC seems to have been considered only in the context of an ARQ-scheme with variable window size [56]. The latter scheme requires acknowledgements, which are not typically used for real-time traffic, and uses feedback, with the concomitant delay problems. Most importantly, packet dropping is performed without regard to the packet's time constraint.

Chapter 3

Packet Loss in Virtual Circuit

This chapter outlines some of the general relations governing the loss of packets in a virtual circuit, independent of the particular queueing model used for the individual nodes. A packet is considered lost if it is either discarded by any of the M nodes it traverses between source and destination or if the time spent in the virtual circuit exceeds a set, fixed deadline d. For FIFO-BS, the time spent will include the service time, that is, time spent equals end-to-end delay or sojourn time; for FIFO-BW, on the other hand, only waiting times will count against the deadline d.

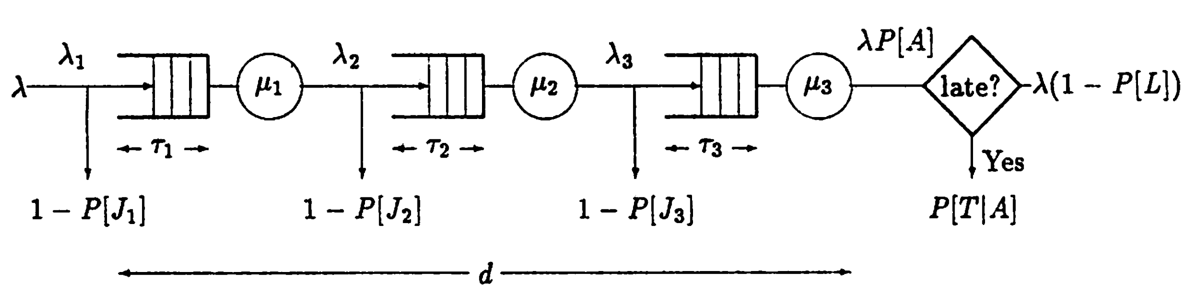

Figure 3.1: Sample virtual circuit

For notation, refer to Fig. 3.1 repeated here for convenience. We define the destination arrival probability $P[A]$ as the probability that a packet is not dropped in any of the M (independent) nodes.

$$P[A] = \prod_{i=1}^{M} P[J_i|J_{i-1}] = \frac{\lambda_M}{\lambda_1}$$

where $P[J_i|J_{i-1}]$ denotes the conditional probability that a packet is admitted to queue i given that is has traversed nodes 1 through $i-1$. (J_0 occurs with certainty.) In other words, $P[A]$ does not include the probability that a packet reaching the destination misses its deadline. According to this definition, $P[A]$ equals one for systems that do not discard packets, such as a tandem $M/M/1$ system. Also, the drop loss, $1 - P[A]$, will be abbreviated as $P[D]$.

In subsequent chapters, we will calculate traffic within the tandem system in two different ways. First, in what will be called *heterogeneous traffic* models, the packets dropped in intermediate nodes is taken into account for λ_i, iteratively computed as $\lambda_i = P[J_{i-1}]\lambda_{i-1}$. For *homogeneous traffic*, the traffic is assumed to be the same throughout the virtual circuit. This assumption may simplify performance computations significantly. If only a small fraction of packets is dropped, the approximation is also rather good, as shown in Chapter 7.

Now suppose that the control policy is applied for all virtual circuits. For simplicity, all nodes are equivalent, receiving an equal fraction of traffic that just has entered the network, has traversed one node, two nodes, and so on. For M nodes and a source intensity of λ/M, each node sees an incoming traffic (before rejection) λ_i of

$$\lambda_i = \frac{\lambda}{M}(1 + P[J] + P[J]^2 + \ldots + P[J]^{M-1}). \tag{3.1}$$

For a given local deadline and an initial value of $\lambda_i = \lambda$, the admittance probability $P[J]$ is computed, yielding a new value for λ_i. This iterative procedure converges to a fix point, the uniform arrival rate at each node in the network. (We will take this viewpoint for the discrete-time analysis in Chapter 8.)

Given that the conditional end-to-end cumulative distribution function of system time S for non-discarded packets is $S(d|A) = P[S < d|A]$, the total loss probability $P[L]$, encompassing both drop loss and late loss, can be written by the law of total probabilities in several different forms:

$$\begin{aligned} P[L] &= P[T] + P[D] \\ &= (1 - S(d|A))P[A] + P[D] \\ &= (1 - S(d|A))P[A] + (1 - P[A]) \\ &= 1 - S(d|A)P[A]. \end{aligned} \tag{3.2}$$

Here, the probability that a packet is late (measured over all packets entering the network) is $P[T]$ (where "T" stands for "tardy"), while the conditional probability $P[T|A] \equiv S(d|A)$ measures only the fraction of packets that are late among those that arrive at the destination. The second and third equations show the contribution of dropped and late packets to the total loss. In the case of voice, $1 - P[L]$ is the fraction of packets that are actually played out at the receiver.

Since each link is assumed to be independent from all others, the end-to-end conditional system time density $s(t|A)$ may be computed as the convolution of the individual system times densities $s_i(t|J_i)$.

$$s(t|A) = s_1(t|J_1) * s_2(t|J_2) * \cdots * s_M(t|J_M)$$

For continuous distributions, we can write, using the convolution property of the Laplace transform:

$$s(t|J) = \mathcal{L}^{-1}\left\{\prod_{i=1}^{M} s_i^*(s|J)\right\} \tag{3.3}$$

where the asterisk superscript and $\mathcal{L}^{-1}$ represent the forward and inverse Laplace transform, respectively. Similarly, for discrete distributions,

$$s(t|J) = \mathcal{F}^{-1}\left\{\prod_{i=1}^{M} s_i^*(s|J)\right\} \tag{3.4}$$

Here, the asterisk superscript and $\mathcal{F}^{-1}$ indicated the forward and inverse discrete Fourier (DFT) transform, respectively. The product is evaluated element-by-element in the complex domain. The DFT is efficiently evaluated using the Fast Fourier Transform (FFT) [57, p. 407]. To avoid wrap-around effect, i.e., computing the circular instead of linear convolution, the two distributions of lengths l_1 and l_2 to be convolved both have to be zero-padded to a length of $l = l_1 + l_2 - 1$. The length of the output sequence is again l. If a standard FFT algorithm is to be used, zero-padding is extended to the next power of two. Evaluation of the product in eq. 3.4 is best done one term at a time unless the random variables are iid. The idea can easily be extended to a combination of discrete and continuous distributions [58, 59].

Equivalent relations hold for the end-to-end waiting time.

Chapter 4

M/M/1/K

4.1 State and Overflow Probabilities

The buffer overflow probability p_K for an $M/G/1/K$ queue can be computed as [60, p. 238]

$$p_K = \frac{\hat{p}_0 + (\rho - 1)\sum_{j=0}^{K-1} \hat{p}_j}{\hat{p}_0 + \rho \sum_{j=0}^{K-1} \hat{p}_j}, \tag{4.1}$$

where ρ denotes the normalized load λ/μ and $\hat{p}_j$ is the probability of finding j customers queued in the corresponding system with infinite waiting room. (Note that for the case of Poisson arrivals, the viewpoint of an outside observer is the same as that of an arriving or departing customer.) For the $M/M/1$ case this probability is known to be

$$\hat{p}_j = (1 - \rho)\rho^j, \tag{4.2}$$

with a corresponding overflow probability for the $M/M/1/K$ system of

$$p_K = \frac{(1-\rho)\rho^K}{1 - \rho^{K+1}}$$

and an acceptance probability

$$1 - p_K = \frac{1 - \rho^K}{1 - \rho^{K+1}}.$$

Here, p_K also follows directly from the flow-equilibrium equations of the birth-death system. The state probabilities p_j are easily shown [60, p. 238][61, p. 104] to be

$$p_j = \frac{(1-\rho)\rho^j}{1 - \rho^{K+1}}$$

From Eq. (4.1), the moments of the number in system can be computed. For the expected number of customers in the system, $E[L] = \sum_{j=0}^{K} j p_j$, we obtain with the expansion

$$\sum_{\nu=0}^{K} \nu \rho = \frac{d}{d\rho} \sum_{\nu=0}^{K} \rho^{\nu+1} - \sum_{\nu=0}^{K} \rho^{\nu}$$

the expression

$$E[L] = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}}.$$

Little's result immediately provides the average system $E[S]$ time as $E[S] = E[L]/\lambda$.

4.2 Distribution of System Time

The distribution of the system time of an admitted customer, $S(y)$, can be derived following the method shown in [61, p. 204]. The system time distribution $S(y|j)$ is conditioned on the number of customers j that an arriving, admitted customer finds in the system, including the one customer currently being served. However, the sequence of admitted customers does not form a Poisson process and thus does not see time averages. The state probability at arrivals, q_k, is related to the state probability seen by a random observer through [62, p. 97] $q_j = p_j/(1-p_K)$ for $j \leq K-1$, which evaluates to

$$q_j = \frac{(1-\rho)\rho^j}{1-\rho^k}$$

is equal to the time-average state probability for an $M/M/1/(K-1)$ system.[1]

As an aside, the previous result is reminiscent of the arrival theorem for closed queueing networks. The connection to the arrival theorem seems less coincidental when one recalls that an $M/M/1/K$ queue has the same state probabilities as the first server in a closed, cyclic queueing network with K customers and two servers with rates μ and λ [64], [65, p. 271]. This relation between open networks with blocking and closed networks can be generalized [65], but is not applicable to networks which reject customers inside the network. For the case of two servers, the Gordon-Newell product form solution is easily written down as

$$p(n_1, K-n_1) = \frac{1}{G}\left(1-\frac{1}{\mu}\right)\left(\frac{1}{\mu}\right)^k\left(1-\frac{1}{\lambda}\right)\left(\frac{1}{\lambda}\right)^{K-k}$$

where the normalization factor G is computed as

$$G = \left(1-\frac{1}{\mu}\right)\left(1-\frac{1}{\lambda}\right)\sum_{k=0}^{K}\left(\frac{1}{\mu}\right)^k\left(\frac{1}{\lambda}\right)^{K-k} = \left(1-\frac{1}{\mu}\right)\left(1-\frac{1}{\lambda}\right)\left(\frac{1}{\lambda}\right)^K\frac{1-\rho^{K+1}}{1-\rho}$$

The $M/M/1/K$ result follows after cancelling terms.

Thus, we can write for the transform of the conditional system time, $s^*(s|j)$,

$$s^*(s|j) \doteq \int_0^\infty e^{-sy}dS(y|j).$$

Let $\hat{b}^*(s)$ be the Laplace transform of the residual service time of the customer found in service on arrival and $b^*(s)$ the Laplace transform of the service times of those waiting and that of the new arrival. Since all RVs are independent,

$$s^*(s|j) = [b^*(s)]\hat{b}^*(s) = \left(\frac{\mu}{s+\mu}\right)^{j+1}$$

[1] A similar result was shown for a finite-buffer system with resume level [63].

The last equality holds since the residual service time in systems with exponential service has the same distribution as the service time itself. The total probability theorem allows us to uncondition $S(s)$ as

$$\begin{aligned}
s^*(s) &= \sum_{j=0}^{K-1} S^*(s|j)q_j \\
&= \sum_{j=0}^{K-1} \left(\frac{M}{s+\mu}\right)^{k+1} \frac{(1-\rho)\rho^k}{1-\rho^K} \\
&= \frac{(1-\rho)\mu}{(1-\rho^K)(s+\mu)} \sum_{j=0}^{K-1} \left(\frac{\lambda}{s+\mu}\right)^k \\
&= \frac{(1-\rho)\mu}{(1-\rho^K)(s+\mu)} \frac{1-\left(\frac{\lambda}{s+\mu}\right)^K}{1-\left(\frac{\lambda}{s+\mu}\right)} \\
&= \frac{1-\rho}{1-\rho^K} \frac{\mu}{(s+\mu)^K} \frac{(s+\mu)^K-\lambda^K}{s+\mu-\lambda}
\end{aligned}$$

The Laplace transform converges for $s > \lambda - \mu$. For $K = 1$, the Erlang loss system, the transform of the system time density simplifies to $\mu/(s+\mu)$, which is simply the transform of the service time density. For the $M/M/1$ system or $K \to \infty$, $s^*(s) = \mu(1-\rho)/(s+\mu-\lambda)$, as required [61, p. 202], since the limit of $\lambda/(s+\mu)$ is zero for values of s in the region of convergence.

4.3 Distribution of Waiting Time

By the same arguments as above, the Laplace transform of the waiting time, $w^*(s)$, is given by

$$\begin{aligned}
w^*(s) &= \sum_{j=0}^{K-1} w^*(s|j)q_j \\
&= \sum_{j=0}^{K-1} \left(\frac{M}{s+\mu}\right)^{k} \frac{(1-\rho)\rho^k}{1-\rho^K} \\
&= \frac{1-\rho}{1-\rho^K} \frac{1}{(s+\mu)^{K-1}} \frac{(s+\mu)^K-\lambda^K}{s+\mu-\lambda}
\end{aligned}$$

The probability distribution (cdf) of the waiting time can now be computed (with the help of the appendix, equations (B.3) and (B.1)) as

$$\begin{aligned}
W(t) &= 1 - Pr\{w > t\} \\
&= 1 - \int_t^\infty \mathcal{L}^{-1} w^*(s)\, dt \\
&= 1 - \int_t^\infty \mathcal{L}^{-1} \left\{ \sum_{j=0}^{K-1} \left(\frac{M}{s+\mu}\right)^k q_j \right\} dt
\end{aligned}$$

$$= 1 - \int_t^\infty \sum_{j=1}^{K-1} \frac{\mu^k t^{k-1} e^{-\mu t}}{(k-1)!} q_k + \delta(t) q_0 \, dt$$

$$= 1 - \sum_{j=1}^{K-1} q_j \sum_{i=0}^{j-1} \frac{(\mu t)^i e^{-\mu t}}{i!} \tag{4.3}$$

which agrees with the expression derived in [62, p. 99] through time-domain means.

4.4 Approximations

For an M-stage system, the Laplace-domain expressions for the end-to-end system and waiting time are difficult to invert explicitly. Boyer *et al.* [23] chose to overcome this difficulty by treating each system time as exponentially distributed, using Kingman's approximation for the tail of $G/G/1$ queues under heavy load, and then using the fact that the sum of M exponentially distributed RVs has an Erlang distribution E_M.

Rather than trying to convolve the system or waiting times, we could find the discrete probability that a customer finds a total of k waiting while traversing the virtual circuit by an M-fold convolution of q_k. The convolution is finite since the total number of customers the test customer has to wait for cannot exceed $M(K-1)$. Thus, efficient means of convolution such as the fast Fourier transform may be used. If all nodes see the same traffic λ, the end-to-end waiting or system time densities could then be computed as for the single-stage case. The cumulative distribution functions would have the same expression as in Eq. (4.3). A similar method will be employed for the discrete-time case.

Another possible approximation is the use of the central limit theorem, which allows us to approximate the pdf of a sum of independent RVs by a normal distribution, regardless of the distribution of the individual RVs if the number of RVs is large. For $M \approx 10$, this approximation may be reasonable.

Chapter 5

M/D/1/K

The derivation of the buffer overflow probability for deterministic packet lengths is more complicated. The generating function of the number of customers in the system is given by the well-known Pollaczek-Khinchin equation

$$\begin{aligned}\Pi(z) &= B^*(\lambda - \lambda z)\frac{(1-\rho)(1-z)}{B^*(\lambda-\lambda z) - z} \\ &= e^{-\lambda\mu}e^{\lambda\mu z}\frac{(1-\rho)(1-z)}{e^{-\lambda\mu}e^{\lambda\mu z} - z}\end{aligned}$$

where $B^*(s)$ is the two-sided Laplace transform of the service time distribution

$$B^*(s) = \int_{-\infty}^{\infty} \delta(t-\mu)e^{-st}dt = e^{-\mu s}$$

The inversion of the expression for $\Pi(z)$ promises to be difficult.

A different approach is outlined in [60, p. 238]: Assume an arbitrary value for Π_0^*, say $\Pi_0^* = 1$, and calculate $\Pi_1^*, \ldots, \Pi_{K-1}^*$ from

$$\Pi_{j+1}^* = e^{\rho}\left(\Pi_j^* - p_j - \sum_{i=1}^{j} p_{j-i+1}\Pi_i^*\right)$$

where, with $H(x)$ as the PDF of the service time,

$$p_j = \int_0^{\infty} \frac{(\lambda x)^j}{j!}e^{-\lambda x}dH(x) = \frac{\rho^j}{j!}e^{-\rho}.$$

Then, set $d = \sum_{i=0}^{K-1}\Pi_i^*$ and calculate

$$\begin{aligned}\Pi_K = q &= \frac{\Pi_0^* + (\rho-1)d}{\Pi_0^* + \rho d} \\ \Pi_j &= \frac{\Pi_j^*}{\Pi_0^* + \rho d}\end{aligned}$$

For a system with deterministic service time $1/\mu$, the residual service time is uniformly distributed. Hence, the PDF of the system time conditioned on the number of customers found on arrival is seen to be

$$\Pr\{S \leq y|k\} = \begin{cases} 0 & \text{if } y < k\mu^{-1} \\ \mu(y - k\mu^{-1}) & \text{if } k\mu^{-1} \leq y \leq (k+1)\mu^{-1} \\ 1 & \text{if } y > k\mu^{-1} \end{cases}$$

Eq. (5.1) provides the unconditioned PDF for the system time.

$$S(y) = \sum_{i=0}^{\lfloor y\mu \rfloor - 1} \Pi_i + \Pi_{\lfloor y\mu \rfloor}(y\mu - \lfloor y\mu \rfloor) \tag{5.1}$$

The corresponding pdf is a piecewise continuous step function, constant over the interval $[m\mu^{-1}, (m+1)\mu^{-1}]$.

As in the $M/M/1/K$ case treated above, we now look at the system time of the tandem link (the virtual circuit) by convolving the individual system times of each node. Again, we assume independence and Poisson output processes. The latter assumption can only be justified if the queue receives packets from a large number of independent sources.

While the Laplace transform of Eq. (5.1) can be easily written down, its multiplication to effect convolution in the original domain becomes quite cumbersome. Thus, it seems advantageous to consider the waiting and service time component of the system time separately. The pdf for the waiting time is of lattice type and has a finite number $(K+1)$ of members. Thus, we can apply the discrete Fourier transform (DFT) on a zero-padded sequence of total length $2K+1$ and then multiply the sequences in the "frequency domain". Usage of the Fast Fourier Transform (FFT) (e.g., [66]) decreases computational complexity from $O(K^2)$ to $O(K \log K)$.

The convolution of the uniformly distributed residual service times is best effected in the Laplace domain. The convolution product

$$\hat{B}^*(s) = \left(\frac{1 - e^{-s/\mu}}{s/\mu}\right)^M$$

can be expanded into $M+1$ terms:

$$\hat{B}^*(s) = \left(\frac{\mu}{s}\right)^M \sum_{k=0}^{M} \binom{M}{k} e^{-ks/\mu},$$

which can be inverted into a sum of shifted powers of y, as shown in Eq. (5.2).

$$B(y) = \mu^M \sum_{k=0}^{M} \frac{(y - k/\mu)^{M-1}}{(M-1)!} u(y - k/\mu) \tag{5.2}$$

As usual, $u(y - y_0)$ denotes the unit step located at y_0.

In order to convolve both parts, the second, continuous distribution needs to be discretized (see [67]). Then, a DFT or FFT can be applied as above, after which the two sequences are multiplied element- by-element. Finally, an inverse FFT restores the pdf of the overall system time.

As in Section 4, approximations by an Erlang or normal pdf can be used. The computation of the second moment of the system time would have to be done numerically.

Chapter 6

M/M/1 with Bounded System Time

Modeling a communication node by a queue with a finite waiting room which can hold at most K packets is usually only an approximation, since in most systems the buffer space is measured in bits rather than packets. If it can be assumed that the service time of a packet depends only on its length and is thus known on entering the system, the model of a queue with bounded system times as treated in [68, Model II] and [69] is more appropriate.

Since the single queue in isolation will form the building block of the virtual circuit analysis, we will commence by analyzing its waiting time distribution in detail, validating and reformulating equations found in the literature.

Let the random variable w_n denote the total amount of work still to be handled by the server at the moment of arrival of the nth packet. This RV is commonly referred to as the virtual waiting time (vwt); its physical representation is the number of bits awaiting transmission. Let the random variable b_n describe the nth customer's service time. Suppose packets are transmitted in the order received (FIFO policy). The nth packet joins the ith queue if and only if

$$w_n + b_n < \tau_i,$$

where τ_n is the node or *local deadline* of the ith node.

6.1 Density Function of Virtual Waiting Time

Before providing the density function, some definitions are in order. The incomplete gamma function $\gamma(a, x)$ is defined as

$$\gamma(a, x) \equiv \int_0^x e^{-u} u^{a-1} \, du.$$

Pochhammer's symbol is denoted as $(x)_n$ and defined as

$$(x)_n = x(x+1)(x+2)\dots(x+n-1)$$

Some of the operator's relevant properties are given below.

$$\begin{aligned}
(x)_0 &= 1 \\
(x)_1 &= x \\
(1)_n &= n! \\
(x)_{n+1} &= (x)_n(x+n)
\end{aligned}$$

Throughout this chapter, we will be using the abbreviations below, with λ and μ refering to arrival and service rate, respectively.

$$\begin{aligned} \rho &\equiv \lambda/\mu \\ b &\equiv e^{-\mu\tau} \\ z &\equiv e^{-\mu(\tau-t)} = be^{\mu t} \\ a &\equiv b^{1-\rho}e^{\rho(b-1)}. \end{aligned}$$

The pdf of the waiting time, $w(t)$, is given in [69, p. 1351] as

$$w(t) = Qh(t). \tag{6.1}$$

where the normalization factor Q is defined as

$$Q = (1-\rho)\{1 - \rho b^{1-\rho}e^{\rho(b-1)} - (\rho b)^{1-\rho}e^{\rho b}[\gamma(\rho,\rho) - \gamma(\rho,\rho b)]\}^{-1}, \rho \neq 1 \tag{6.2}$$

while $h(t)$ has the form

$$\boxed{h(t) = \begin{cases} \lambda b^{1-\rho}(z^{\rho-1} - z^{\rho})e^{\rho b - \rho z}, & \text{for } 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases}}$$

Note that $h(0) = \lambda(1-b)$.

In the limit as $\tau \to \infty$, which is equivalent to $b \to 0$, $w(t)$ should converge to the pdf of the waiting time for a simple $M/M/1$ system:

$$w_{\mathrm{M/M/1}}(t) = (1-\rho)\delta(t) + \lambda(1-\rho)e^{-\mu(1-\rho)t} \tag{6.3}$$

Since $\lim_{b\to 0}\gamma(\rho,\rho b) = 0$,

$$Q_{\mathrm{M/M/1}} = \lim_{b\to 0} Q = 1 - \rho.$$

By expanding z and cancelling the terms containing b, it is easy to show that $h(t)$ indeed converges to $\lambda e^{-\mu x(1-\rho)}$ as required.

The incomplete gamma function $\gamma(x,y)$ can be expanded into an infinite series, so that the bracketed term and its factor, Q', from Eq. (6.2) can be written as

$$\begin{aligned} Q' &= -(\rho b)^{1-\rho}e^{\rho b}[\gamma(\rho,\rho) - \gamma(\rho,\rho b)]\}^{-1} \\ &= -(\rho b)^{1-\rho}e^{\rho b}\left[e^{-\rho}\sum_{n=0}^{\infty}\frac{\rho^{n+\rho}}{(\rho)_{n+1}} - e^{-\rho b}\sum_{n=0}^{\infty}\frac{(\rho b)^{n+\rho}}{(\rho)_{n+1}}\right] \\ &= +(\rho b)^{1-\rho}e^{\rho b}\left[e^{-\rho b}\sum_{n=1}^{\infty}\frac{(\rho b)^{n-1+\rho}}{(\rho)_n} - e^{-\rho}\sum_{n=1}^{\infty}\frac{(\rho)^{n-1+\rho}}{(\rho)_n}\right] \\ &= \frac{(\rho b)^{1-\rho}}{(\rho b)^{1-\rho}}\sum_{n=1}^{\infty}\frac{(\rho b)^n}{(\rho)_n} - \frac{(\rho b)^{1-\rho}}{\rho^{1-\rho}}e^{\rho(b-1)}\sum_{n=1}^{\infty}\frac{\rho^n}{(\rho)_n} \\ &= \sum_{n=1}^{\infty}\frac{(\rho b)^n}{(\rho)_n} - b^{1-\rho}e^{\rho(b-1)}\sum_{n=1}^{\infty}\frac{\rho^n}{(\rho)_n} \\ &= \sum_{n=1}^{\infty}\frac{\rho^n(b^n - a)}{(\rho)_n}. \end{aligned}$$

Using the above, Q can be written in computable form as (see [69, eq. (10)])

$$\boxed{Q = (1-\rho)\left[1 - \rho a + \sum_{n=1}^{\infty} \frac{\rho^n (b^n - a)}{(\rho)_n}\right]^{-1}, \rho \neq 1.} \tag{6.4}$$

6.2 Laplace Transform of the Pdf of the Virtual Waiting Time

Eq. (4.16) in [68] provides the Laplace transform of the pdf of the vwt. It can be immediately simplified with the identity $\Gamma(i+1) = i!$, yielding in our notation[1]:

$$w^*(s) = w^*(-1) b^{1+s} \left\{ 1 + (1+s) \sum_{i=0}^{\infty} \rho^i \frac{\Gamma(\rho - s - 1)}{\Gamma(\rho - s + i)} \right\} - Q(1+s) \sum_{i=0}^{\infty} (\rho b)^i \frac{\Gamma(\rho - s - 1)}{\Gamma(\rho - s + i)} \tag{6.5}$$

Q is the factor defined in Eq. (6.2).

The Γ-function has poles at 0, -1, -2, ... Since $w^*(s)$ has to be an entire function, i.e., holomorphic without singularities, Q can be determined by letting $s = \rho - 1$ and forcing the two infinite sums containing the offending $\Gamma(0)$ to cancel each other:

$$\rho w^*(-1) b^{\rho} \sum_{i=0}^{\infty} \rho^i \frac{\Gamma(0)}{\Gamma(i+1)} = \rho Q \sum_{i=0}^{\infty} (\rho b)^i \frac{\Gamma(0)}{\Gamma(i+1)}$$

Using the well-known series expansion of the exponential function, we can simplify this to

$$b^{\rho} w^*(-1) \rho \Gamma(0) e^{\rho} = \rho Q \Gamma(0) e^{\rho b},$$

from which

$$Q = e^{\rho(1-\tau) - \rho b} w^*(-1) \tag{6.6}$$

follows immediately.

Next, the constant $w^*(-1)$ is to be computed. The norming condition requires that $w^*(0) = 1$. (This can be easily verified by recalling the integral property

$$w^*(0) = \int_{0^-}^{\infty} w(t) dx$$

of the Laplace transform.) From

$$w^*(0) = 1 = w^*(-1) e^{-\tau} \left\{ 1 + \sum_{i=0}^{\infty} \rho^i \frac{\Gamma(\rho - 1)}{\Gamma(\rho + i)} \right\} - e^{\rho(1-\tau) - \rho b} w^*(-1) \sum_{i=0}^{\infty} (\rho b)^i \frac{\Gamma(\rho - 1)}{\Gamma(\rho + i)}$$

eq. (4.18) in Cohen's paper,

$$w^*(-1) = \left[\left\{ 1 + \sum_{i=0}^{\infty} \rho^i \frac{\Gamma(\rho - 1)}{\Gamma(\rho + i)} \right\} b - e^{\rho(1-\tau) - \rho b} \sum_{i=0}^{\infty} (\rho b)^i \frac{\Gamma(\rho - 1)}{\Gamma(\rho + i)} \right]^{-1},$$

[1] Asterisks will be used to denote the transform operation

follows immediately.

The last equation for $w^*(-1)$ can be simplified considerably by making use of the two identities

$$\begin{aligned}\Gamma(\rho+i) &= \Gamma(\rho)(\rho)_i \\ \Gamma(\rho-1) &= \frac{\Gamma(\rho)}{\rho-1},\end{aligned}$$

leading to (for any constant a)

$$\sum_{i=0}^{\infty} a^i \frac{\Gamma(\rho-1)}{\Gamma(\rho+i)} = \sum_{i=0}^{\infty} a^i \frac{\Gamma(\rho)}{(\rho-1)(\rho)_i\Gamma(\rho)} = \sum_{i=0}^{\infty} \frac{a^i}{(\rho-1)_{i+1}} = \left\{1+\sum_{i=1}^{\infty}\frac{a^i}{(\rho)_i}\right\}\frac{1}{\rho-1} \tag{6.7}$$

Note that the terms of the summation can be generated recursively. This simplification is believed to be new.

To check our computations, we compare the expression for Q shown in Eq. (6.6) with that derived earlier, Eq. (6.4). With Eq. (6.7), we can write

$$Q = b^\rho e^{\rho(1-b)}(\rho-1)\left[\left\{\rho+\sum_{i=1}^{\infty}\frac{\rho^i}{(\rho)_i}\right\}b - b^\rho e^{\rho(1-b)}\sum_{i=0}^{\infty}\frac{(\rho b)^i}{(\rho)_i},\right]^{-1} \tag{6.8}$$

which indeed begins to look very similar to the expression found earlier. By noting that $b^\rho e^{\rho(1-b)} = b/a$ we can rewrite Eq. (6.8) as

$$Q = (\rho-1)b/a\left[\left\{\rho-1+\sum_{i=0}^{\infty}\frac{\rho^i}{(\rho)_i}\right\}b - \frac{b}{a}\sum_{i=0}^{\infty}\frac{(\rho b)^i}{(\rho)_i}\right]^{-1}$$

which simplifies to

$$Q = (\rho-1)\left[a(\rho-1)+a\sum_{i=0}^{\infty}\frac{\rho^i}{(\rho)_i} - \sum_{i=0}^{\infty}\frac{(\rho b)^i}{(\rho)_i}\right]^{-1}. \tag{6.9}$$

The last expression is easily seen to be equivalent to the one given by Gavish and Schweitzer, Eq. (6.4).

After this digression, let us return to the computation of $w^*(s)$. Replacing the Γ-function is even more important in the expression for $w^*(s)$ since here the Γ-function takes complex arguments. In addition, numerical inversion requires repeated evaluations of the expression. Using Eq. (6.7), we can write

$$w^*(s) = w^*(-1)e^{-(1+s)\tau}\left\{1+\frac{1+s}{\rho-s-1}\sum_{i=0}^{\infty}\frac{\rho^i}{(\rho-s)_i}\right\} + Q\frac{1+s}{\rho-s-1}\sum_{i=0}^{\infty}\frac{(\rho b)^i}{(\rho-s)_i},$$

which can also be written without using $w^*(-1)$ as

$$\boxed{w^*(s) = Qab^s\left\{1+\frac{1+s}{\rho-s-1}\sum_{i=0}^{\infty}\frac{\rho^i}{(\rho-s)_i}\right\} + Q\frac{1+s}{\rho-s-1}\sum_{i=0}^{\infty}\frac{(\rho b)^i}{(\rho-s)_i}.} \tag{6.10}$$

This more computable form is believed to be novel.

6.3 Cumulative Distribution Function of Virtual Waiting Time

The cumulative distribution function (cdf) of the virtual waiting time is necessary to determine the probability that a packet is excessively delayed. Through two substitutions, a closed-form expression for this cdf can be found, extending the results given in the references. Taking into account the finite probability of an empty system, write

$$w'(t) = Qh(t) + Q\delta(t) = Q\lambda b^{1-\rho}e^{\rho b}[g_1(t) - g_2(t)] + Q\delta(t),$$

where

$$g_1(t) = \exp(-\mu(\tau - t)(\rho - 1) - \rho b e^t)$$

and

$$g_2(t) = \exp(-\mu(\tau - t)\rho - \rho b e^t).$$

Now, taking note of the probability mass at the origin, we have for $0 \le t \le \tau$,

$$\begin{aligned} W(t) &= \int_0^t w(y)dy + Q \\ &= Q\lambda b^{1-\rho}e^{\rho b}\left[\int_0^t g_1(y)dy - \int_0^t g_2(y)dy\right] + Q \\ &= Q\lambda b^{1-\rho}e^{\rho b}\left[W_1(t) - W_2(t)\right] + Q. \end{aligned}$$

Substituting $z = \tau - y$, with $dz/dy = -1$,

$$W_1(t) = \int_{\tau-t}^{\tau} g_1(z)dz = \frac{1}{\lambda}\int_{\tau-t}^{\tau} \exp\left[-\mu z(\rho - 1) - \rho e^{-\mu z} + \mu z\right]\lambda e^{-\mu z}dz$$

Now we can substitute $u = \rho e^{-\mu z}$, with $du/dz = -\lambda e^{-\mu z}$ and the limits $l_1 = \rho b e^{\mu x}$, $l_2 = \rho b$. A few steps separate us from a closed-form solution.

$$\begin{aligned} W_1(t) &= \frac{1}{\lambda}\rho^{2-\rho}\int_{\tau}^{\tau-t}\left[(\rho e^{-\mu z})^{\rho-1}(\rho e^{-\mu z})^{-1}e^{-\rho e^{-\mu z}}\right]\lambda e^{-\mu z}dz \\ &= \frac{\rho^{1-\rho}}{\mu}\int_{l_2}^{l_1} u^{\rho-1}u^{-1}e^{-u}du \\ &= \frac{\rho^{1-\rho}}{\mu}\int_{l_2}^{l_1} u^{\rho-2}e^{-u}du \\ &= \frac{\rho^{1-\rho}}{\mu}\left[\int_0^{l_1} u^{\rho-2}e^{-u}du - \int_0^{l_2} u^{\rho-2}e^{-u}du\right] \end{aligned}$$

Since the incomplete gamma function[2] is defined as

$$\gamma(a, x) \equiv \int_0^x e^{-u}u^{a-1}du,$$

[2]A good survey of its important properties and relationships is contained in [70, Chapter 45].

we can write

$$W_1(t) = \frac{\rho^{1-\rho}}{\mu}\left[\gamma(\rho-1,l_1) - \gamma(\rho-1,l_2)\right]. \tag{6.11}$$

Similarly, for

$$W_2(t) = \int_{\tau-t}^{x} g_2(z)dz,$$

with

$$g_2(z) = \exp(-\lambda z - \rho e^{-\mu z}),$$

we can write

$$W_2(t) = \frac{1}{\lambda}\int_{\tau-t}^{x} \exp\left[-\lambda z - \rho e^{-\mu z} + \mu z\right] \lambda e^{-\mu z} dz.$$

Using the same substitutions as before, we obtain

$$\begin{aligned} W_2(t) &= \frac{1}{\lambda}\frac{\rho}{\rho^{\rho}}\int_{\tau-t}^{\tau}\left[(\rho e^{\mu z})^{\rho}(\rho e^{-\mu z})^{-1}e^{-\rho e^{-\mu z}}\right]\lambda e^{-\mu z}dz \\ &= \frac{1}{\mu\rho^{\rho}}\int_{l_2}^{l_1} u^{\rho}u^{-1}e^{-u}du \\ &= \frac{1}{\mu\rho^{\rho}}\int_{l_2}^{l_1} u^{\rho-1}e^{-u}du \\ &= \frac{1}{\mu\rho^{\rho}}\left[\gamma(\rho,l_1) - \gamma(\rho,l_2)\right]. \end{aligned} \tag{6.12}$$

Finally, Eq. (6.11) and Eq. (6.12) can be combined into

$$\boxed{W(t) = Qb^{1-\rho}e^{\rho b}\rho^{1-\rho}\left\{\rho[\gamma(\rho-1,l_1) - \gamma(\rho-1,l_2)] - \gamma(\rho,l_1) + \gamma(\rho,l_2)\right\} + Q} \tag{6.13}$$

As in the case of the pdf, it is instructive to take the limit as τ approaches infinity (or b approaches zero). We remember that $lim_{b\to 0}Q = 1-\rho$. Using l'Hospital's rule, we obtain the partial results

$$\lim_{b\to 0}\frac{\gamma(\rho-1,l_1)}{b^{\rho-1}} = \frac{\rho^{\rho-1}}{\rho-1}e^{x(\rho-1)}$$

and

$$\lim_{b\to 0}\frac{\gamma(\rho-1,l_2)}{b^{\rho-1}} = \frac{\rho^{\rho-1}}{\rho-1},$$

which when combined with the other factors yield the expression for the distribution of the waiting time in an $M/M/1$ queue:

$$W_{\mathrm{M/M/1}}(t) = 1 - \rho e^{-\mu(1-\rho)t}$$

In applying l'Hospital's rule, we made use of the fact [70, p. 441] that $d\gamma(\nu,t)/dt = t^{\nu-1}e^{-t}$.

As in section 6.1, the difference of the two incomplete gamma functions can be expressed as an infinite sum, which, truncated appropriately, may serve to evaluate $W(t)$:

$$\gamma(\nu,l_1) - \gamma(\nu,l_2) = c^{\nu-1}\sum_{j=1}^{\infty}\frac{\exp(\mu x(\nu-1) - ce^{\mu x} - \mu xj)c^j - \exp(-c)c^j}{(\nu)_j},$$

where $c = \rho b$ and $\nu = \rho - 1$ for the first γ-difference in Eq. (6.13) and $\nu = \rho$ for the second difference.

6.4 Probability of Admission

The equilibrium probability $P[J]$ that a random arrival joins the system is computed in [69, eq. 12] as

$$\boxed{P[J] = 1 - Qb^{1-\rho}e^{\rho(b-1)}.} \tag{6.14}$$

6.5 Density Function of System Time of Admitted Customers

The system time t of customers admitted to the system is the sum of their virtual waiting time and their own service time. By assumption, the service time is distributed exponentially as $b(t) = \mu e^{-\mu x}$. Eq. (20) in [69] provides the joint probability density function for the customer's waiting time x and service time y conditioned on the customer joining the system:

$$f(x, y|J) = \begin{cases} b(y)[Q\delta(x) + w(x)]/P[J] & \text{if } x + y \leq \tau \\ 0 & \text{otherwise} \end{cases} \tag{6.15}$$

The pdf of the sum of x and y is given by the integral

$$s(x|J) = \int_0^x f(x-y, y|J)dy.$$

Since the densities for both waiting and service time are zero left of the origin, we could replace the infinite limits on the integrals by finite ones [71, p. 135]. Making use of the definition of the Dirac δ-function, the integral simplifies to

$$s(x|J) = \frac{1}{P[J]}\left(Qb(x) + \int_0^x b(y)w(x-y)dy\right). \tag{6.16}$$

The approach shown in section 6.3 can be used here as well to derive a closed-form expression for $s(x|J)$. For brevity, only the integral from Eq. (6.16) will be evaluated.

$$\begin{aligned} &\int_0^x b(y)w(x-y)dy \\ &= Qb^{1-\rho}\lambda \int_0^x \mu e^{-\mu y}(z^{\rho-1} - z^{\rho})e^{\rho b-\rho z}dy \\ &= Qb^{1-\rho}\lambda e^{\rho b}\left[e^{x(\lambda-\mu)}b^{\rho-1}\int_0^x \mu\exp(-\mu y\rho - \rho b e^{\mu x}e^{-\mu y})dy \right. \\ &\quad \left. -e^{\lambda x}b^{\rho}\int_0^x \mu\exp(-\mu y(\rho+1) - \rho b e^{\mu x}e^{\mu y}dy\right] \end{aligned}$$

With the substitution $u = \rho b e^{\mu x}e^{-\mu y}$, we have $du = -\mu\rho b e^{\mu x}e^{-\mu y}dy$. The lower limit becomes $l_1 = \rho b e^{\mu x}$, the upper limit $l_2 = \rho b$. With this,

$$\begin{aligned} &\int_0^x b(y)w(x-y)dy \\ &= Qb^{1-\rho}\lambda e^{\rho b}\left[-\frac{e^{x(\lambda-\mu)}}{(\rho b)^{\rho}e^{\lambda x}}\int_{l_1}^{l_2} u^{\rho-1}e^{-u}du + \frac{be^{\lambda x}}{\rho b^{\rho+1}e^{x(\lambda+\mu)}}\int_{l_1}^{l_2} u^{\rho}e^{-u}du\right] \end{aligned}$$

$$= \frac{Qb^{1-\rho}\lambda e^{\rho b}}{(\rho b)^\rho e^{\lambda x}} \left\{ -e^{-\mu x} \left[\gamma(\rho, l_2) - \gamma(\rho, l_1)\right] + \frac{b}{\rho b e^{\mu x}} \left[\gamma(\rho+1, l_2) - \gamma(\rho+1, l_1)\right] \right\}$$

$$= \frac{Q\lambda e^{\rho b} e^{-\mu x}}{(\rho b)^\rho} \left\{ \gamma(\rho, l_1) - \gamma(\rho, l_2) + \frac{e^{-\mu x}}{\rho} \left[\gamma(\rho+1, l_2) - \gamma(\rho+1, l_2)\right] \right\}$$

In the Laplace domain, convolution becomes simple multiplication:

$$s^*(s) = \frac{1}{P[J]} \frac{\mu}{s+\mu} w^*(s) \tag{6.17}$$

6.6 System Time CDF for Admitted Customers

The cumulative distribution function $S(x|J)$ of the system time for admitted customers could naively be computed by numerically integrating the expression for $s(x|J)$, Eq. (6.16), found in the preceding section. However, the double numerical integration is both slow and may lack accuracy for the tail of the distribution. It seems advantageous to make use of the explicit, closed-form expression for the cumulative distribution function for the virtual waiting time $W(x)$ that we derived earlier in section 6.3. The results in this section extend those by Cohen and Gavish *et al.*.

The distribution of a sum Z of two random variables X and Y is given by [71, eq. (6-37)]

$$F_Z(z) = \int_{-\infty}^{z-y} \int_{-\infty}^{\infty} f(x,y)\, dx dy.$$

Plugging in Eq. (6.15) and remembering that the densities vanish left of the origin yields

$$S(x|J) = \left[\int_0^x b(y) \int_0^{x-y} w(x)\, dx dy + Q \int_0^x b(y)\, dy\right] \frac{1}{P[J]},$$

which becomes

$$S(x|J) = \left[\int_0^x b(y)(W(x-y) - W(0))dy + Q(1 - e^{-\mu x})\right] \frac{1}{P[J]}, \tag{6.18}$$

where $W(0) = Q$.

It is possible to find a closed-form expression for the integral in Eq. (6.18) by using [72, eq. 1.2.1(2)]:

$$I(x,\alpha,\beta) = \int_0^x \gamma(\alpha, \beta y) dy = x\gamma(\alpha, \beta x) - \frac{1}{\beta}\gamma(\alpha+1, \beta x) \tag{6.19}$$

With the substitution $z = e^{-\mu y}$, where $dz/dy = -\mu e^{-\mu y}$, we can define a new function

$$K(x,\alpha,\beta) = \int_0^x \mu e^{-\mu y} \gamma(\alpha, \beta e^{-\mu y}) dy = I(1,\alpha,\beta) - I(e^{-\mu x}, \alpha, \beta).$$

In our application, $\alpha = \rho$ or $\alpha = \rho - 1$, and $\beta = \rho b e^{\mu x}$. Noting that only $l_1 = \rho b e^{\mu(x-y)}$ depends on the variable of integration, we can expand the integral from Eq. (6.18) into

$$\int_0^x b(y)[W(x-y) - Q]dy =$$

$$Qb^{1-\rho} e^{\rho b} \rho^{1-\rho} \left[\rho \int_0^x \mu e^{-\mu y} \gamma(\rho-1, l_1) dy - \rho\gamma(\rho-1, l_2) \int_0^x \mu e^{-\mu y} dy \right.$$

$$\left. - \int_0^x \mu e^{-\mu y} \gamma(\rho, l_1) dy - \gamma(\rho, l_2) \int_0^x \mu e^{-\mu y} dy \right],$$

giving immediately the desired closed-form expression for $S(x|J)$

$$S(x|J) = \frac{Qb^{1-\rho}e^{\rho b}\rho^{1-\rho}}{P[J]}\left[\rho K(x,\rho-1,\rho b e^{\mu x}) - \rho\gamma(\rho-1,\rho b)(1-e^{-\mu x}) - K(x,\rho,\rho b e^{\mu x}) + \gamma(\rho,\rho b)(1-e^{-\mu x})\right]. \tag{6.20}$$

6.7 Maximum On-Time Throughput

If customers traversing a queueing system are subject to deadlines, throughput, that is, the number of customer arrivals per time unit, is no longer a suitable figure of merit. Rather, "goodput", i.e., the rate of customers arriving *on time*, is of interest. To allow for some numerical results, we will limit ourselves to a single queueing system rather than a tandem link.

6.7.1 $M/M/1$ without Bounds on Waiting Time

Here, the goodput is simply the product of the CDF at deadline and the arrival rate λ:

$$T(\tau|J)\lambda = (1 - e^{-\mu(1-\frac{\lambda}{\mu})\tau})\lambda$$

Although not directly related to the systems and policies considered in the balance of this report, it is of some interest to determine the offered load that maximizes the goodput. It is clear from intuition, that beyond the certain load added some traffic increase the queue length such that an even larger fraction of the packets does not make the deadline, effectively decreasing the goodput.

To determine λ_{opt}, the derivative w.r.t λ

$$\frac{d}{d\lambda}T(\tau|J)\lambda = 1 - (1+\lambda\tau)e^{(\lambda-\mu)\tau}$$

is set to zero. No closed-form solution exists; however, the root can easily be determined numerically. For $\mu = 1$, $\tau = 23$, $\lambda_{\text{opt}} = 0.87$, with $T_{\text{opt}}(\tau) = 0.9522$ and a goodput of 0.8284. (The value of τ was chosen to yield a success rate of 0.99 at $\rho = 0.8$.)

6.7.2 $M/M/1$ with Bounded System Time

In the case of waiting times bounded by the constant τ, it is of interest to find the optimal τ, i.e., that value of τ that maximizes the goodput for a given λ. The probability that a customer passes through the system in time is equal to the product of its admittance probability $P[J]$ and the probability that its system time (conditioned on admittance) is less than τ, which we labeled as $T(\tau|J)$. Since λ is held constant, $P[J]T(\tau|J)$ is to be maximized with respect to τ.

Differentiating the closed-form expression Eq. (6.20) for $S(x|J)$ w.r.t. τ^3 is rather tedious, Therefore, the IMSL function ZXGSP is used for optimization. ZXGSP works with a golden-section search method and does not require the evaluation of derivatives.

As may be suspected in this single-queue case, the maximum goodput is achieved when only packets that have no chance of making the deadline, that is, those which on arrival find that their service time plus their waiting time exceeds d, are discarded ($\tau = d$). Discarding additional packets

[3] or b, since τ is uniquely determined by b

to lower the arrival rate is therefore not effective. This result was confirmed by running examples for $\tau = 23$ and $\tau = 10$ for a range of offered loads. Tab. 6.1 compares the loss probabilities of an $M/M/1$ queueing system with no admittance restriction to that of the bounded-waiting-time case. Note that at $\tau = d$, $p_{\mathrm{loss,opt}}$ simply becomes $1 - P[J]$.

τ	ρ	$p_{\mathrm{loss},M/M/1}$, %	$p_{\mathrm{loss,opt}}$, %
23	0.80	0.8000	0.0900
	0.90	9.020	0.4600
	0.95	30.08	0.9000
	0.97	48.65	1.160

Table 6.1: Losses for $M/M/1$ queue with and without bounded system time

6.8 Numerical Example for Single Stage

As a numerical example to verify Eq. (6.13), take $\rho = 0.8$, $\mu = 1$, $\tau = 2$ and $x = 1$. Then, $\gamma(\rho-1, l_1) = -6.82669$, $\gamma(\rho-1, l_2) = -8.00568$, $\gamma(\rho, l_1) = 0.41379$ and $\gamma(\rho, l_2) = 0.20130$, yielding $W(1.0) = 0.9028$.

Probability densities and distributions for the virtual waiting time and the system time of admitted customers were computed numerically for the parameter values $\rho = 0.8$, $\mu = 1$ and $\tau = 2$. The values for Q and $P[J]$ were computed using Eq. (6.2) and Eq. (6.14), giving $Q = 0.5932$ and $P[J] = 0.8009$. Eq. (6.1) yields $w(x)$, which is then numerically integrated using the IMSL routine `DCADRE`, giving the distribution function $W(x)$ of the virtual wait. Evaluation of the closed-form expression for $W(x)$, Eq. (6.13), provided identical results, thus verifying the equation. The pdf of the system time, $s(x|J)$, is derived by convolution through numerical integration with `DCADRE` using Eq. (6.16). Since `DCADRE` has difficulties integrating $s(x|J)$ at the origin, a "dumber" quadrature routine, `QROMB` [57], is used. Finally, another numerical integration on $s(x|J)$ delivers the distribution of the system time, $S(x|J)$. As mentioned, this approach is rather inefficient, so we also programmed Eq. (6.18), which gave the same results in a fraction of the time. The time savings becomes especially significant when $S(x|J)$ is evaluated repeatedly while searching for an optimum τ. Tab. 6.2 summarizes the results.

The same results (to within the fourth digit) can also be obtained by starting out from Eq. (6.5). First, the IMSL routine `FLINV` numerically inverts the expression to find $w(x)$. It is then integrated with `QROMO` [57], a routine employing Romberg integration on an open interval. Experience showed that the inversion of $w^*(s)$ near the origin caused problems, tripping `DCADRE`. To avoid some of these boundary problems, the value of the function at the origin is returned rather than the exact evaluation if the argument is very close to zero or τ. The system time of admitted customers is now computed by numerically inverting Eq. (6.17), with `QROMO` again integrating to get $S(x|J)$, the system time CDF. The integration routines are time-consuming since they have to perform one call to `FLINV` for each sample point on the curve. Also, errors in the inversion seem to make the function noisier, tripping especially sophisticated adaptive quadrature routines such as `DCADRE`.

t	$w(t)$	$W(t)$	$s(t\|J)$	$S(t\|J)$
0.00	0.4103	0.5932	0.7406	0.0000
0.20	0.3716	0.6714	0.6947	0.1435
0.40	0.3315	0.7417	0.6482	0.2778
0.60	0.2901	0.8039	0.6009	0.4028
0.80	0.2475	0.8577	0.5527	0.5182
1.00	0.2039	0.9028	0.5034	0.6238
1.20	0.1599	0.9392	0.4532	0.7195
1.40	0.1162	0.9668	0.4021	0.8050
1.60	0.0740	0.9858	0.3506	0.8803
1.80	0.0347	0.9966	0.2991	0.9452
2.00	0.0000	1.0000	0.2486	1.0000

Table 6.2: Densities and distributions for $M/M/1$ queue with bounded wait

ρ	15	17	19	21	23	25	27	29	31	∞
0.10	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
0.20	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.30	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.40	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
0.50	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.60	0.5999	0.5999	0.6000	0.6000	0.6000	0.6000	0.6000	0.5999	0.5999	0.6000
0.70	0.6996	0.6998	0.6999	0.6999	0.6999	0.6997	0.6995	0.6994	0.6994	0.6995
0.80	0.7986	0.7991	0.7994	0.7996	0.7993	0.7975	0.7958	0.7946	0.7937	0.7936
0.85	0.8475	0.8482	0.8487	0.8491	0.8482	0.8435	0.8384	0.8344	0.8315	0.8271
0.90	0.8959	0.8968	0.8975	0.8980	0.8960	0.8842	0.8705	0.8590	0.8496	0.8188
0.95	0.9433	0.9444	0.9453	0.9460	0.9416	0.9149	0.8825	0.8536	0.8288	0.6642

Table 6.3: Influence of τ on goodput of $M/M/1$ queue with bounded wait

6.9 Numerical Analysis for Tandem Queueing System

After obtaining results for the single queue in isolation, we are now equipped to predict the loss performance of the tandem system. First, to build confidence in the accuracy of the convolution and numerical inversion, the convolution, inversion and integration was performed for a tandem-$M/M/1$ system using Eq. (A.7). Eq. (3.3) was implemented in double precision complex arithmetic, with IMSL routines `FLINV` and `DCADRE` handling inversion and integration. For comparison, the Romberg quadrature routine `QROMO` [57] was used, but it required about three times the number of function invocations.

Next, the FIFO-BS system was evaluated similarly. Tab. 6.4 shows the analytical results for the same system simulated in the following section. The virtual circuit consists of ten identical nodes with $\mu = 1$. Packets arrive at the first node at a rate of $\lambda = 0.8$. A packet is allowed $d = 78.5$ time units to traverse the virtual circuit or is considered lost. Each node discard packets with a system time of τ and above. The column $P[L]$ shows the fraction of packets that either did not make the deadline or were discarded en route. The probability of being dropped at an intermediate node is labeled $P[D]$. At the last node, the arrival rate has been reduced to λ_M. For comparison, the last column shows the loss rate for an $M/M/1$ system that does not discard packets. As expected, the system under study approaches the tandem $M/M/1$-link asymptotically as τ goes to infinity (see Section A.5).

It should be noted that the evaluation of the loss probabilities for one value of τ consumes between 5 and 15 minutes of MicroVAX II CPU time.

τ	$P[L]$,%	$P[D]$,%	λ_M	$M/M/1$
8.00	16.296	16.296	0.67890	0.05013
12.00	7.152	7.150	0.74736	0.05013
16.00	3.486	3.304	0.77585	0.05013
20.00	2.578	1.537	0.78883	0.05013
24.00	2.989	0.710	0.79486	0.05013
28.00	3.665	0.325	0.79766	0.05013
32.00	4.207	0.147	0.79894	0.05013
36.00	4.563	0.067	0.79952	0.05013
40.00	4.713	0.030	0.79978	0.05013
44.00	4.982	0.014	0.79990	0.05013
48.00	4.952	0.006	0.79996	0.05013
52.00	4.985	0.003	0.79998	0.05013
56.00	4.999	0.001	0.79999	0.05013
60.00	5.007	0.001	0.80000	0.05013
64.00	5.011	0.000	0.80000	0.05013
68.00	5.011	0.000	0.80000	0.05013
72.00	5.012	0.000	0.80000	0.05013
76.00	5.013	0.000	0.80000	0.05013
80.00	5.013	0.000	0.80000	0.05013

Table 6.4: Analysis for VC with 10 nodes; $\lambda_{VC} = 0.80$; $d = 78.50$

6.10 Simulation

In order to judge the impact of our simplifying modeling assumptions and catch problems with the numerical evaluation, the tandem queueing system was simulated using the author's discrete event package [73]. The simulator counts the number of packets discarded for each node and tallies the total packet loss due to discarding or tardiness.

A virtual circuit consisting of ten identical nodes with exponential servers was simulated. The service time of each packet was drawn independently at each node with mean $\mu = 1$ (Kleinrock's independence assumption). At the first node, packets arrived with exponential interarrival times with mean $\lambda = 0.8$ (i.e., $\rho = 0.8$ at the first node). For this system, 5.01% of the packets will not clear the given end-to-end deadline of $d = 78.5$ if no packets are dropped by intermediate nodes (see Eq. (A.5)). The expected end-to-end travel time is 50.

The effect of transients is controlled by disregarding packets with sequence numbers less than

2000. The simulation was terminated after 30,000, 100,000 and 1,000,000 packets. The same seed for the random number generator was used in each case, resulting in an identical sequence of interarrival times and service times. The results are collected in Tab. 6.5.

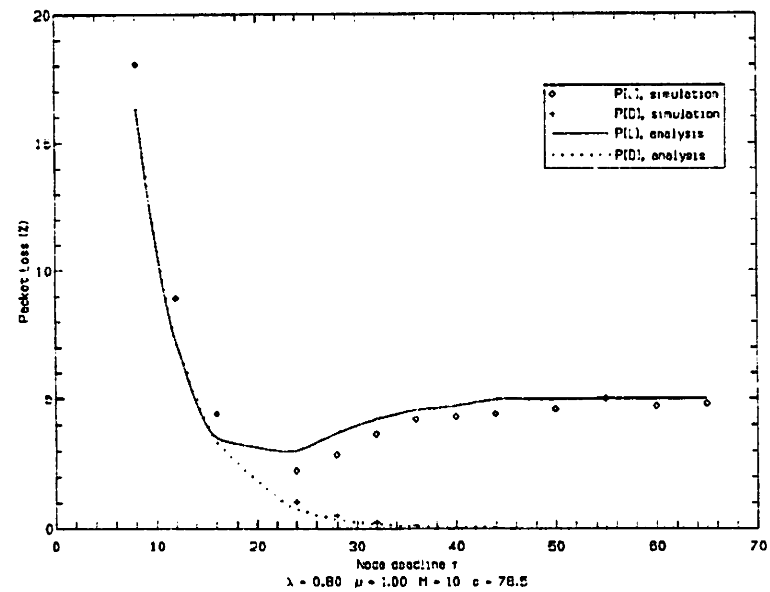

Figure 6.1: Comparison of Analysis and Simulation Results

Simulation and analysis results for this particular case are compared in Fig. 6.1. It can be seen that the analysis tracks the simulation results fairly closely. In the low-loss region of interest, the analysis tends to be pessimistic. For this particular set of parameters, the queue control method reduces end-to-end packet loss by a factor of roughly two.

τ	30000 packets $\bar{x}$	σ	$P[L]$, %	$P[D]$, %	10^5 packets $P[L]$, %	$P[D]$, %	10^6 packets $P[L]$, %	$P[D]$, %
8					17.68	17.68	18.07	18.07
10					12.28	12.28	12.57	12.57
12					8.65	8.65	8.937	8.937
14					6.11	6.11	6.224	6.224
16					4.23	4.23	4.427	4.410
18	42.15	10.60	3.1	3.030	2.95	2.91	3.332	1.549
24	46.37	13.50	2.3	1.053	2.31	0.969	2.225	1.039
26	47.49	14.40	3.2	0.690	2.39	0.563	2.661	0.7077
28	48.53	14.70	3.7	0.600	2.22	0.438	2.851	0.4862
30	47.69	14.90	4.0	0.483	3.36	0.332	3.137	0.3507
32	48.52	15.20	4.4	0.180	3.03	0.167	3.634	0.2296
34	48.27	15.00	4.0	0.037	2.92	0.112	3.869	0.1406
36	48.44	15.40	3.9	0.083	3.92	0.068	4.204	0.1075
38	47.84	15.70	4.4	0.067	3.69	0.058	4.445	0.0749
40	48.87	16.20	6.0	0.053	5.00	0.040	4.302	0.0434
42	47.44	14.80	3.1	0.040	3.45	0.020	4.475	0.0315
44	47.23	14.80	3.3	0.047	3.52	0.045	4.408	0.0160
46	47.27	14.80	3.0	0.050	4.28	0.026	4.569	0.0160
50	47.11	14.60	2.9	0.030	3.13	0.030	4.607	0.0091
55	48.28	15.50	4.3	0.017	3.57	0.009	4.999	0.0034
60	47.52	15.10	3.7	0.000	4.07	0.002	4.708	0.0007
65	47.77	15.01	3.4	0.000	3.47	0.000	4.803	0.0006
∞	47.06	14.46		0.000		0.000	4.712	0.0000

Table 6.5: Simulation of FIFO discipline with uniform deadlines and Kleinrock's independence assumption

Chapter 7

M/M/1 Queue with Bounded Waiting Time

This queueing system, denoted as **FIFO-BW** for short, has a single exponential server that serves customers in the order of their arrival. It has an infinite buffer so that every arriving customer joins the buffer but will leave after waiting τ time units if it is still in the buffer at that time. Once a customer enters service, he will complete service. Thus, the waiting time is bounded from above by τ. Arrivals are from a Poisson process, while service times are exponentially distributed. The system is equivalent to one that rejects arriving customers if the virtual work (*not* including the work that the customer brings along) exceeds τ. Since an actual implementation would look at the amount of virtual work (untransmitted bits, in the case of a communication link), we will adopt the latter viewpoint. Results for this system are provided by [74]; the original derivation of [75] was corrected by [76, p. 46–58]. The latter also solve the $M/M/c$ case and extend the result to exponential deadlines. The general case of a $GI/M/c$ queue is treated by Swensen [77].

This rather lengthy chapter is broken into three parts. The first derives analytic expressions for loss-related performance meaasures, while the second applies these results to investigate the possible improvements in end-to-end loss as a function of the length of the virtual circuit, the tolerable loss and the source traffic. Also, the effect of simplifying assumptions on traffic and local deadlines is studied. The third section presents some simulation results to gauge the accuracy of the model and the impact of the simplifying assumptions underlying the analysis.

7.1 Single-Stage Performance Measures

7.1.1 Probability of Admission

A customer joins the system (and thus eventually receives service within the deadline τ) with probability

$$\boxed{P[J] = \begin{cases} \dfrac{\alpha}{1-\rho^2 e^{-\nu\tau}} & \rho \neq 1 \\ \dfrac{1+\mu\tau}{2+\mu\tau} & \rho = 1 \end{cases}} \tag{7.1}$$

where

$$\alpha = 1 - \rho e^{-\nu\tau}.$$

Note that α equals the cumulative distribution of the $M/M/1$ waiting time at point τ. For infinite deadlines τ, i.e., the simple $M/M/1$ queue, α equals one, leading to $P[J] = 1$, as required for that queue. For $\tau = 0$, we obtain the single-server Erlang loss system or $M/M/1/1$ queue and the correct admission probability of $1/(1+\rho)$ as $\alpha = 1 - \rho$.

7.1.2 Single-Stage Waiting Time

Density

The conditional waiting time is distributed with density

$$w(t|J) = \begin{cases} \dfrac{\nu\rho e^{-\nu t}}{\alpha} u(\tau - t)u(t) + \dfrac{1-\rho}{\alpha}\delta(t) & \rho \neq 1, 0 \leq t \leq \tau \\ \dfrac{\mu}{1+\mu\tau} & \rho = 1, 0 \leq t \leq \tau \\ 0 & t \notin [0, \tau] \end{cases}$$

Recognizing $u(\tau - t) = 1 - u(t - \tau)$, the waiting time in the Laplace domain is derived for $\rho \neq 1$.

$$\boxed{w^*(s|J) = \frac{1}{\alpha}\left[\frac{\nu\rho}{s+\nu}\left\{1 - e^{-(s+\nu)\tau}\right\} + (1-\rho)\right]} \tag{7.2}$$

As $\tau \to \infty$, the $M/M/1$ case

$$w^*(s) = (1-\rho) + \frac{\nu\rho}{s+\nu}$$

emerges. Note that $w(0|J) = (\nu\rho + 1 - \rho)/\alpha$ is not equal to the probability of an empty system, $(1-\rho)/\alpha$.

Distribution

Integration yields the corresponding distribution function

$$W(t|J) = \begin{cases} 0 & t < 0 \\ \dfrac{1 - \rho e^{-\nu t}}{\alpha} & \rho \neq 1, 0 \leq t \leq \tau \\ \dfrac{1+\mu t}{1+\mu\tau} & \rho = 1, 0 \leq t \leq \tau \\ 1 & t > \tau \end{cases}$$

The waiting time distribution tends towards that of an $M/M/1$ queue as $\tau \to \infty$: $W(t) = 1 - \rho e^{-\nu t}$.

Moments

The moment-generating function of the waiting time is $w^*(-s)$ or

$$E[e^{sw}] = \frac{1}{\alpha}\left[\frac{\nu\rho}{\nu - s}\{1 - e^{s\tau}e^{-\nu\tau}\} + (1-\rho)\right]$$

As required, $w^*(0) = 1$.

With the aid of Section B.13 in the appendix, the rth derivative of the moment-generating function is written as

$$E[e^{sw}] = \frac{\nu\rho r!}{\alpha}\left[\frac{1}{(\nu - s)^{r+1}} - e^{-\nu\tau}e^{\tau s}\sum_{i=1}^{r+1}\frac{\tau^{r+1-i}}{\nu^i(r+1-i)!}\right]$$

which yields the nth moment when evaluated at $s = 0$:

$$E[w^n] = \frac{\nu\rho n!}{\alpha}\left[\frac{1}{\nu^{n+1}} - e^{-\nu\tau}\sum_{i=1}^{n+1}\frac{\tau^{n+1-i}}{\nu^i(n+1-i)!}\right] = \frac{\rho n!}{\alpha\nu^n}\left[1 - e^{-\nu\tau}\sum_{i=0}^{n}\frac{\tau^i}{\nu^{n-i}i!}\right]$$

Using [78, item 10, p. 604]

$$\sum_{k=0}^{n}\frac{x^k}{k!} = \frac{1}{n!}e^x\Gamma(n+1,x),$$

this can be rewritten in closed form as

$$E[w^n] = \frac{\rho}{\alpha\nu^n}\left[n! - \Gamma(n+1,\nu\tau)\right]$$

where $\Gamma(n,x)$ is the complementary incomplete Gamma function defined as

$$\Gamma(n+1,x) = \int_x^\infty t^n e^{-t}\,dt$$

For numerical evaluation, a recursive expression for the expectation is more useful and can be found after some algebra to be

$$E[w^n] = \frac{n}{\nu}E[w^{n-1}] - \frac{\rho\tau e^{-\nu\tau}}{\alpha}$$

or as backwards iteration

$$E[w^{n-1}] = \frac{(E[w^n] + \rho\tau e^{-\nu\tau}/\alpha)\nu}{n}.$$

The recursion is valid for $E[w^2]$ and above. If the random variable is normalized with respect to τ, the moments are divided by τ^n and therefore computed recursively as

$$E[w^n] = \frac{n}{\nu\tau}E[w^{n-1}] - \frac{\rho e^{-\nu\tau}}{\alpha}$$

and

$$E[w^{n-1}] = \frac{(E[w^n] + \rho e^{-\nu\tau}/\alpha)\nu\tau}{n}$$

The first moment follows immediately:

$$E[w] = \frac{\rho}{\alpha}\left[\frac{1}{\nu} - e^{-\nu\tau}\left(\frac{1}{\nu} + \tau\right)\right] = \frac{\rho}{\alpha\nu}\left[1 - e^{-\nu\tau}(1 + \nu\tau)\right]$$

Note that for the $M/M/1$ case ($\alpha = 1$, $\tau = \infty$),

$$E[w^n_{M/M/1}] = \frac{n!}{\nu^n}\rho.$$

7.1.3 Approximation of Single-Stage Distribution Function

Asymptotic Inversion of Laplace Transform

In preparation for multistage problems, the approximation of the distribution function by the numerical inversion technique described in section A.3.3 of the appendix was attempted. Here,

$$\frac{\rho\nu}{\alpha}\sum_{k=1}^{\lfloor st\rfloor} s^k \left[\frac{1}{(\nu+s)^{k+1}} - e^{-(s+\nu)\tau}\sum_{i=1}^{k+1}\frac{\tau^{k+1-i}}{\nu^i(k+1-i)!}\right] + \frac{1}{\alpha}\left[\frac{\rho\nu}{\nu+s}\left(1-e^{-(s+\nu)}\right)+(1-\rho)\right] \to F(t)$$

For computational purposes, it is convenient to split up this expression into an $M/M/1$ term, as derived in the appendix (scaled by $1/\alpha$), a "constant" term and a term consisting of the double summation.

$$\frac{1}{\alpha}\left[1-\rho\left(\frac{s}{s+\nu}\right)^{\lfloor st\rfloor+1}\right] - \frac{\rho\nu}{\alpha}e^{-(s+\nu)\tau}\left[\sum_{k=1}^{\lfloor st\rfloor} s^k \sum_{i=0}^{k}\frac{\tau^i}{\nu^{k+1-i}i!}\right] - \frac{\rho\nu}{\alpha}\left[\frac{e^{-(s+\nu)}}{s+\nu}\right] \to F(t)$$

The expression in the double summation is computable only if expressed logarithmically, with outside factors pulled in as

$$\exp(i\ln\tau - \ln i! - (k+1-i)\ln\nu + k\ln s - (s+\nu)\tau)$$

For some reason still unknown, the expression diverges as soon as $\lfloor st\rfloor > \tau$.

Moment Approximation

If the random variable t has finite support on the interval $[0,1]$, its cumulative distribution function can be derived from its moments through the inversion formula [79, p. 227]

$$\sum_{j=0}^{\lfloor nt\rfloor}\binom{n}{j}(-1)^{n-j}\Delta^{n-j}E[t^j] = \sum_{j=0}^{\lfloor nt\rfloor}\binom{n}{j}(-1)^{n-j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^{n-j-k}E[t^{j+k}] \to F(t).$$

Indeed, for a uniformly distributed random variable t, $E[t^n]=1/(n+1)$ and with the summation [78, item 43, p. 611]

$$\sum_{k=0}^{n}\frac{(-1)^k}{a+k}\binom{n}{k} = \frac{1}{a}\binom{a+n}{n}^{-1}$$

the argument to the outer sums becomes

$$\binom{n}{j}\sum_{k=0}^{n-j}\binom{n-j}{k}(-1)^k\frac{1}{j+k+1} = \frac{1}{n+1}$$

from which the approximation is seen to be

$$\frac{\lfloor nt\rfloor+1}{n+1} \to t \equiv F(t).$$

Unfortunately, even in this simple case the numerical approximation shows errors of greater than 10^{-5} (and rapidly increasing from then on) for $n > 21$ ($t = 0.5$), despite the use of quadruple precision arithmetic with 33 digits of numerical precision and magnitude-sorted addition.

Similar difficulties befall the waiting time distribution for the system under consideration, even for a single queue. For the parameters $\tau = 40$, $t = 20$, $\rho = 0.8$, $\nu = 0.2$, the best approximation to the exact value $1.4388 \cdot 10^{-2}$ is obtained for $n = 22$ as $1.7104 \cdot 10^{-2}$, slightly better than the result obtained by the approximate inversion of the Laplace transform.

7.2 M-Stage Waiting Time by Analytic Inversion

In this section, we evaluate the end-to-end waiting time of the tandem system by analytic convolution, either in the time domain through integration or in the Laplace domain by expanding products of sums into sums of products. In the following, we will investigate four cases:

1. traffic λ_i and local deadlines τ_i are assumed to be equal throughout the network (homogeneous traffic and deadlines), i.e., $\lambda_i = \lambda$ and $\tau_i = \tau$;
2. the restriction on the local deadline is removed, allowing τ_i to assume any value
3. heterogeneous traffic and deadlines, however with the restriction that $\lambda_i \neq \lambda_j$ for $i \neq j$;
4. completely arbitrary λ_i; this incurs a complexity not compensated for by additional insight and is thus omitted.

Allowing heterogeneous traffic, but enforcing homogeneous local deadlines does not offer any simplification compared to heterogeneous deadlines.

In each of the three cases treated below, the expression for the waiting time density was subjected to two rough sanity checks. First, for the zero point of the cdf we must have

$$P\{w = 0\} = \prod_{i=1}^{M} \frac{1 - \rho_i}{\alpha_i}$$

Also, $P\{w \leq M\tau\} = 1$.

7.2.1 Homogeneous Traffic and Deadlines, 2 Nodes

This case ignores the change in traffic intensity along the virtual circuit caused by discarded packets. Both local deadlines are assumed to be equal to τ.

For $0 \leq y \leq \tau$, the convolution integral in Eq. (B.8) is instantiated as

$$w(t) * w(t) = \left(\frac{\nu\rho}{\alpha}\right)^2 \int_0^y e^{-\nu t} e^{-\nu(y-t)} dt + 2\frac{1-\rho}{\alpha}\frac{\nu\rho}{\alpha} e^{-\nu y} + \left(\frac{1-\rho}{\alpha}\right)^2 \delta(y),$$

where $\nu = \mu - \lambda$. This evaluates to

$$w(t) * w(t) = \frac{\rho\nu}{\alpha^2} e^{-\nu y}[\rho\nu y + 2(1-\rho)] + \left(\frac{1-\rho}{\alpha}\right)^2 \delta(y).$$

Since the waiting time pdf is zero for $t > \tau$, the convolution integral ranges from $y - \tau$ to τ for the case $\tau \leq y \leq 2\tau$, with the result

$$w(t) * w(t) = \frac{1}{\alpha^2}\nu\rho e^{-\nu\rho}(\nu\rho)^2(2\tau - y)$$

which appropriately decays to zero for $y \to 2\tau$.

After integration, the probability distribution function for $0 \leq y \leq \tau$ is written as

$$W(t) = \frac{1}{\alpha^2}\left[1 - (\nu\rho y - \rho + 2)\, e^{-\nu y}\right].$$

Homogeneous Traffic and Deadlines, M Nodes

For more than two stages, the evaluation of the convolution integral becomes hopelessly tedious because of the numerous subregions to be considered. For the case of homogeneous traffic and deadlines τ, a closed-form expression for the density can be more easily derived from the Laplace transform. Here, the polynomial coefficient is helpful, defined in analogy to the binomial coefficient:

$$\binom{n}{k_1, k_2, \ldots, k_r} = \frac{n!}{k_1!k_2!\ldots k_r!}$$

Thus, with Eq. (7.2), the M-stage convolution is

$$\begin{aligned}
&\mathcal{L}^{-1}\left[\frac{1}{\alpha}\frac{\nu\rho}{s+\nu}(1 - e^{-(s+\nu)\tau}) + \frac{1-\rho}{\alpha}\right]^M \\
&= \mathcal{L}^{-1}\left\{\frac{1}{\alpha^M}\sum_{k+l+m=M}\binom{M}{k, l, m} e^{-l\nu\tau}(\nu\rho)^{k+l}(1-\rho)^m(-1)^l\frac{e^{-l\tau s}}{(s+\nu)^{k+l}}\right\}.
\end{aligned}$$

For $k + l > 0$, the fraction containing terms in s can be readily inverted as

$$\frac{(t - l\tau)^{k+l-1}}{(k+l-1)!}e^{-\nu(t-l\tau)}u(t - l\tau)$$

For $k + l = 0$, that is, $k = l = 0$, the summand reduces to $(1-\rho)^m$.

The cdf at point d can be computed by integrating each term in the sum over the interval $l\tau$ to $d - l\tau$, using equation B.2, as the integral is zero for other values of t.

Even in this case where the parameters λ and μ (and, therefore, ρ, r and ν) are independent of the node index i, the inversion of the transformed convolved waiting time is tedious, since the Mth power of a sum of n (here, 3) terms unfolds into

$$\binom{M+n-1}{M} = \frac{(M+n-1)!}{M!(n-1)!} = \frac{M^2+3M+2}{2}$$

terms in the sum-of-products form (combinations of n elements taken M at a time with repetition).

Homogeneous Traffic, Heterogeneous Local Deadlines

In this case, the τ_i *may* depend on the node index, but the traffic indicator ν is independent of it. In the Laplace domain, we can expand the product of single-stage waiting time transforms into a sum

$$\prod_{i=1}^{M} \frac{1}{\alpha^i} \left[\frac{\nu\rho}{s+\nu}(1-e^{-(s+\nu)\tau}) + (1-\rho) \right]$$
$$= \left[\prod_{i=1}^{M} \frac{1}{\alpha_i} \right] \sum_{k=0}^{M} \sum_{\vec{l}} \left[\frac{\nu\rho}{s+\nu} + (1-\rho) \right]^k (-1)^{M-k} \left[\frac{\nu\rho}{s+\nu} \right]^{M-k} \exp\left(-(s+\nu) \sum_{i=1}^{M} l_i \tau_i \right),$$

where l_i is restricted to either zero or one. The inner sum iterates over all binary vectors $\vec{l}$ of length M where $M-k$ components l_i are one and the remaining k are zero.

Each term in the inner sum can be readily inverted for a given k as

$$(-1)^{M-k} e^{-\nu\tau'} \sum_{j=0}^{k} \binom{k}{j} (1-\rho)^{k-j} (\nu\rho)^{j+M-k} A_{k,j}$$

where

$$A_{k,j} = \frac{(t-\tau')^{M+j-k-1}}{(M+j-k-1)!} e^{-\nu(t-\tau')} u(t-\tau')$$

for $j+M-k>0$ and $A_{k,j} = \delta(t-\tau')$ for $j=k-M$. Here, we define $\tau' = \sum_i l_i \tau_i$. The sum has a total of approximately

$$\sum_{k=0}^{M} \binom{M}{M-k} (k+1) = \sum_{k=0}^{M} \binom{M}{k} (k+1) = 2^{M-1} \cdot M + 2^M = 2^M(1+M/2)$$

terms (see [78, p. 609, items 6 and 7]). The exact term count depends on the relationship between the τ_i's and t, since the $u(t-\tau')$ will suppress a whole innermost summation for negative arguments. The count does not take into account subsummations (in the current implementation, there are $M+1$).

Heterogeneous Traffic, Heterogeneous Local Deadlines, 2 Nodes

For brevity, let us define the constant $r = 1-\rho$. For the first region of $0 \le y \le \tau_{1,2}$, the convolution integral evaluates to

$$\begin{aligned} w_{12}(y) &= w_1(y) * w_2(y) \\ &= \frac{1}{\alpha_1\alpha_2} \left\{ \frac{\nu_1\nu_2\rho_1\rho_2}{\nu_2-\nu_1} \left[e^{-\nu_1 y} - e^{-\nu_2 y} \right] + \nu_2 r_1 \rho_2 e^{-\nu_2 y} + \nu_1 r_2 \rho_1 e^{-\nu_1 y} + r_1 r_2 \delta(y) \right\} \end{aligned}$$

The second region spans $\tau_1 \le y \le \tau_2$, with a value of

$$w_{12}(y) = w_1(y) * w_2(y) = \frac{\nu_1\nu_2\rho_1\rho_2}{\alpha_1\alpha_2(\nu_2-\nu_1)} \left[e^{-\nu_2 y + \nu_2\tau_1 - \nu_1\tau_1} - e^{-\nu_2 y} \right].$$

The third and final region, $\tau_2 \leq y \leq \tau_1 + \tau_2$ yields

$$w_{12}(y) = w_1(y) * w_2(y) = \frac{\nu_1\nu_2\rho_1\rho_2}{\alpha_1\alpha_2(\nu_2 - \nu_1)}\left[e^{-\nu_2 y + \nu_2\tau_1 - \nu_1\tau_1} - e^{-\nu_1 y - \nu_2\tau_2 + \nu_1\tau_2}\right].$$

The density is zero outside the three regions.

The same solution is obtained by inverting the Laplace transform. Integration yields the cdf as

$$\begin{aligned}
W_{12}(t) &= r_1 + r_2 \\
&+ r_1\rho_2\left(1 - e^{-\nu_2(t\wedge\tau_2)}\right) \\
&+ r_2\rho_2\left(1 - e^{-\nu_1(t\wedge\tau_1)}\right) \\
&+ c\left(\frac{1}{\nu_1} - \frac{e^{-\nu_1 t}}{\nu_1} - \frac{1}{\nu_2} + \frac{e^{-\nu_2 t}}{\nu_2}\right) \\
&+ ce^{-\nu_2\tau_2}\left(\frac{1}{\nu_1} - \frac{e^{-\nu_1(t-\tau_2)}}{\nu_1} - \frac{1}{\nu_2} + \frac{e^{-\nu_2(t-\tau_2)}}{\nu_2}\right) \\
&+ ce^{-\nu_1\tau_1}\left(\frac{1}{\nu_1} - \frac{e^{-\nu_1(t-\tau_1)}}{\nu_1} - \frac{1}{\nu_2} + \frac{e^{-\nu_2(t-\tau_1)}}{\nu_2}\right) \\
&+ ce^{-\nu_1\tau_1-\nu_2\tau_2}\left(\frac{1}{\nu_1} - \frac{e^{-\nu_1(t-\tau_1-\tau_2)}}{\nu_1} - \frac{1}{\nu_2} + \frac{e^{-\nu_2(t-\tau_1-\tau_2)}}{\nu_2}\right)
\end{aligned}$$

where the constant $\frac{\nu_1\rho_1\nu_2\rho_2}{\nu_2-\nu_1}$ has been abbreviated as c.

7.2.2 Heterogeneous Traffic, Heterogeneous Local Deadlines, M Nodes

Since the number of cases to consider increases compared to the uniform traffic case, a direct convolution is even less feasible than before. Also, an approximation cannot be made by taking the reduced traffic into account through the α factors only since while $1/\alpha$ increases with λ, the "body" of the distribution function decreases.

However, with the restriction that $\nu_i \neq \nu_j$ for $i \neq j$, the expression for the Laplace transform of the waiting time of the tandem system can be expanded from a product of sums into a sum of products. Each summand is a readily invertible rational expression multiplied by an exponential in $s\tau$. Restricting the ν_i's to be unequal avoids multiple poles and the concommittant complications in the partial fraction expansion.

Analogous to the $M/M/1$ case (see eq. (A.4)), the pdf is epressed in the time-domain as

$$\begin{aligned}
w_M(t) &= \sum_{k=1}^{2^M-1}\left[\prod_{i=1}^{M}\frac{1}{\alpha_i}\right]\left\{\left[\prod_{j=1}^{M} r_j^{1-b(k)_j}(\rho_j\nu_j)^{b(k)_j}\right]\sum_{j\in b(k)}\frac{\sum_{m=0}^{2^{|b(k)|}-1} e^{-\beta_m}e^{-\nu_j(t-\tau'_m)}u(t-\tau'_m)}{\prod_{i\in b(k),\, i\neq j}\nu_i-\nu_j}\right\} \\
&+ \prod_{j=1}^{M} r_j\delta(t).
\end{aligned}$$

Here, $b(k)_i \in \{0,1\}$ stands for the ith digit of k written as a base-2 (binary) number. (We will arbitrarily count starting with 1 at the least significant digit.) For brevity, $i \in b(k)$ denotes all indices i such that $b(k)_i$ is 1. The number of ones in $b(k)$ is shown as $|b(k)|$, hinting at a notion of a (Hamming) "distance". The computation of the quantities β_m and τ'_m is based on the expansion of the product of the $|l_k|$ terms of the form $1 - \exp(-(s + \nu_i)\tau_i)$. Define the mapping function $h(k)$, whose ith component $h(k)_i$ is the index of the ith "one" bit in $b(k)$. Then,

$$\tau'_m = \sum_{i=1, i \in b(i)}^{|b(k)|} \tau_{h(k)_i}$$

and

$$\beta_m = \sum_{i=1, i \in b(i)}^{|b(k)|} \nu_{h(k)_i} \tau_{h(k)_i}.$$

The cumulative distribution function is readily computed by replacing the terms in the summation over m by

$$\frac{1}{\nu_j}\left[1 - e^{-\nu_j(t-\tau'_m)}\right] e^{-\beta_m} u(t - \tau'_m)$$

The number of summands in the triple summation is computed by noting that if $|b(k)| = m$, the two inner summation have a total of $m \cdot 2^m$ terms. Since there are $\binom{M}{m}$ ways of selecting m "one"-bits out of a total of M bits, there are a total of

$$\sum_{m=0}^{M} \binom{M}{m} m \cdot 2^m = 2M \cdot 3^{M-1}$$

terms in the expansion (see [78, p. 612, item 18 and p. 608, item 3]). For example, for $M = 5$, a manageable 810 additions are required, while for $M = 10$, the number of terms climbs to 393660. The computation of the β_m and τ'_m is not included in these counts, however. This number represents an upper bound for all values of t. The number of terms will be lower for small t since the $u(t - \tau'_m)$ step function will force most terms to zero. The actual implementation uses intermediate sums, incurring $\sum_{m=0}^{M} \binom{M}{m} m+1 = 2^{M-1} \cdot M+1$ additional operations, but reducing storage requirements to $\sum_{m=0}^{M} m \binom{M}{m} = M \cdot 2^{M-1}$ for the outer loop and 2^M in the inner loop.

It was attempted to make the numerical evaluation stable by using sorted summation, where terms are added in order of absolute values, minimizing the loss of accuracy involved in adding numbers of disparate magnitude. The computation runs into trouble when two or more of the ν_i are almost equal, since we divide by their difference. This occurs for small loss probabilities (approximately 0.005) and low traffic.

Table 7.1 compares the various approaches to finding the end-to-end delay distribution, showing that numeric convolution (numeric quadrature of the convolution integral) offers poor accuracy even for a small two-node system. On the other hand, the results for the remaining methods agree very closely, giving some credence to the analytic derivations and their computer implementations.

M	λ	d	τ	$\lambda_i = \lambda$	$P[D]$	$P[T]$	$P[L]$	method
2	0.8	21.5441	∞		0.000	$5.000 \cdot 10^{-2}$		
			10	no	$4.377 \cdot 10^{-2}$	$-8.252 \cdot 10^{-5}$	$4.369 \cdot 10^{-2}$	C
				no		$1.381 \cdot 10^{-7}$	$4.377 \cdot 10^{-2}$	L
				yes	$4.685 \cdot 10^{-2}$	0.000	$4.685 \cdot 10^{-2}$	F
			15	no	$1.578 \cdot 10^{-2}$	$8.005 \cdot 10^{-3}$	$2.365 \cdot 10^{-2}$	C
				yes	$1.639 \cdot 10^{-2}$	$8.472 \cdot 10^{-3}$	$2.472 \cdot 10^{-2}$	C
				yes		$8.174 \cdot 10^{-3}$	$2.443 \cdot 10^{-2}$	F
			20	yes	$5.922 \cdot 10^{-3}$	$2.408 \cdot 10^{-2}$	$2.986 \cdot 10^{-2}$	F,L
				yes		$2.397 \cdot 10^{-2}$	$2.975 \cdot 10^{-2}$	C
				no	$5.808 \cdot 10^{-3}$	$2.397 \cdot 10^{-2}$	$2.913 \cdot 10^{-2}$	C
				no		$2.357 \cdot 10^{-2}$	$2.924 \cdot 10^{-2}$	L
			30	yes	$7.943 \cdot 10^{-3}$	$4.622 \cdot 10^{-2}$	$4.700 \cdot 10^{-2}$	F,C,L
				no	$7.910 \cdot 10^{-4}$	$4.608 \cdot 10^{-2}$	$4.684 \cdot 10^{-2}$	C,L
5	0.8	40.466	∞		0.000	$5.000 \cdot 10^{-2}$		
			20	no	$1.369 \cdot 10^{-2}$	$1.768 \cdot 10^{-2}$	$3.113 \cdot 10^{-2}$	L

Table 7.1: Loss probabilities for uniform deadlines, derived from numeric convolution (C), numeric inversion of the Laplace transform (L), closed-form expression (F) or closed-form inversion of the Laplace transform (I)

7.3 The Effectiveness of the Control Policy — Numerical Results

In this section, we will investigate aspect of the performance of overload control using bounded waiting times for "typical" communication networks. In Section 7.3.1, we ignore that dropping packets thins traffic towards the destination node. Thus, only the local overload control effect of the policy is taken into account. In the following section 7.3.2, the traffic decrease due to dropping is figured in, however, the local deadlines are the same throughout the virtual circuit. Then, we look at the best performance achievable by tuning the local deadline, finding that maintaining the same local deadlines throughout the virtual circuit often incurs little penalty.

An actual implementation would probably maintain the same local deadline over a wide traffic range since it is difficult to measure traffic accurately. Section 7.3.3 compares different approaches in finding a compromise between optimal performance and potential degradation if the system moves away from the design parameters.

The performance in real systems is often determined by bottlenecks. The effect of a heavily loaded node on the performance of the control policy is described through some examples in Section 7.3.4.

Finally, it is shown in Section 7.3.5 that the control policy can improve the goodput of a tandem system, the amount of traffic reaching the destination within the deadline.

Unless otherwise noted, all results were computed by the closed-form (analytic) inversion of the Laplace transform, as described above. Also, throughout "uniform" and "homogeneous" are to be taken as synomymous and refer to the independence of the traffic (λ) or local deadline (τ) from the node index. Thus, in the remainder of this chapter, "uniform" does not refer to the uniform distribution of a random variable.

7.3.1 Homogeneous Traffic and Deadlines

Influence of Local Deadline on Loss

Tables D.1 through D.3 in Appendix D and graphs 7.1 through 7.12 in this section provide loss values for different combinations of VC length M, input traffic intensity λ, end-to-end deadlines d and local deadlines τ. The end-to-end deadlines d were chosen so that the uncontrolled system (i.e., with no packet dropping at intermediate nodes) would achieve a set level of delay loss performance, here five, two, one and one-tenth of a percent. The uncontrolled loss is shown in columns marked $M/M/1$.

For homogeneous traffic, the local deadline $\tau_i = \tau$ needs to be varied only between d/M and d. For values of $\tau \leq d/M$, all packets which are not dropped will make their deadline. Thus, tightening the deadline beyond d/M can only worsen the result. For $\tau \geq d$, any packets dropped at an intermediate node would be late on arrival at the end. Loosening the deadline above d only generates additional traffic downstream, but cannot increase the fraction of packets within deadline.

In the graphs, the line labeled "dropped packets" graphs the values for $P[D]$ in the tables, "late packets" corresponds to $P[T]$, "late arrivals" to $P[T|A]$ and "total packet loss" to $P[L]$. For brevity, the term *uncontrolled loss* will be understood to mean the total end-to-end loss incurred if the nodes admit all packets, equivalent to an infinite local deadline. The term *controlled loss* denotes the loss seen after applying the bounded-wait policy.

The graphs suggest a number of conclusions. First, the general shape of the performance curves is very similar regardless of M and λ. In all cases, if the local deadline is tightened below the optimal value, the total loss increases rapidly above the optimal point and soon beyond the value without queue control. For looser-than-optimal deadlines, the degradation in performance is much less severe, suggesting erring on the side of higher values for τ.

In general, this queue control policy performs best for high traffic and higher uncontrolled losses, as shown by the contour plot of Fig. 7.13[1]. It depicts combinations of VC length and traffic resulting in equal gain, where *gain* (for the lack of a better term) is defined as the ratio of controlled to uncontrolled total loss. This suggests the use of the policy to control the deleterious effects of overload rather than to improve delays and losses under light load.

The components of the total loss depend on the local deadline in the expected fashion. For tight deadlines, most of the loss is made up of packets discarded at the individual nodes. As only packets that experience little delay at the nodes reach the destination, most of them will make the end-to-end deadline. For loose deadlines, on the other hand, most all of the packets reach the destination and have to be discarded there.

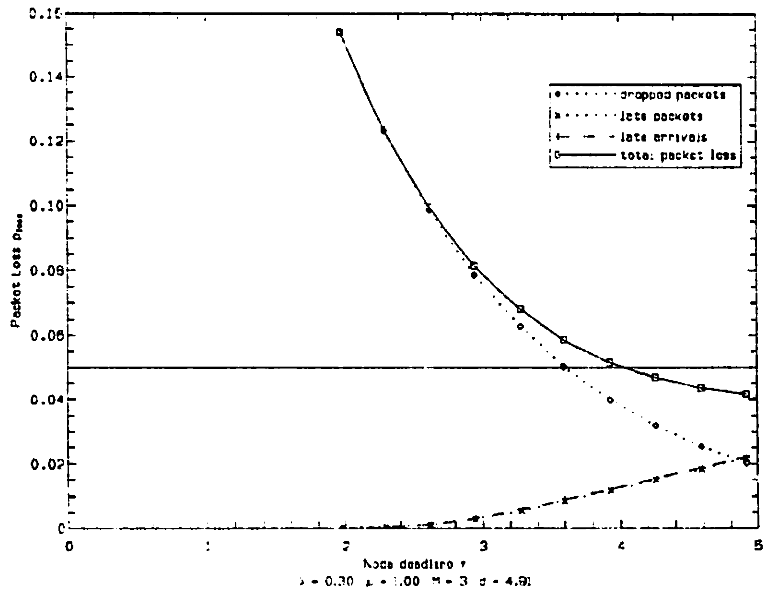

Figure 7.1: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 3$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.3$

The queue control processor and the virtual circuit setup could be simplified if the local deadline could be set regardless of the length of the virtual circuit and the current node load. The contour graph in Fig. 7.14 shows how the optimal deadline varies with VC length and traffic. Surprisingly, for high loads, the optimal deadline is almost independent of the number of stages in the virtual circuit, suggesting that a single dynamic buffer size for all circuits with similar loss constraint may be feasible. (Note, however, that the end-to-end deadline varies for each parameter tuple, as described at the beginning of this section.)

Some of the above figures plotting loss components vs. local deadline seem to suggest that

[1]Detailed values are collected in Tab. D.10 through D.14 in the Appendix.

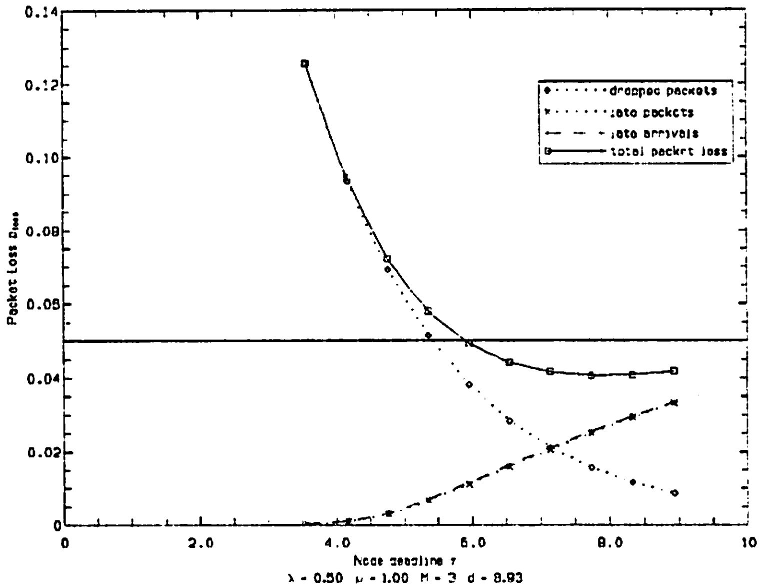

Figure 7.2: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 3$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.5$

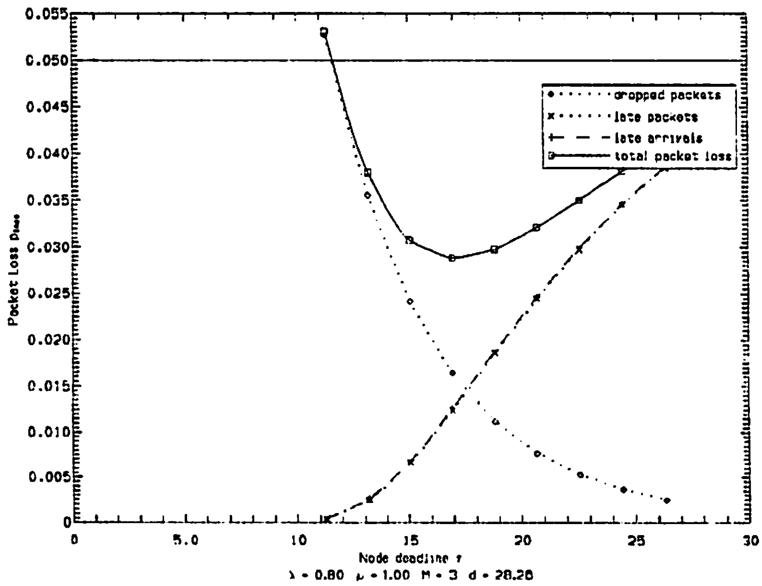

Figure 7.3: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 3$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.8$

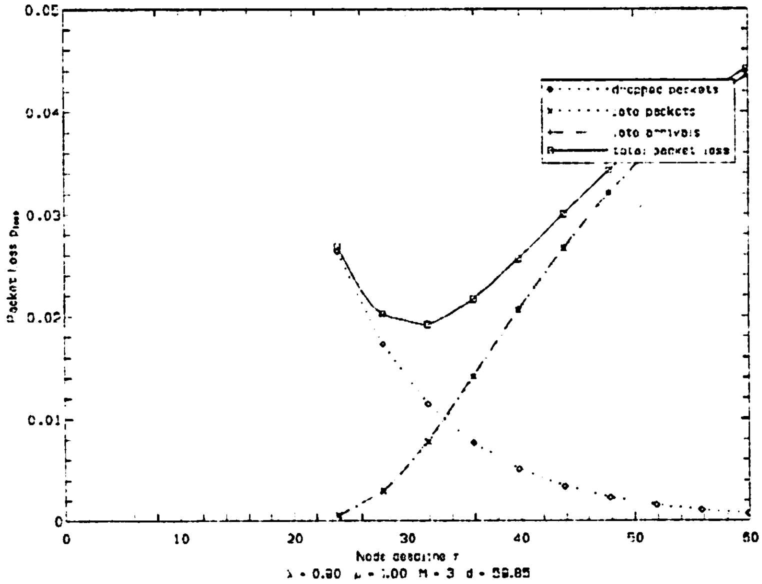

Figure 7.4: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 3$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.9$

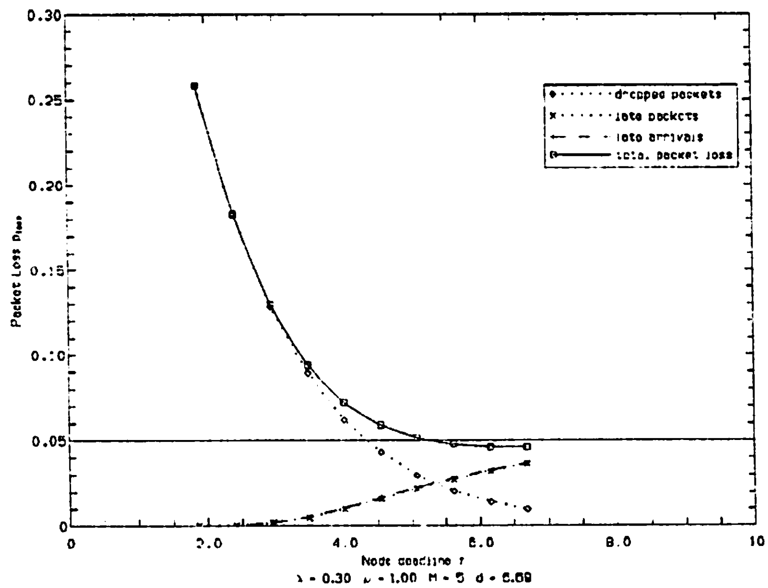

Figure 7.5: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.3$

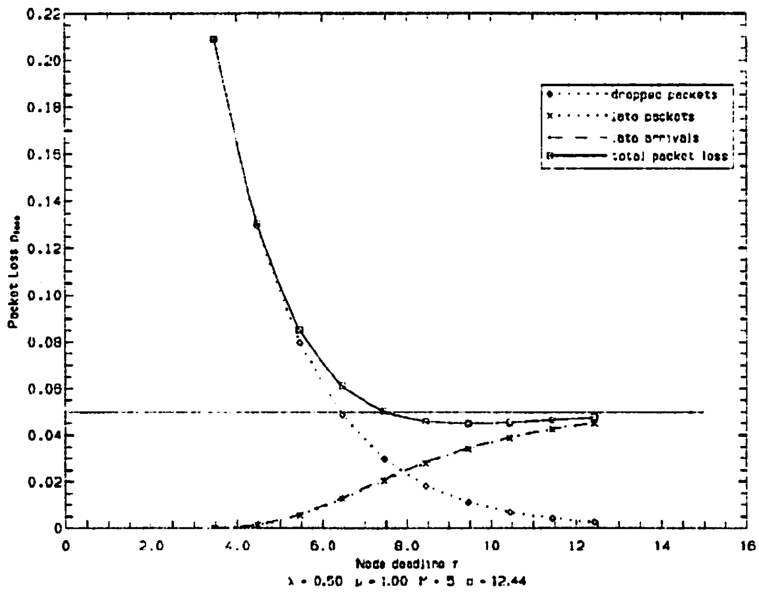

Figure 7.6: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.5$

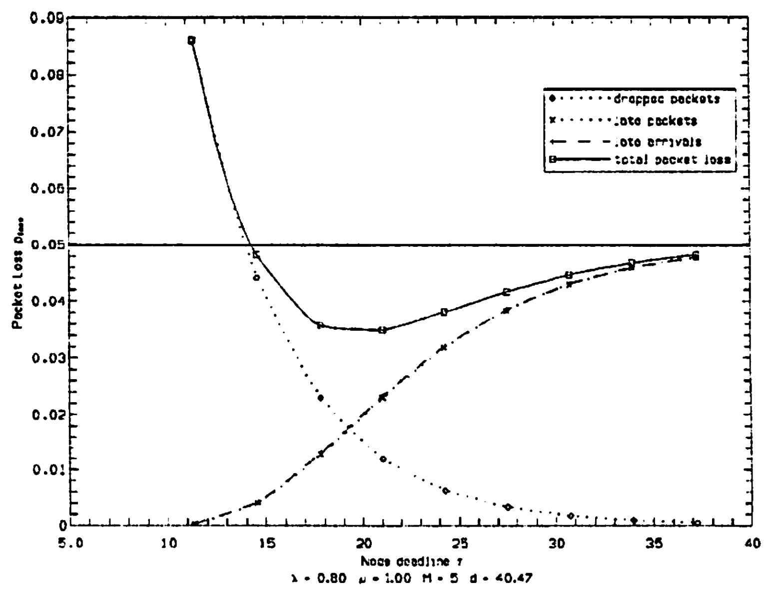

Figure 7.7: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $5{\cdot}10^{-2}$, arrival rate $\lambda = 0.8$

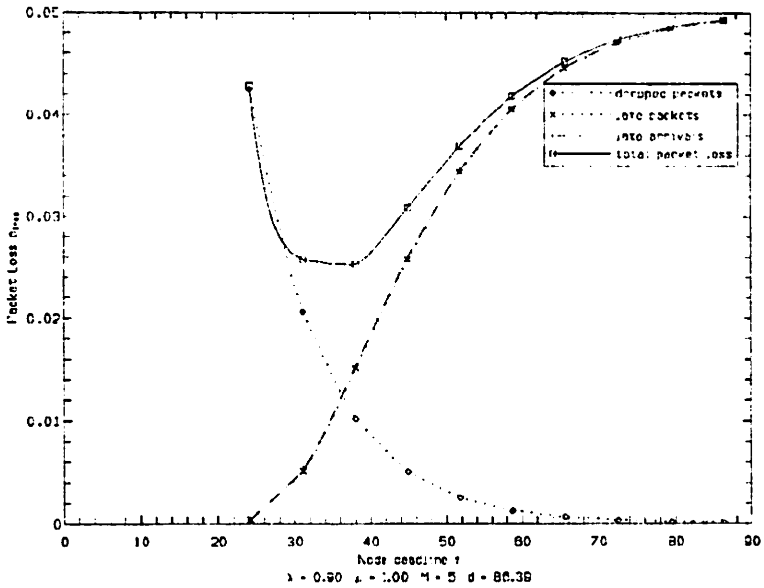

Figure 7.8: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $5 \cdot 10^{-2}$, arrival rate $\lambda = 0.9$

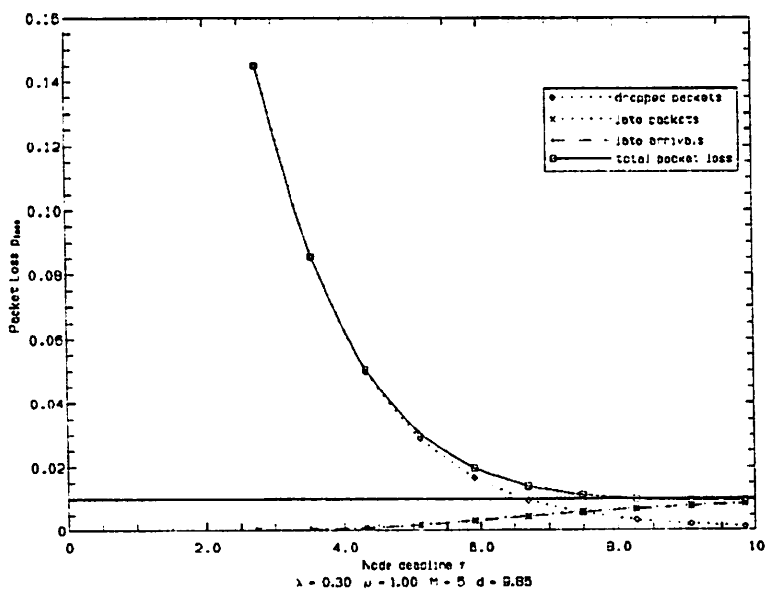

Figure 7.9: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $1 \cdot 10^{-2}$, arrival rate $\lambda = 0.3$

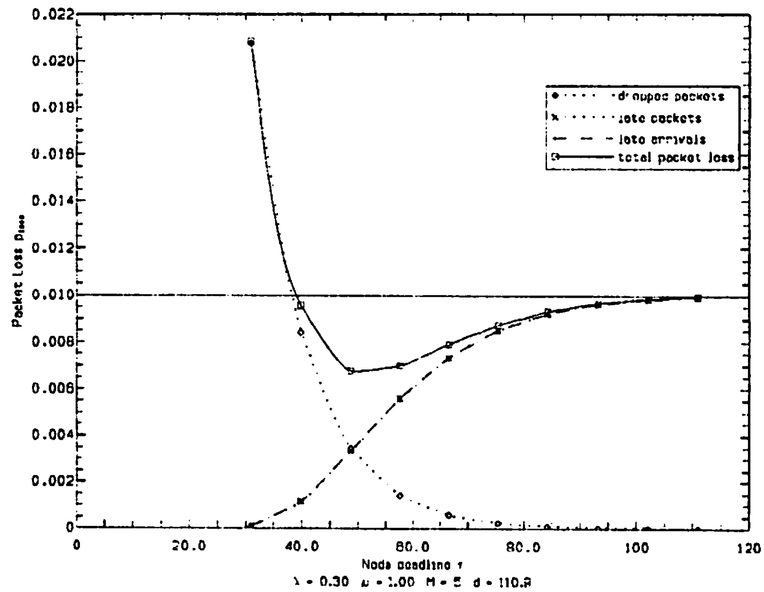

Figure 7.10: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $1{\cdot}10^{-2}$, arrival rate $\lambda = 0.9$

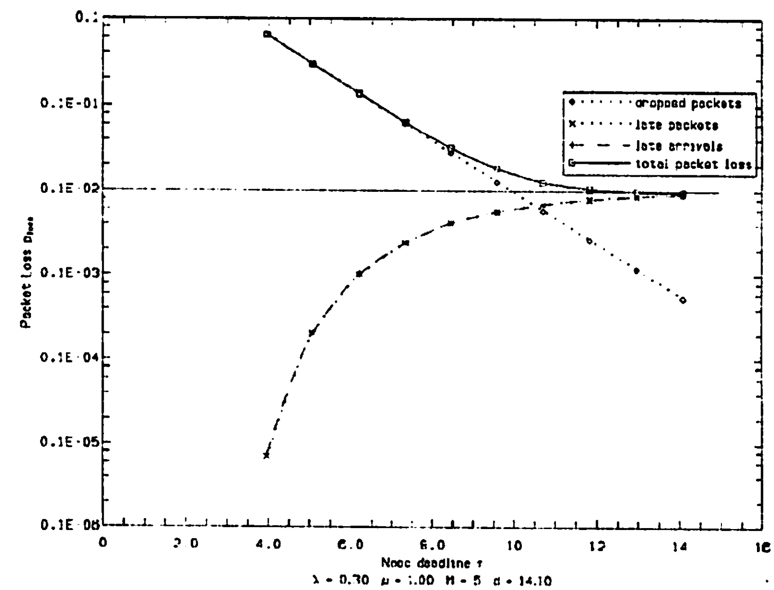

Figure 7.11: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $1{\cdot}10^{-3}$, arrival rate $\lambda = 0.3$

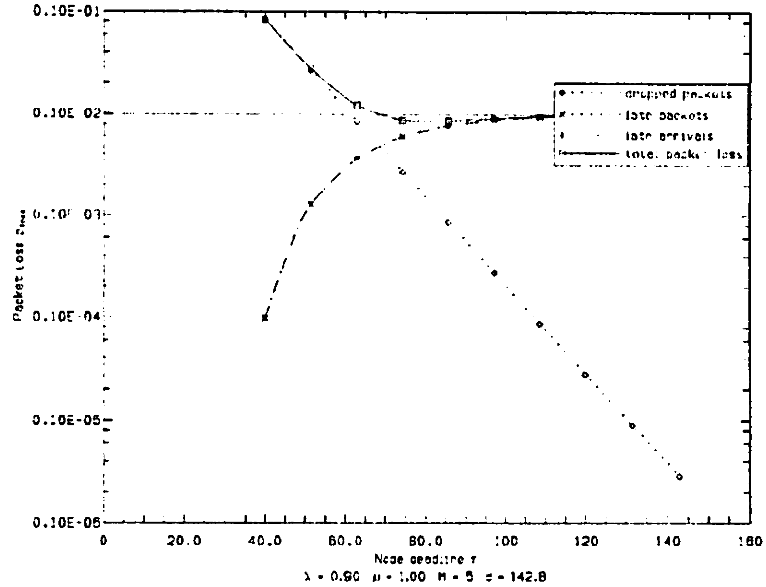

Figure 7.12: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 5$, uncontrolled loss of $1 \cdot 10^{-3}$, arrival rate $\lambda = 0.9$

at the optimal operating point, the fraction of packets dropped and the fraction discarded at the destination is close to equal. However, the contour graph 7.15 dispels any hope of using the relation between the two loss components for establishing the optimal deadline.

7.3.2 Decreasing Traffic, Homogeneous Deadlines

In this subsection, we will drop one of our simplifying assumptions, namely that of homogeneous traffic. Thus, the customers lost at a node will be reflected in reduced traffic downstream, with better loss performance for the remaining customers. It is intuitively clear that these assumptions should yield better performance than those made in the above. Numerical results are collected in Appendix D as Tab. D.15 to Tab. D.22.

The optimal local deadlines τ_i can be established in a variety of ways, with different trade-offs between computational complexity and loss performance. The numerical examples presented in tables 7.2 through 7.4 consider the following approaches, starting with given M, ρ and nominal (that is, $M/M/1$) loss.

uni λ Through one-dimensional minimization[2], a single local deadline τ_u is established for all nodes, assuming that the traffic is the same as the source traffic throughout. (That is, we are making the homogeneous-traffic assumption once again.)

uni τ As in the previous case, a single uniform deadline τ_v is found for all nodes the system; however, the traffic intensity λ_i takes into account that packets have been dropped at nodes

[2] *Numerical Recipes* routine GOLDEN with a tolerance of 10^{-4}

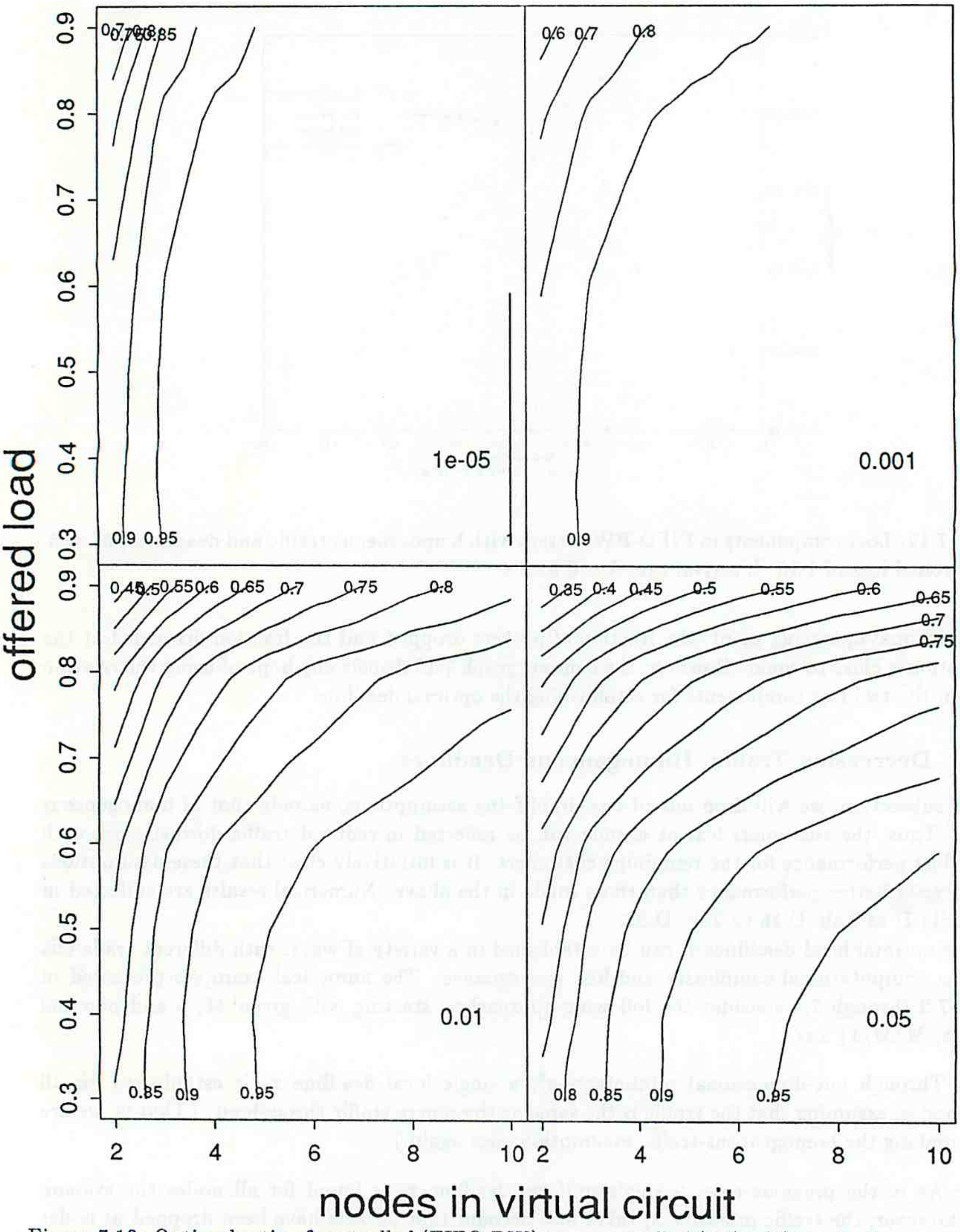

Figure 7.13: Optimal ratio of controlled (FIFO-BW) to uncontrolled (M/M/1) loss for homogeneous traffic and deadlines; nominal losses of $1 \cdot 10^{-5}$, 0.001, 0.01 and 0.05

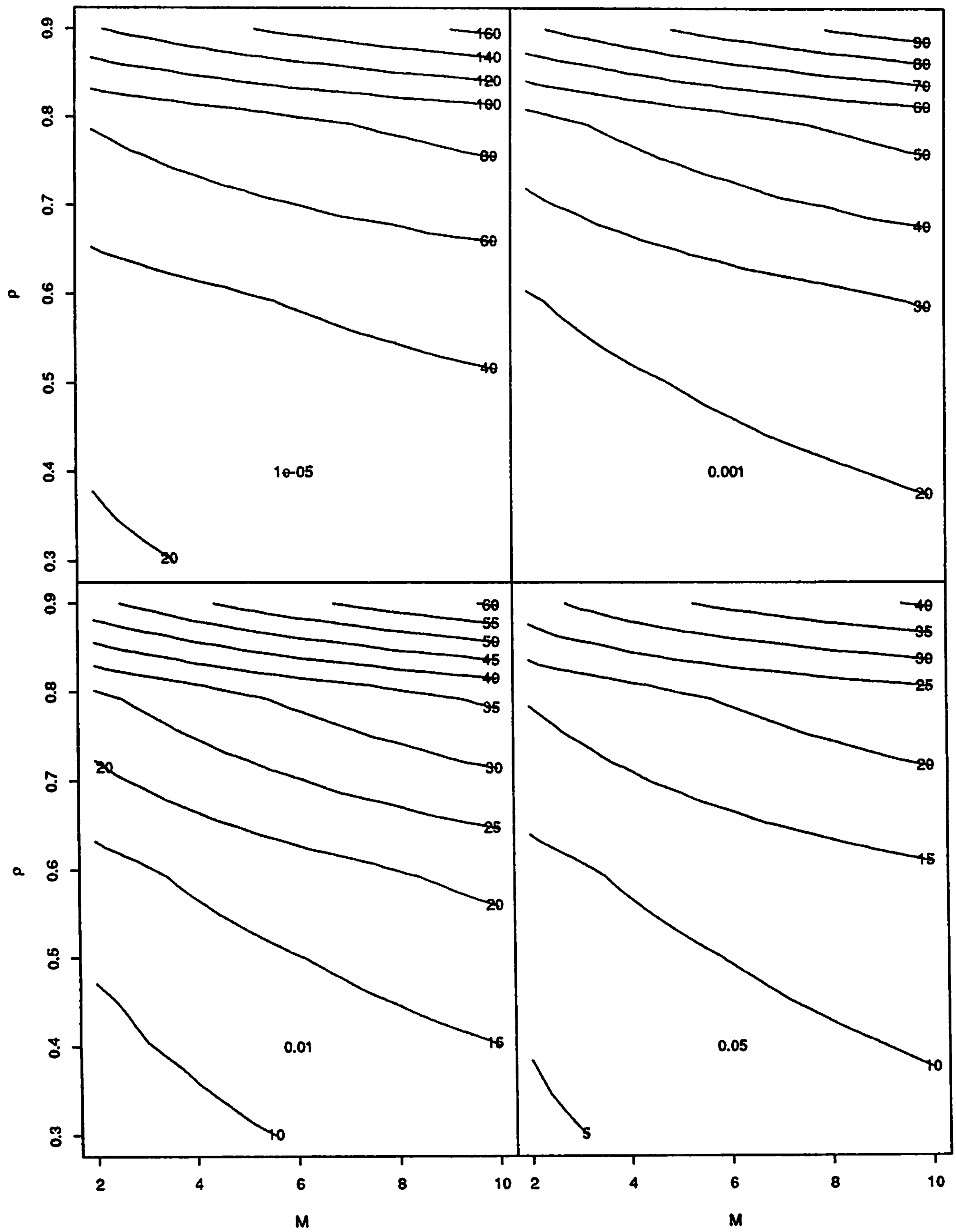

Figure 7.14: Optimal homogeneous deadlines for FIFO-BW system with homogeneous traffic; uncontrolled losses of $1{\cdot}10^{-5}$, 0.001, 0.01 and 0.05

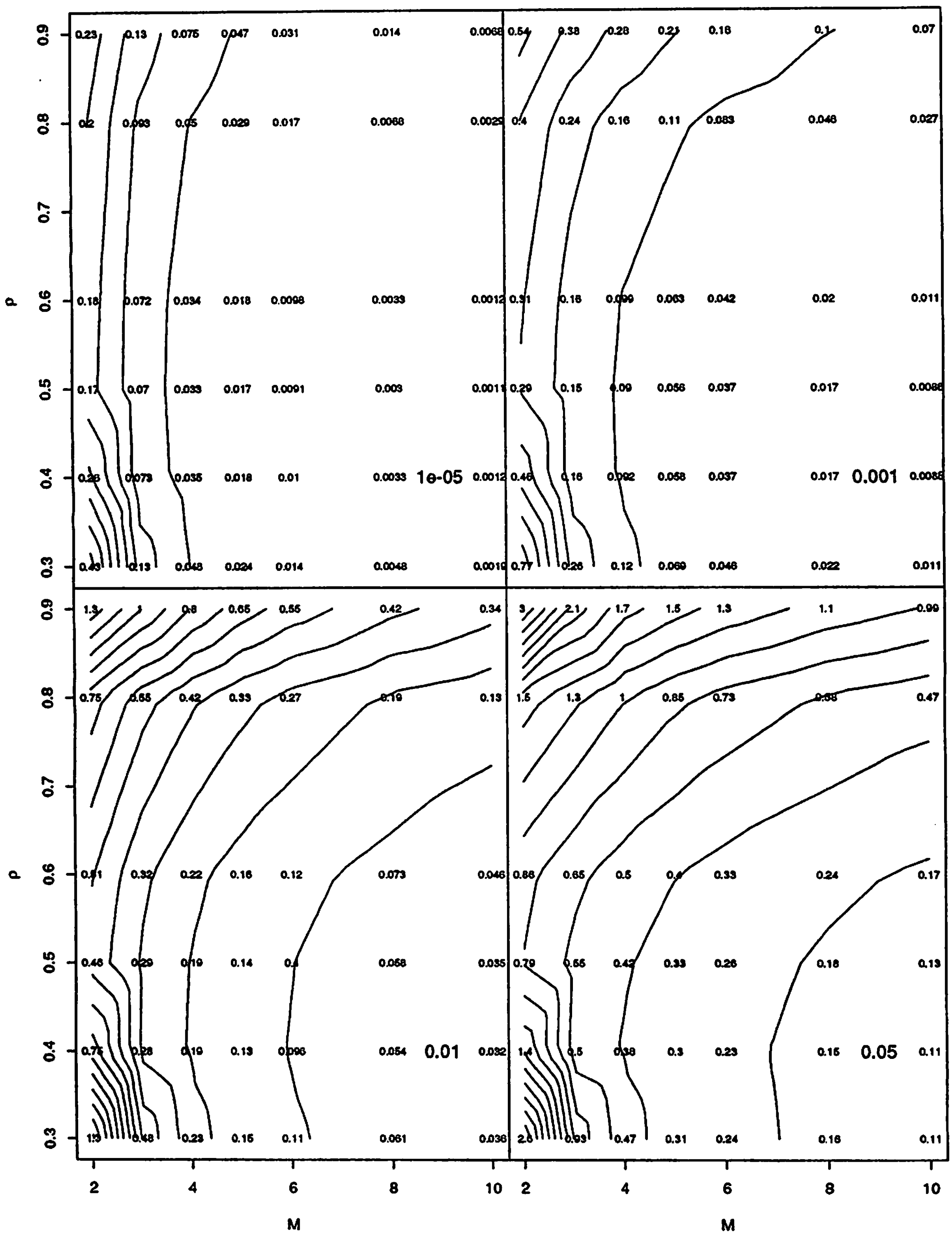

Figure 7.15: Optimal ratio of drop to late loss for uniform traffic and deadlines; uncontrolled losses of $1\cdot10^{-5}$, 0.001, 0.01 and 0.05

$j < i$. The closed-form expression derived in section 7.2.2 was implemented in PASCAL quadruple precision since a large number of terms spanning a range of more than 10^{14} needs to be added. Negative and positive terms are added separately, in order of magnitude, to combat loss of accuracy incurred by adding numbers of very different magnitude.

non-uni Via M-dimensional minimization (as described earlier), an M-tuple of local deadlines τ_i was found that minimizes the overall end-to-end loss, taking decreasing traffic along the VC into account.

bound Since the numerical accuracy of the previous results cannot be assumed without verification, a lower bound on the end-to-end loss was computed by making the traffic arriving at the last node under optimal non-uniform deadlines the uniform traffic throughout the virtual circuit. Thus, this system has to perform no worse than the previous case even for keeping the same (non-uniform) deadlines. Then, a uniform optimal deadline is computed for this system. It is clear that a system with uniform traffic has to have optimal local deadlines that are uniform; thus, this system can only improve upon the previous case.

For all optimizations, the optimal deadline can be bracketed between d/M and d, as discussed above, with a starting value halfway in-between.

Two multidimensional optimization routines were given the task to find the optimal set of τ_i minimizing the total loss $P[L]$. Surprisingly, the routine AMOEBA, drawn from [57, p. 280], required far fewer function evaluations than the IMSL routine DUMINF. The former uses a simplex method without gradient estimation, while the latter is based on the quasi-Newton method with finite-difference estimation of gradients.[3] The bounds on the domain of the distribution function are enforced by setting the function value to one for arguments outside the domain. As an indication of potential numerical difficulties, the traffic impacting the last node was computed under non-uniform deadlines. This traffic was declared to be the VC-wide load and the best loss under uniform traffic assumptions was recomputed, providing a lower bound on the loss probability of the original system. It was seen that summing all terms rather than using intermediate sums in the inner loop brought no improvement. Also, modifying the sorted summation to keep track of negative and positive terms separately did not help. Due to these numerical problems and excessive run times, the tables show gaps for higher values of M and ρ.

The tables show that in general the difference between the cases "uni τ" and "non-uni" is negligible at below 0.5% relative performance improvement. Only for the case $M = 5$, $\lambda = 0.9$ and an uncontrolled loss of 5% decreases the loss from 2.02 to 1.99%, a relative improvement of 1.4%. *Thus, in general the effort of finding and implementing node-dependent, non-homogeneous deadlines seems wasted.*

Fig. 7.16 through 7.19 show how the achievable loss depends on the length of the virtual circuit and the optimization assumptions. The solid lines depict the optimal loss for homogeneous traffic and deadlines, while the dotted lines show homogeneous (uniform) deadlines, but decreasing traffic. Note that the case of non-uniform deadlines and traffic is not shown on the graphs since the values do not differ visibly from those for uniform deadlines. The nominal losses of 5, 2 and 1% are indicated by horizontal lines, with curve pairs associated with the nominal losses in the obvious manner. The graphs show, as in the case of homogeneous traffic above, that in general performance

[3] For a system with $M = 4, \lambda = 0.8, p_{M/M/1} = 0.05$, the simplex method took 90 evaluations, while the quasi-Newton method required 305 function values.

gains are largest for short circuits with heavy traffic and high loss tolerance. The difference between the traffic assumptions is visible only for the case of 5% uncontrolled loss and increases slightly with the length of the virtual circuit. Thus, the additional computational complexity of accounting for packet loss is justified only for long VCs with high loss and traffic.

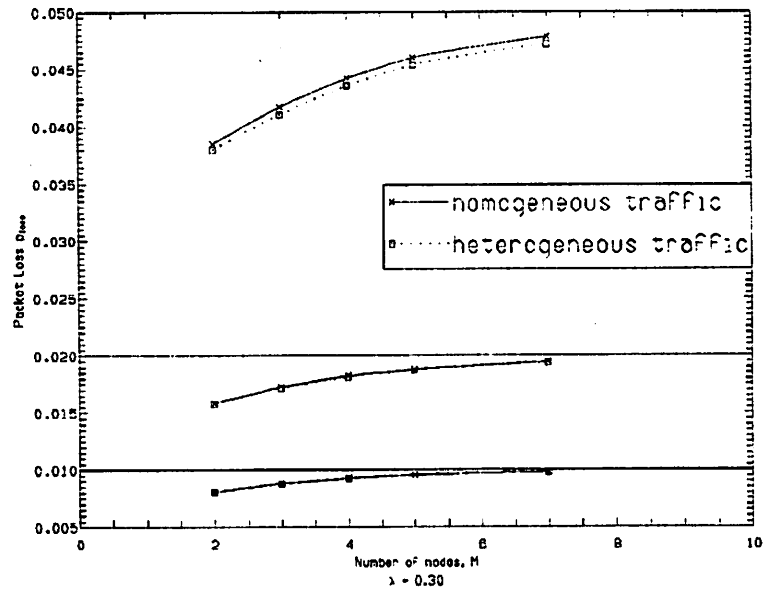

Figure 7.16: Total packet loss vs. number of nodes, for optimal uniform deadlines based on uniform or decreasing traffic, $\lambda = 0.30$

7.3.3 Congestion Control with Robust Local Deadlines

On-line adaptive congestion faces the hurdles of measurement and control parameter adaptation. Especially under high loads, accurate (i.e., low-variance) estimates of even first-order statistics may be hard to come by, for statistical as well as computational reasons[4]. Certainly, an on-line optimization algorithm, evaluating the optimal local deadline for the estimated traffic for each VC, imposes an unbearable computational burden on the node queue controller.

Given the computational constraints, we propose the following static control scheme. The scheme maintains a single, load-independent local deadline for the life time of a virtual circuit. At call setup, the end-to-end deadline (and, therefore, the playout delay buffer) is set so that the desired loss under non-overload conditions can be met. During the commit phase of the call setup, the local deadlines are established (for example by consulting a node-resident table), taking the length of the virtual circuit and its loss tolerance into account and making a tradeoff between overload protection and additional loss incurred under lighter loads (see discussion below). The value of τ computed is stored with the virtual circuit control block in the switching nodes. During

[4]For a load of 0.9, about 300 samples of the interarrival time are necessary to estimate the mean within 10% of the true mean with a confidence of 90%.

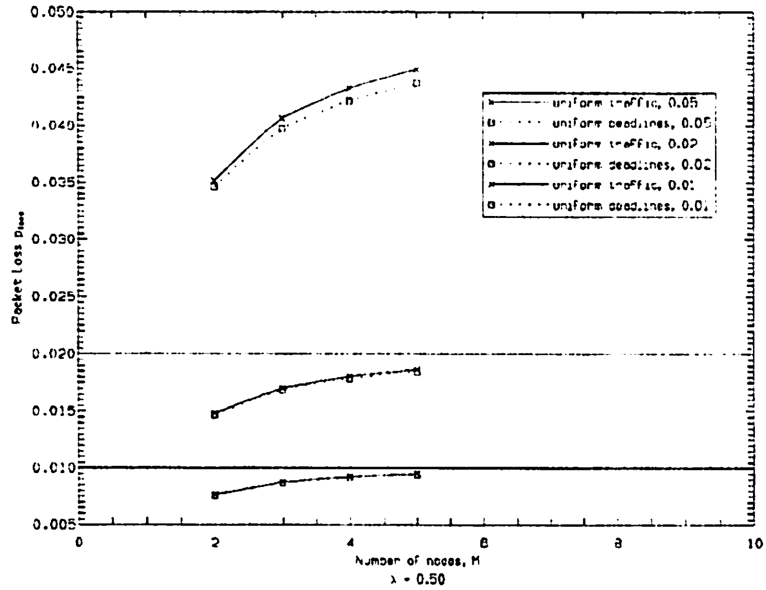

Figure 7.17: Total packet loss vs. number of nodes, for optimal uniform deadlines based on uniform or decreasing traffic, $\lambda = 0.50$

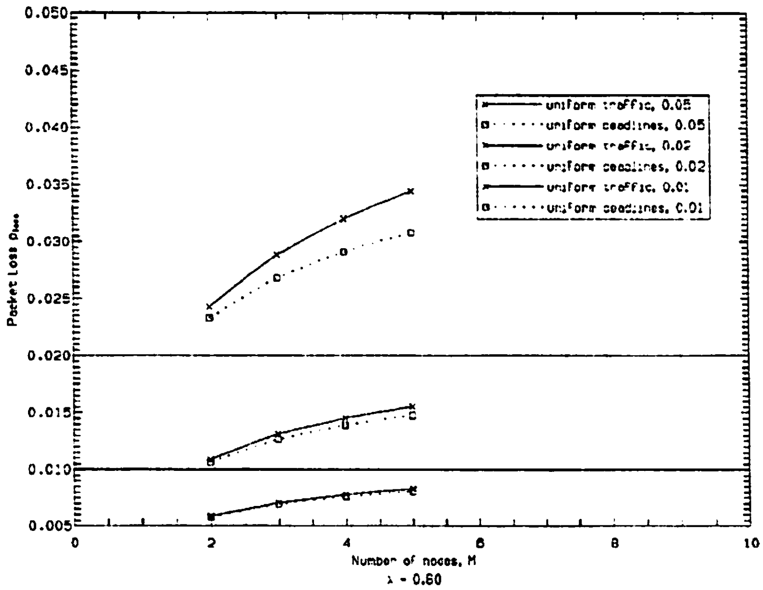

Figure 7.18: Total packet loss vs. number of nodes, for optimal uniform deadlines based on uniform or decreasing traffic, $\lambda = 0.80$

M	ρ	$M/M/1$	uni λ	uni τ	non-uni	bound	τ_u	τ_v	τ_1	τ_2	τ_3
2	0.30	5.0^{-2}	3.856^{-2}	3.803^{-2}	3.803^{-2}	3.750^{-2}	3.9	3.9	3.9	3.9	
		2.0^{-2}	1.582^{-2}	1.572^{-2}	1.572^{-2}	1.563^{-2}	5.4	5.4	5.4	5.4	
		1.0^{-2}	8.053^{-3}	8.026^{-3}	8.026^{-3}	8.000^{-3}	6.5	6.5	6.5	6.5	
		1.0^{-3}	8.428^{-4}	8.424^{-4}	8.425^{-4}	8.421^{-4}	10.1	10.1	10.1	10.1	
	0.50	5.0^{-2}	3.516^{-2}	3.463^{-2}	3.464^{-2}	3.409^{-2}	7.0	7.0	6.9	6.9	
		2.0^{-2}	1.479^{-2}	1.469^{-2}	1.469^{-2}	1.458^{-2}	9.1	9.1	9.1	9.1	
		1.0^{-2}	7.622^{-3}	7.595^{-3}	7.595^{-3}	7.567^{-3}	10.7	10.7	10.6	10.6	
		1.0^{-3}	8.169^{-4}	8.165^{-4}	8.166^{-4}	8.162^{-4}	15.8	15.8	15.8	15.8	
	0.80	5.0^{-2}	2.430^{-2}	2.330^{-2}	2.327^{-2}	2.212^{-2}	15.6	15.4	15.1	15.7	
		2.0^{-2}	1.087^{-2}	1.064^{-2}	1.064^{-2}	1.039^{-2}	20.5	20.3	20.2	20.6	
		1.0^{-2}	5.835^{-3}	5.764^{-3}	5.763^{-3}	5.690^{-3}	24.3	24.2	24.1	24.3	
		1.0^{-3}	6.827^{-4}	6.817^{-4}	6.818^{-4}	6.807^{-4}	37.1	37.0	37.1	37.2	
	0.90	5.0^{-2}	1.541^{-2}	1.449^{-2}	1.448^{-2}	1.346^{-2}	27.9	27.8	27.4	28.6	
		2.0^{-2}	7.401^{-3}	7.136^{-3}	7.131^{-3}	6.838^{-3}	36.5	36.4	35.9	36.9	
		1.0^{-2}	4.180^{-3}	4.087^{-3}	4.086^{-3}	3.987^{-3}	43.6	43.4	43.1	43.6	
		1.0^{-3}	5.511^{-4}	5.494^{-4}	5.494^{-4}	5.477^{-4}	68.4	68.3	68.4	68.6	
3	0.30	5.0^{-2}	4.173^{-2}	4.105^{-2}	4.105^{-2}	4.038^{-2}	4.9	4.9	4.9	4.9	4.9
		2.0^{-2}	1.725^{-2}	1.713^{-2}	1.713^{-2}	1.702^{-2}	6.5	6.5	6.5	6.5	6.5
		1.0^{-2}	8.794^{-3}	8.765^{-3}	8.765^{-3}	8.736^{-3}	7.7	7.7	7.7	7.7	7.7
		1.0^{-3}	9.181^{-4}	9.178^{-4}	9.178^{-4}	9.175^{-4}	11.6	11.6	11.6	11.6	11.6
	0.50	5.0^{-2}	4.062^{-2}	3.971^{-2}	3.968^{-2}	3.868^{-2}	7.9	7.8	7.6	7.8	7.9
		2.0^{-2}	1.701^{-2}	1.686^{-2}	1.686^{-2}	1.670^{-2}	10.2	10.1	10.0	10.3	10.3
		1.0^{-2}	8.729^{-3}	8.690^{-3}	8.690^{-3}	8.652^{-3}	11.9	11.9	11.9	11.9	11.9
		1.0^{-3}	9.200^{-4}	9.196^{-4}	9.198^{-4}	9.192^{-4}	17.4	17.4	17.6	17.4	17.6
	0.80	5.0^{-2}	2.886^{-2}	2.683^{-2}	2.674^{-2}	2.446^{-2}	17.1	16.8	16.1	16.8	17.5
		2.0^{-2}	1.310^{-2}	1.263^{-2}	1.262^{-2}	1.209^{-2}	22.2	21.9	21.4	21.9	22.5
		1.0^{-2}	7.056^{-3}	6.921^{-3}	6.918^{-3}	6.771^{-3}	26.3	26.0	25.8	26.0	26.5
		1.0^{-3}	8.184^{-4}	8.169^{-4}	8.169^{-4}	8.153^{-4}	40.1	40.0	40.0	40.1	39.9
	0.90	5.0^{-2}	1.901^{-2}	1.705^{-2}	1.698^{-2}	1.496^{-2}	30.7	30.5	29.4	30.9	31.8
		2.0^{-2}	9.274^{-3}	8.692^{-3}	8.674^{-3}	8.052^{-3}	39.2	38.8	37.7	39.2	39.9
		1.0^{-2}	5.286^{-3}	5.079^{-3}	5.074^{-3}	4.856^{-3}	46.3	45.9	45.1	46.2	46.7
		1.0^{-3}	6.999^{-4}	6.965^{-4}	6.966^{-4}	6.931^{-4}	72.5	72.4	72.0	72.7	73.1

Table 7.2: Losses for different optimization strategies, $M = 2 \dots 3$

M	ρ	$M/M/1$	uni λ	uni τ	non-uni	bound	τ_u	τ_v	τ_1	τ_2	τ_3	τ_4	τ_5
4	0.30	5.0^{-2}	4.423^{-2}	4.357^{-2}	4.358^{-2}	4.290^{-2}	5.8	5.8	5.8	5.8	5.8	5.8	
		2.0^{-2}	1.822^{-2}	1.812^{-2}	1.813^{-2}	1.802^{-2}	7.6	7.6	7.5	7.5	7.6	7.6	
		1.0^{-2}	9.262^{-3}	9.238^{-3}	9.239^{-3}	9.214^{-3}	8.8	8.8	8.8	8.8	8.8	8.8	
		1.0^{-3}	9.567^{-4}	9.565^{-4}	9.567^{-4}	9.563^{-4}	12.9	12.9	12.9	12.8	12.9	12.8	
	0.50	5.0^{-2}	4.329^{-2}	4.216^{-2}	4.212^{-2}	4.087^{-2}	8.7	8.5	8.3	8.5	8.6	8.7	
		2.0^{-2}	1.803^{-2}	1.786^{-2}	1.786^{-2}	1.766^{-2}	11.2	11.0	10.8	11.1	11.0	11.3	
		1.0^{-2}	9.212^{-3}	9.171^{-3}	9.171^{-3}	9.125^{-3}	13.0	12.9	12.9	12.7	12.9	13.0	
		1.0^{-3}	9.582^{-4}	9.578^{-4}	9.580^{-4}	9.575^{-4}	18.9	18.9	18.6	18.8	18.8	18.7	
	0.80	5.0^{-2}	3.203^{-2}	2.912^{-2}	2.895^{-2}	2.572^{-2}	18.4	18.0	16.8	17.8	18.6	19.2	
		2.0^{-2}	1.452^{-2}	1.388^{-2}	1.385^{-2}	1.312^{-2}	23.8	23.3	22.5	23.3	23.5	24.2	
		1.0^{-2}	7.794^{-3}	7.616^{-3}	7.611^{-3}	7.401^{-3}	28.1	27.7	27.0	27.5	27.8	28.6	
		1.0^{-3}	8.864^{-4}	8.848^{-4}	8.849^{-4}	8.832^{-4}	42.8	42.7	42.6	42.8	43.0	43.2	
	0.90	5.0^{-2}	2.194^{-2}	1.891^{-2}	1.874^{-2}	1.557^{-2}	32.9	32.5	30.2	32.0	33.5	35.2	
		2.0^{-2}	1.070^{-2}	9.813^{-3}	9.773^{-3}	8.831^{-3}	41.7	41.1	39.3	40.9	42.1	43.2	
		1.0^{-2}	6.075^{-3}	5.767^{-3}	5.756^{-3}	5.427^{-3}	49.0	48.3	47.2	48.2	48.8	50.1	
		1.0^{-3}	7.877^{-4}	7.834^{-4}	7.835^{-4}	7.791^{-4}	76.6	76.3	75.7	76.3	76.9	76.4	
5	0.30	5.0^{-2}	4.600^{-2}	4.535^{-2}	4.534^{-2}	4.464^{-2}	6.5	6.4	6.3	6.3	6.4	6.4	6.6
		2.0^{-2}	1.885^{-2}	1.876^{-2}	1.876^{-2}	1.866^{-2}	8.3	8.3	8.2	8.2	8.3	8.2	8.5
		1.0^{-2}	9.542^{-3}	9.522^{-3}	9.522^{-3}	9.499^{-3}	9.6	9.6	9.6	9.6	9.6	9.5	9.7
		1.0^{-3}	9.764^{-4}	9.762^{-4}	9.763^{-4}	9.761^{-4}	13.9	13.9	13.8	13.9	14.0	13.9	14.0
	0.50	5.0^{-2}	4.495^{-2}	4.372^{-2}	4.365^{-2}	4.222^{-2}	9.5	9.1	8.8	9.0	9.2	9.4	9.6
		2.0^{-2}	1.863^{-2}	1.845^{-2}	1.845^{-2}	1.825^{-2}	12.1	11.8	11.5	11.8	11.8	12.1	12.0
		1.0^{-2}	9.478^{-3}	9.440^{-3}	9.440^{-3}	9.400^{-3}	14.0	13.9	13.6	13.9	13.9	14.1	14.1
		1.0^{-3}	9.762^{-4}	9.759^{-4}	9.761^{-4}	9.757^{-4}	20.2	20.1	20.1	20.4	20.5	20.2	20.2
	0.80	5.0^{-2}	3.447^{-2}	3.080^{-2}	3.051^{-2}	2.630^{-2}	19.6	19.0	17.3	18.3	19.2	19.9	20.7
		2.0^{-2}	1.555^{-2}	1.477^{-2}	1.472^{-2}	1.376^{-2}	25.2	24.5	23.2	23.8	24.5	25.1	25.8
		1.0^{-2}	8.296^{-3}	8.091^{-3}	8.083^{-3}	7.844^{-3}	29.8	29.2	28.5	28.6	29.1	29.5	30.1
		1.0^{-3}	9.253^{-4}	9.238^{-4}	9.239^{-4}	9.223^{-4}	45.2	45.1	45.0	45.0	45.5	45.3	46.0
	0.90	5.0^{-2}	2.425^{-2}	2.019^{-2}	1.991^{-2}	1.570^{-2}	34.6	34.0	30.7	32.6	34.4	36.3	38.1
		2.0^{-2}	1.182^{-2}	1.065^{-2}	1.057^{-2}	9.265^{-3}	43.8	42.8	40.3	41.6	43.0	44.3	45.6
		1.0^{-2}	6.680^{-3}	6.283^{-3}	6.263^{-3}	5.814^{-3}	51.4	50.4	48.5	49.4	50.4	51.4	52.3
		1.0^{-3}	8.450^{-4}	8.403^{-4}	8.403^{-4}	8.353^{-4}	80.3	79.9	79.2	79.6	79.6	80.3	80.0

Table 7.3: Optimal losses for FIFO-BW, $M = 4 \ldots 5$

M	ρ	$M/M/1$	uni λ	uni τ	non-uni	bound	τ_u	τ_v	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7
7	0.3	5^{-2}	4.78^{-2}	4.72^{-2}	4.72^{-2}	4.64^{-2}	7.5	7.2	7.1	7.1	7.2	7.2	7.3	7.4	7.6
		2^{-2}	1.94^{-2}	1.94^{-2}	1.94^{-2}	1.93^{-2}	9.4	9.3	9.2	9.3	9.3	9.5	9.4	9.4	9.5
		1^{-2}	9.79^{-3}	9.78^{-3}	9.78^{-3}	9.76^{-3}	10.9	10.8	11.0	10.8	10.9	10.8	10.9	11.0	10.8
	0.5	5^{-2}	4.69^{-2}	4.56^{-2}	4.55^{-2}	4.37^{-2}	10.7	10.1	9.4	9.8	9.9	9.9	10.4	10.5	
		2^{-2}	1.93^{-2}	1.91^{-2}	1.91^{-2}	1.89^{-2}	13.6	13.2	12.8	12.9	13.1	13.4	13.3	13.6	13.8
		1^{-2}	9.74^{-3}	9.71^{-3}	9.72^{-3}	9.68^{-3}	15.7	15.5	15.4	15.2	15.2	15.5	15.9	15.9	15.7
		1^{-3}	9.91^{-4}	1.00^{-3}											
	0.8	5^{-2}	3.80^{-2}	3.30^{-2}											
	0.9		2.78^{-2}	2.18^{-2}											
10	0.3	5^{-2}	4.89^{-2}												
	0.5		4.84^{-2}												

Table 7.4: Optimal losses for FIFO-BW, $M = 7 \ldots 10$

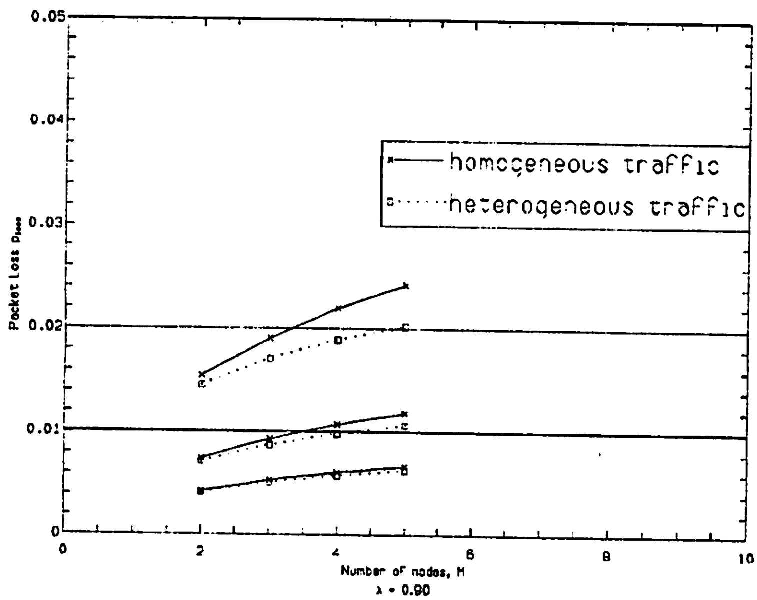

Figure 7.19: Total packet loss vs. number of nodes, for optimal uniform deadlines based on uniform or decreasing traffic, $\lambda = 0.90$

the VC data transmission phase, the queue controller determines the applicable τ, measured in transmission units, for each packet from a look-up table and compares it to the current number of transmission units queued, including those of the packet in transmission. (Here we assume a fixed-rate channel.) The value for τ could also be attached to the packet on entering the switch, just as the output port identifier. However, since, unlike the port identifier, τ is not needed until the packet reaches the output buffer, switch complexity and internal speed-up would probably be lower if τ was determined at the output port. In either case, these operations are comparable to the address lookup in complexity and readily implementable in hardware at packet rate.

We will elaborate on the determination of the per-call value of τ by means of an example. Suppose that a 5-node connection can tolerate a loss of 1% without noticeable grade-of-service impairment. A call is allowed into the network only if, among other constraints, the average load does not exceed $\rho = 0.8$ at the time of call setup. In our case, the playout buffer is set up for an end-to-end deadline of $d = 52.7$.

Depending on the criteria used, there are a numerous choices for a "good" local deadline τ. In table D.23, we summarize losses for a five-node system where the source traffic load varies while the end-to-end and local deadlines are held constant. The column labels reflect the system variation evaluated:

M/M/1 Uncontrolled (nominal) loss of tandem-M/M/1 system

uni λ Traffic reduction towards the end of the virtual circuit is not taken into account.

uni τ Traffic at each node takes dropped packets into consideration.

red. net. The traffic at the end of the network for the previous case is declared to be the traffic throughout the network, implying a homogeneous system where every node applies the same controls. Clearly, this is only a very rough approximation to the network equilibrium traffic.

The effect of several choices for τ on the end-to-end loss is evident from Fig. 7.21. A first attempt for τ uses the value optimal for a load of 0.9, yielding $\tau = 18.3$. Assume that during the call, the network load rises to 0.9. If no queue control were applied, the losses would reach an intolerable 32%. With the queue control and $\tau = 18.3$, losses are reduced to 7%, which an interpolation method might be able to cope with. However, this deadline is unduly restrictive for lower loads. For example, at $\rho = 0.8$, losses are double those incurred without control. Tightening the deadline to $\tau = 15$ does not improve overload performance, but further hampers GOS at $\rho = 0.8$, pushing losses to 3.5%. A more conservative choice of $\tau = 23.3$, ensures that losses at design load (0.8) and below never exceed 1%, while still limiting overload losses to 9.2%. If load should rise momentarily to $\rho = 0.95$, the overload mechanism will cut losses from an uncontrolled 83% to 20%. As mentioned earlier, actual network performance will most likely be better as all VCs can be assumed to apply similar control mechanisms.

7.3.4 Bottlenecks

In this section, we examine the sensitivity of the mechanism and the analysis to "bottlenecks", that is, virtual circuits where one node (here, arbitrarily the middle one) is more or less congested than the others. Again, the end-to-end deadline d is fixed so that the VC with no control and no bottleneck incurs a fixed loss (case 1 in the table). Then, the following variations of determining τ are analyzed, with numbers corresponding to the "case n" column:

1. M/M/1 with no control and no bottleneck; the end-to-end deadline is based on this case;
2. M/M/1 with no control, but bottleneck as specified;
3. FIFO-BW with *homogeneous* deadlines τ_1 optimized for system *without* bottleneck;
4. FIFO-BW with *homogeneous* deadlines τ_2 optimized for system *with* bottleneck;
5. FIFO-BW with *non-homogeneous* deadlines optimized for system *with* bottleneck;
6. FIFO-BW using τ_1 as *homogeneous* local deadline, but in sytem *with* bottleneck.

The last option corresponds to the case where a bottleneck develops after a deadline for a homogeneous system has been determined (say, at call setup).

Table 7.5 shows that in all cases, even mismatched local deadlines outperform the uncontrolled system. Using the deadline derived for the system without bottleneck yields satisfactory performance even when one node is close to saturation. For the five-node example, loss increases only from 7.2% to 8.0% by using a deadline not optimized for the system with the bottleneck.

7.3.5 Goodput

Tab. D.24 and Fig. 7.22 compare the goodput, defined as $\lambda(1 - P[L])$, of the uncontrolled and controlled system. Beyond a certain load, $\rho = 0.83$ in the example, the goodput for the uncontrolled

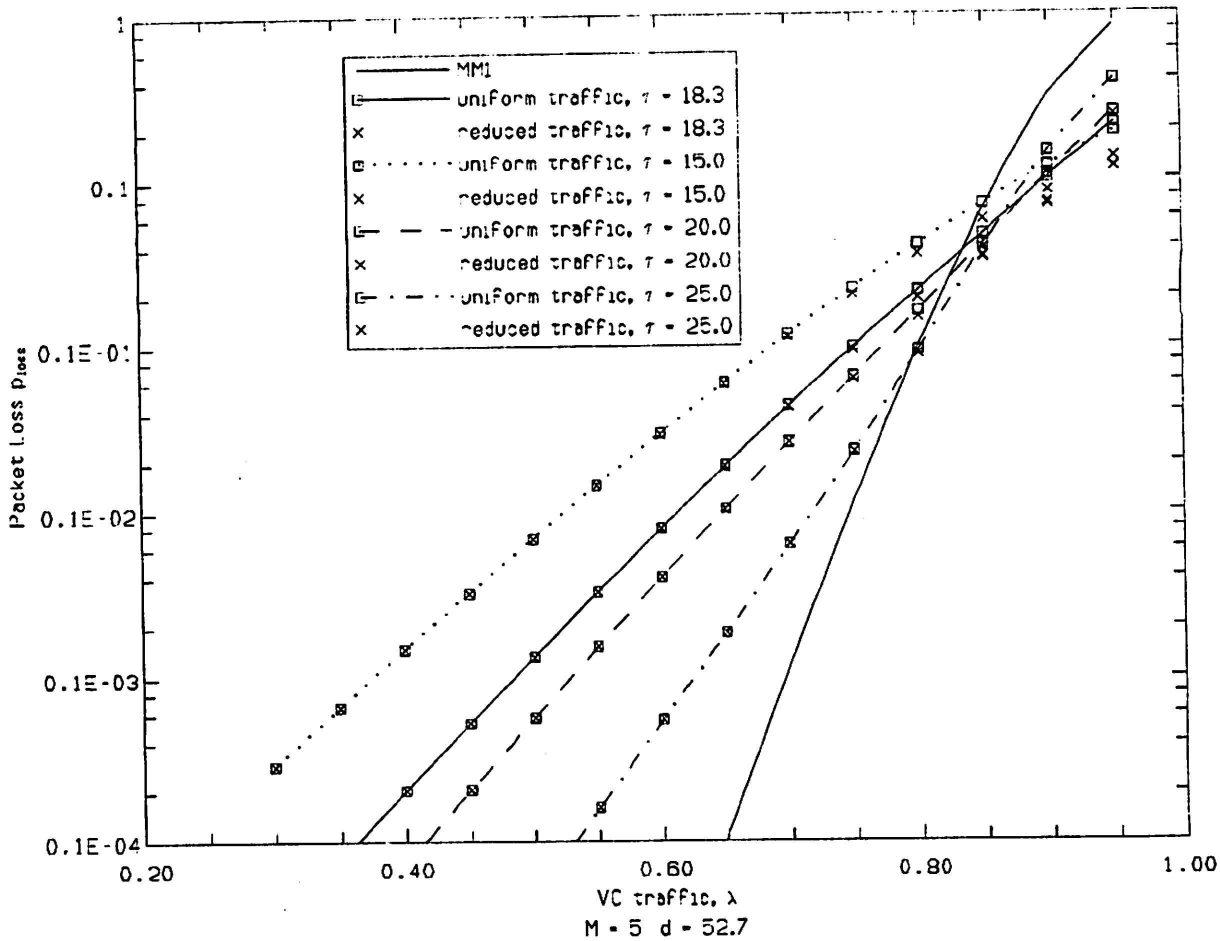

Figure 7.20: Comparison of overload performance of FIFO-BW to that of uncontrolled system

M	ρ	b	ρ_b	case 1	case 2	case 3	case 4	case 5	case 6
3	0.80	2	0.60	$5.000 \cdot 10^{-2}$	$2.303 \cdot 10^{-2}$	$2.886 \cdot 10^{-2}$	$1.706 \cdot 10^{-2}$	$1.701 \cdot 10^{-2}$	$1.388 \cdot 10^{-2}$
			0.70		$2.943 \cdot 10^{-2}$		$1.706 \cdot 10^{-2}$	$1.701 \cdot 10^{-2}$	$1.747 \cdot 10^{-2}$
			0.85		$8.027 \cdot 10^{-2}$		$3.720 \cdot 10^{-2}$	$3.717 \cdot 10^{-2}$	$3.779 \cdot 10^{-2}$
			0.90		$1.559 \cdot 10^{-1}$		$5.095 \cdot 10^{-2}$	$5.091 \cdot 10^{-2}$	$5.366 \cdot 10^{-2}$
			0.95		$3.659 \cdot 10^{-1}$		$6.919 \cdot 10^{-2}$	$6.915 \cdot 10^{-2}$	$7.655 \cdot 10^{-2}$
5	0.80	3	0.60	$5.000 \cdot 10^{-2}$	$2.881 \cdot 10^{-2}$	$3.447 \cdot 10^{-2}$	$1.900 \cdot 10^{-2}$	$1.829 \cdot 10^{-2}$	$1.939 \cdot 10^{-2}$
			0.70		$3.428 \cdot 10^{-2}$		$2.257 \cdot 10^{-2}$	$2.238 \cdot 10^{-2}$	$2.268 \cdot 10^{-2}$
			0.85		$7.233 \cdot 10^{-2}$		$4.038 \cdot 10^{-2}$	$4.020 \cdot 10^{-2}$	$4.117 \cdot 10^{-2}$
			0.90		$1.306 \cdot 10^{-1}$		$5.364 \cdot 10^{-2}$	$5.339 \cdot 10^{-2}$	$5.662 \cdot 10^{-2}$
			0.95		$3.140 \cdot 10^{-1}$		$7.200 \cdot 10^{-2}$	$7.170 \cdot 10^{-2}$	$7.996 \cdot 10^{-2}$

Table 7.5: Effect of bottleneck of M-stage VC with traffic ρ_b at node b on loss performance of FIFO-BW; cases explained in text

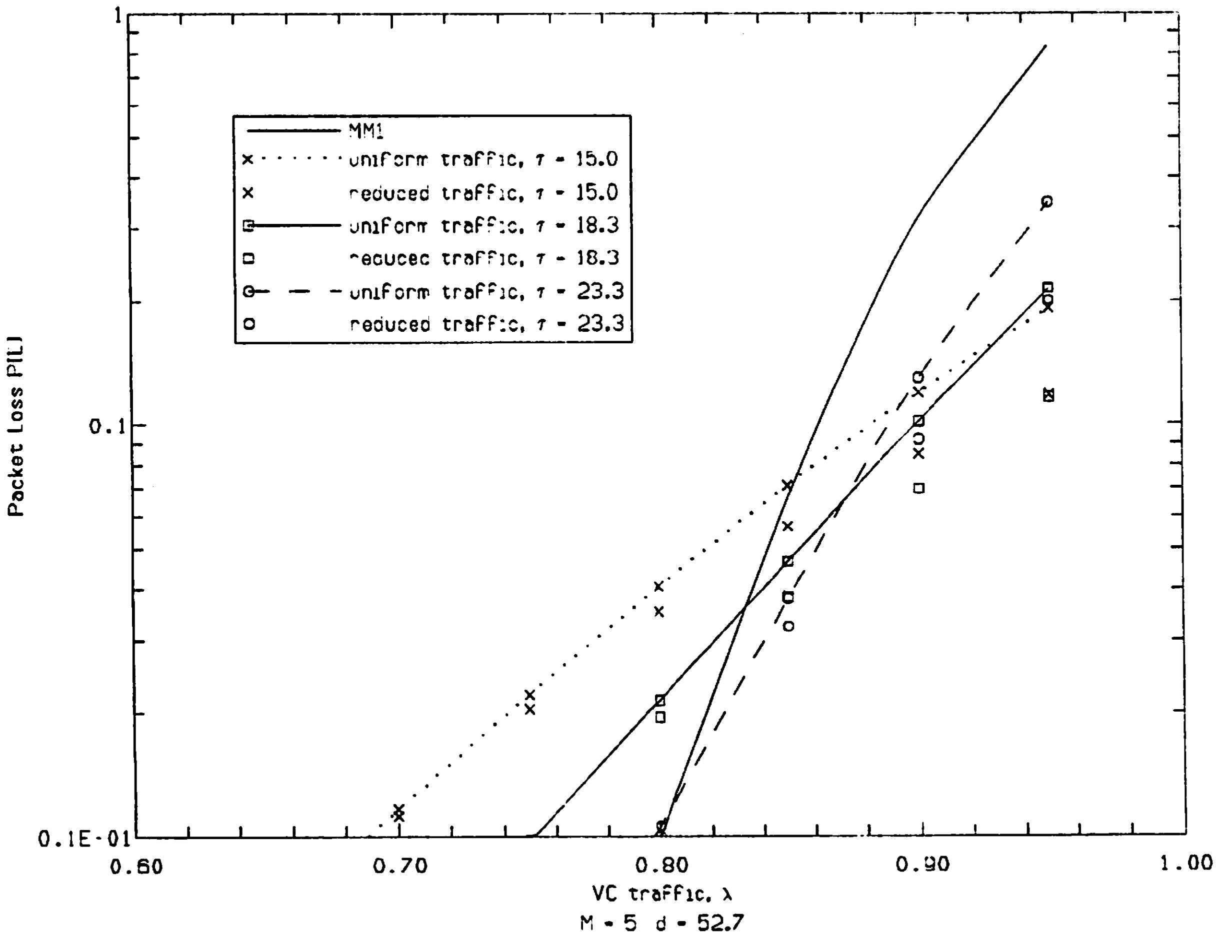

Figure 7.21: Comparison of overload performance of FIFO-BW to that of uncontrolled system; overload region

system actually decreases with increasing load. Optimal random discarding as proposed in [80] can hold the goodput at the peak value even under overload, as indicated by the horizontal line in the figure. It throttles the input traffic to the point of optimal goodput by randomly discarding packets at the source. Optimal random discarding requires, however, on-line traffic or gradient estimation ($\partial P[D]/\partial\lambda$) to determine the fraction of packets to be discarded. Note also that if we allow the deadline to be adjusted on-line according to the current traffic, FIFO-BW offers a higher goodput than optimal random discarding.

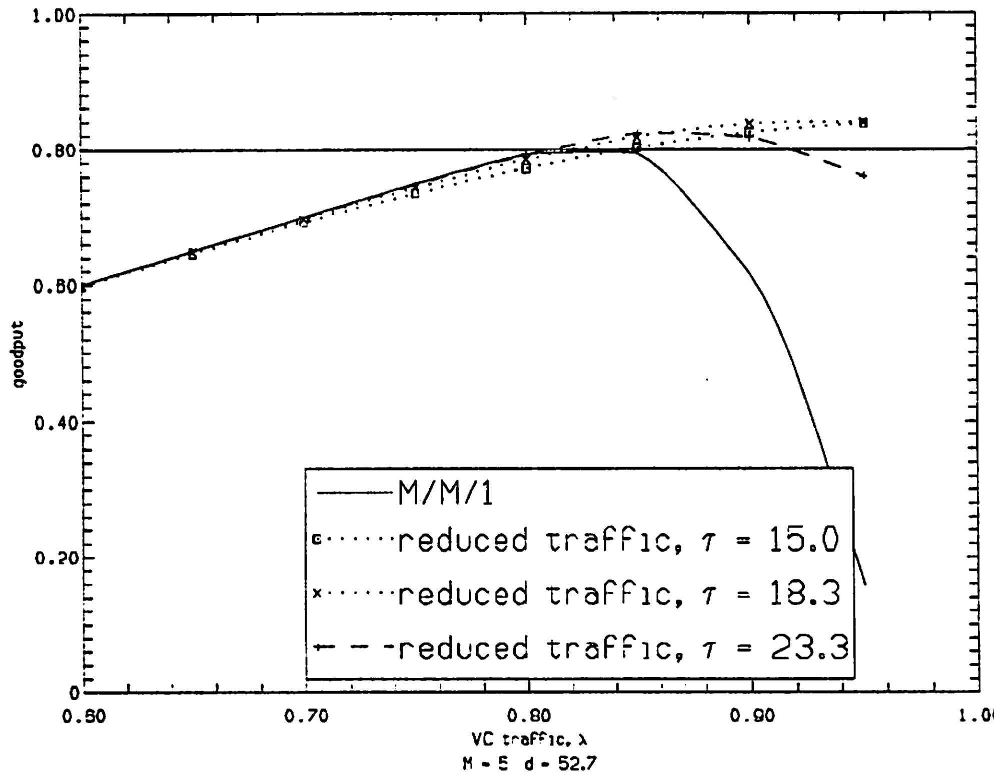

Figure 7.22: Comparison of goodput: tandem-M/M/1 vs. FIFO-BW with various local deadlines, $M = 5$, $\mu = 1$

7.4 Simulation

A set of simulation experiments was performed to validate the preceding analysis and investigate the effect of the simplifying modeling assumptions. The simulation maintains the Kleinrock assumption, namely that packets draw a new service time at each node visited, but approaches reality by making no assumptions on the input traffic of interior nodes. (It would be interesting to study the effect of dropping the Kleinrock assumption. To avoid pipelining, packets would have to take a random path through the network.)

In general, the simulation confirms that the analysis can serve as a usable performance predictor.

In general, the analysis overestimates the actual system losses, in all its components. Since the aberration from a product-form tandem network becomes less as the number of dropped packets decreases, the analysis tends to track the analysis more closely as τ increases. Fortunately, the value of the local deadline yielding minimal losses predicted by numerical analysis agrees closely with the one observed in the simulation experiements, allowing numerical computation of the best local deadline. Since mean values are typically less sensitive to model simplifications, it does not surprise to see a very close correlation between analysis and simulation for that particular measure.

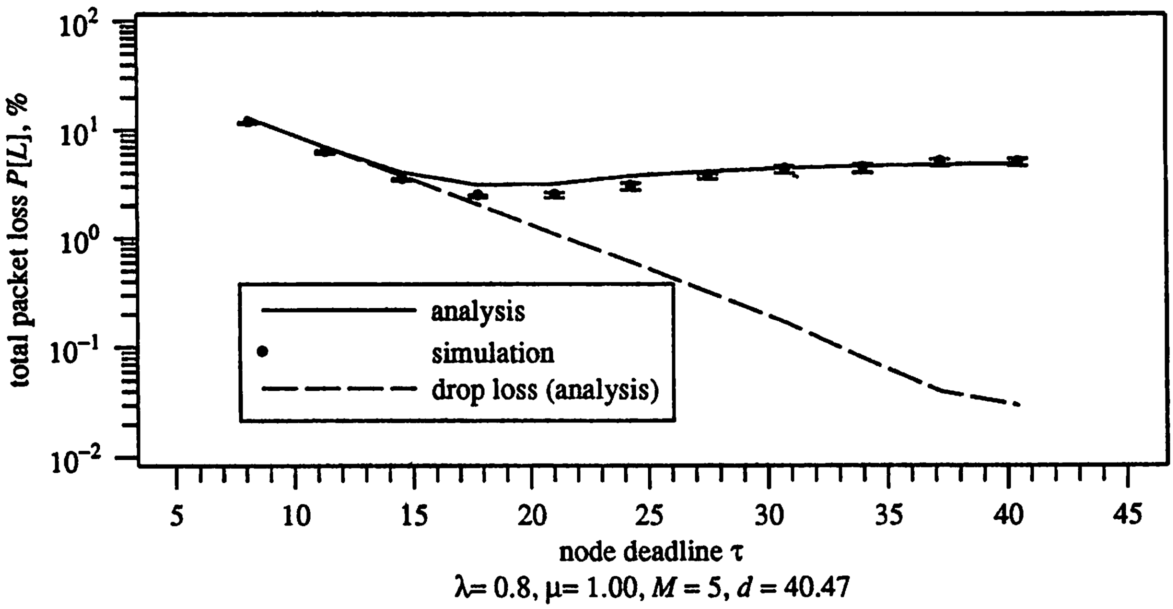

Figure 7.23: Total loss; analysis and simulation

7.5 Interloss Statistics

A first attempt was made to characterize the interloss periods. Their first moments are tabulated in Tab. 7.6. To gauge the correlation among successive interloss periods, the one-step correlation coefficient appeared to be a good first indicator. The von Neumann statistic

$$q = \frac{\sum_{i=1}^{n-1}(x_i - x_{i+1})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

reflects the first-order autocorrelation ρ_1 [81, p. 67]. If the x_i are uncorrelated, $E[q] = 2$, regardless of their distribution. If the x_i are correlated with $\rho_1 \neq 0$ and normally distributed, then $E[q] = 2 - 2/n - 2E[\rho_1]$.

For $M = 5$, $\lambda = 0.80$, $d = 52.67$, $\tau = 14.75$, 3117 losses were tabulated, with a mean of 40.14 and a standard deviation of 105.18. Here, $q = 1.954$, $\hat{\rho}_1 = 0.02$. For a sample of 3200 uniformly distributed random numbers, we obtained $q = 1.985$. Thus, any correlation between interloss times seems to be very slight and within the range of statistical uncertainty.

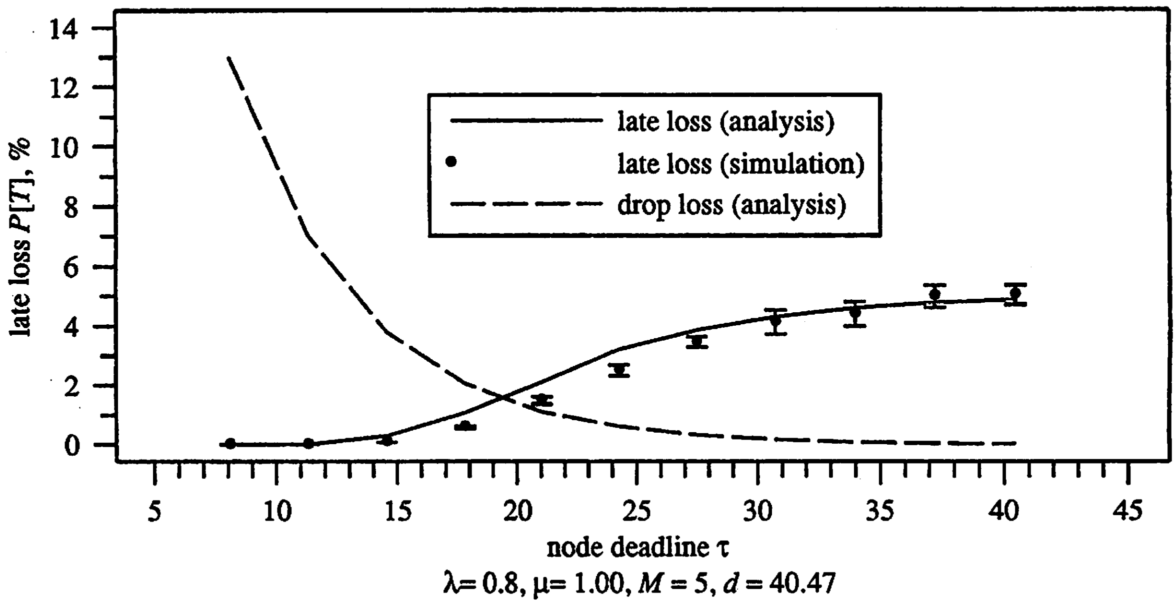

Figure 7.24: Late loss; analysis and simulation

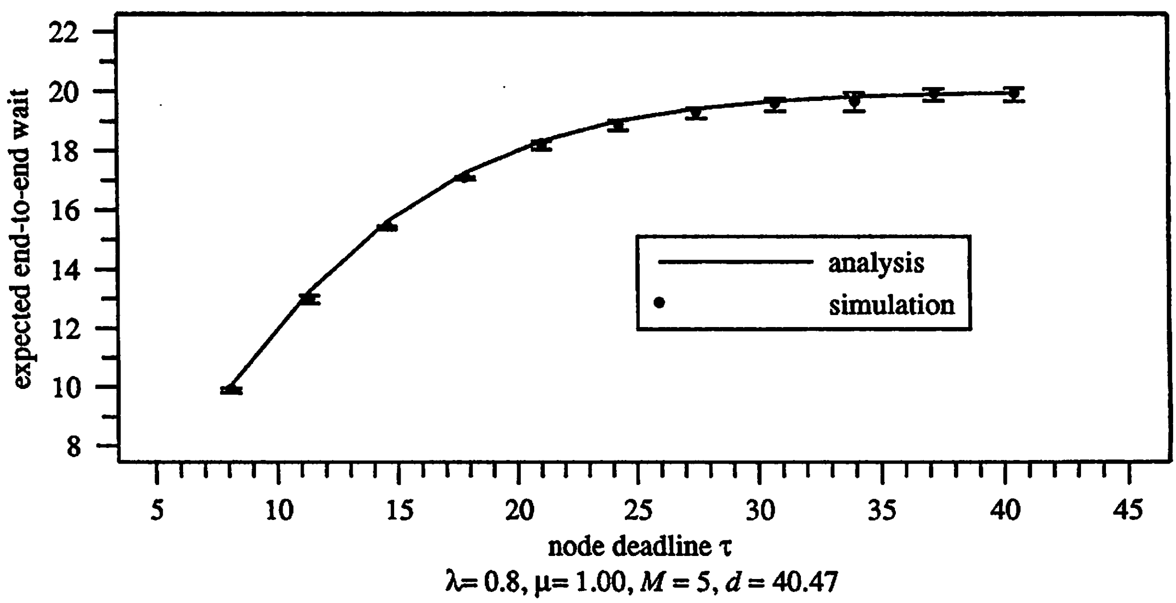

Figure 7.25: Expected end-to-end waiting time; analysis and simulation

M	ρ	d	τ	samples	average	standard deviation	coefficient of variation	3rd moment
5	0.80	40.47	8.09	58172	10.76	21.59	2.01	$5.532 \cdot 10^4$
			11.33	31090	20.13	47.12	2.34	$5.858 \cdot 10^5$
			14.57	132374	35.85	93.54	2.61	$4.589 \cdot 10^6$
			17.81	40635	51.94	164.80	3.17	$2.724 \cdot 10^7$
			21.04	41384	50.99	219.10	4.30	$8.021 \cdot 10^7$
			24.28	33422	42.11	246.80	5.86	$1.497 \cdot 10^8$
			27.52	39947	35.18	243.10	6.91	$1.585 \cdot 10^8$
			30.75	47921	29.33	224.40	7.65	$1.440 \cdot 10^8$
			33.99	33514	27.99	222.90	7.97	$1.422 \cdot 10^8$
			37.23	54282	25.93	216.20	8.34	$1.482 \cdot 10^8$
			40.47	55439	25.38	207.90	8.19	$1.219 \cdot 10^8$
			∞	53136	26.45	219.10	8.28	$1.443 \cdot 10^8$

Table 7.6: Interloss times (simulation)

Chapter 8

Fixed-Length Packets

While we have so far considered queueing systems in continuous time, we will now shift our focus to those systems where arrivals and service take place at discrete time intervals. We are motivated by slotted multiplexing and switching systems with fixed packet sizes, such as the proposed ATM-based networks [82]. While the analysis techniques differ, the gain achievable by selective discarding is of the same magnitude as in the continuous-time case. Also, the relationship of loss vs. local deadline appears to be similar.

8.1 System Description

The basic assumptions for the system remain unchanged from the continuous-time cases investigated above. Again, queues are coupled only through the traffic rates of the VC under study. Here, each of the M nodes in a VC is modeled as an independent $D^{[X]}/D/1/K$ queue, that is, batch arrivals with size X coming from a Poisson distribution, a deterministic single server and a finite system capacity K. (Thus, K takes the role that τ played previously.)

The number of arrivals per slot is assumed to follow a Poisson distribution since in many packet networks, switches receive packets from a large number of independent external sources and other switches. For example, Hluchyj and Karol [83] show the Poisson assumption models a single queue with thirty or more Bernoulli input sources quite accurately. For this case, the losses for Bernoulli sources approach the Poisson case asymptotically from below as the number of sources approaches infinity.

It should be noted, however, that for certain bursty traffic types, such as remote paging or graphics buffer transmission, the Bernoulli source model may not be adequate.

In the terminology of Hunter [84], we describe an early arrival system, where arrivals occur at the beginning of a time slot and departures at the end[1]. In a communication context, this corresponds to the situation where a packet is completely buffered, i.e., converted from bit- or byte-serial format to packet-parallel format, before being transmitted again. For cut-through switching, other definitions may be more appropriate.

We define q_k as the system occupancy pmf seen by the first customer in an arriving batch. For general batch size probability mass function (pmf) a_k, the q_k's are described by the following

[1]Thus, for a system of size K, an arriving batch can see at most $K-1$ customers. Also, at least one customer of each batch always joins the system.

recursive equations [83]

$$
\begin{aligned}
q_1 &= \frac{q_0}{a_0}(1 - a_0 - a_1) \\
q_n &= \frac{1}{a_0}\left[q_{n-1} - \sum_{k=1}^{n} a_k q_{n-k}\right], \; 2 \le n < K \\
q_0 &= \left[1 + \sum_{n=1}^{K-1} q_n/q_0\right]^{-1}
\end{aligned}
$$

The probability $P[J]$ that a packet joins the queue is given by

$$P[J] = \frac{1 - q_0 a_0}{\lambda},$$

since $1 - a_0 q_0$ is the normalized throughput. Closed-form expressions for q_k can be written down for some batch distributions, but they are generally numerically unstable.

We are interested not only in $P[J]$, but also the distribution of delay. Let us focus on a random arriving customer. Since a random customer is more likely to enter the system as part of a large batch than as part of a small one, we must take renewal effects into account in computing waiting time pmf's for a random customer. Let a_k be the probability that a batch consists of k customers, $p_I(k)$ the probability that a randomly chosen customer arrives as part of a batch of size k and $p_P(k)$ that a randomly chosen customer is the kth member ($k = 1, 2, \ldots$) of its batch. Then [85],

$$
\begin{aligned}
p_I(k) &= \frac{k a_k}{E[A]} \\
p_P(k) &= \frac{1}{E[A]} \sum_{j=k}^{\infty} a_j = \frac{1}{E[A]}\left(1 - \sum_{j=0}^{k-1} a_k\right),
\end{aligned}
$$

where $E[A]$ is the expected batch size. The waiting time pmf $p_W(k)$ seen a by randomly chosen customer can now be computed readily. The wait consists of that seen by the first customer in the batch and the service time of the customers within the same batch that are served before the randomly chosen customer, yielding

$$p_W(k) = \sum_{j=0}^{k} q_j p_P(k - j + 1) \text{ for } k \in [0, K-1]. \tag{8.1}$$

Note that $p_W(k)$ is the waiting time pmf for all customers, regardless of whether they join the queue, thus for a buffer size of K, the probability that a random customers joins the queue is given by $P[J] = \sum_{i=0}^{K-1} p_W(k)$. Therefore, the (conditional) waiting time pmf of packets that join the queue is $p_W(k)/P[J]$.

On a related note, Halfin [86] showed for very general systems that for geometric batch sizes, the waiting time of a random customer is equal to the waiting time of the last customer in a batch, assuming that the service policy does not discriminate based on batch size or position within a batch.

8.2 Packet Loss for Tandem Link

Once the single-queue waiting time probabilities have been computed, discrete convolution is used to arrive at the end-to-end loss through the waiting time distribution.

Tables 8.1 through 8.3 show the relationship between the loss components and the local deadline K for our standard example with $M = 5$ nodes and an incoming traffic intensity of $\lambda = 0.8$. For comparison with the continuous-time case, we aim for an uncontrolled loss of around 5%. With a loss of 4.02%, an end-to-end deadline of $d = 20$ comes closest. Table 8.1 describes the case of heterogeneous traffic, where the down-stream traffic reduction due to packet dropping is figured in. The second case, whose results appear in Table 8.2, assumes that the control policy is applied uniformly through the network, as described in chapter 3, Eq. (3.1), so that all nodes see the same, reduced traffic. The third table (Table 8.3) ignores the traffic reduction due to packet dropping altogether, so that each node sees a traffic intensity of λ.

K	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	gain
2	$2.378{\cdot}10^{-01}$	$4.703{\cdot}10^{-08}$	$6.171{\cdot}10^{-08}$	$2.378{\cdot}10^{-01}$	5.92
4	$1.010{\cdot}10^{-01}$	$-9.597{\cdot}10^{-10}$	$-1.068{\cdot}10^{-09}$	$1.010{\cdot}10^{-01}$	2.51
6	$4.536{\cdot}10^{-02}$	$7.858{\cdot}10^{-04}$	$8.231{\cdot}10^{-04}$	$4.614{\cdot}10^{-02}$	1.15
8	$2.033{\cdot}10^{-02}$	$7.333{\cdot}10^{-03}$	$7.485{\cdot}10^{-03}$	$2.766{\cdot}10^{-02}$	0.69
10	$8.958{\cdot}10^{-03}$	$1.794{\cdot}10^{-02}$	$1.810{\cdot}10^{-02}$	$2.690{\cdot}10^{-02}$	0.67
12	$3.884{\cdot}10^{-03}$	$2.708{\cdot}10^{-02}$	$2.719{\cdot}10^{-02}$	$3.096{\cdot}10^{-02}$	0.77
14	$1.665{\cdot}10^{-03}$	$3.318{\cdot}10^{-02}$	$3.323{\cdot}10^{-02}$	$3.484{\cdot}10^{-02}$	0.87
16	$7.087{\cdot}10^{-04}$	$3.675{\cdot}10^{-02}$	$3.677{\cdot}10^{-02}$	$3.746{\cdot}10^{-02}$	0.93
18	$3.006{\cdot}10^{-04}$	$3.863{\cdot}10^{-02}$	$3.864{\cdot}10^{-02}$	$3.893{\cdot}10^{-02}$	0.97
20	$1.272{\cdot}10^{-04}$	$3.952{\cdot}10^{-02}$	$3.952{\cdot}10^{-02}$	$3.964{\cdot}10^{-02}$	0.99

Table 8.1: Tandem $D^{[M]}/D/1/K$ system with $M = 5$ stages, $\lambda = 0.8$, $d = 20$, Poisson-distributed batch size, uncontrolled loss of 4.018%; heterogeneous traffic

Figure 8.1 compares the relative gain for the (continuous-time) FIFO-BW case of chapter 7 and the discrete-time cases discussed above; gain is defined as the ratio of controlled to uncontrolled loss. The end-to-end deadline d for the continuous was chosen so that its no-discarding loss matches those of the discrete-time system exactly. Despite the fact that the end-to-end deadlines for the two systems differ markedly, with 40.71 in the continuous-time, variable-length-packet case and 20 in the discrete-time, fixed-length-packet case, the gain curve plotted against relative local deadlines (τ/d and K/d) show a remarkable similarity, with only marginally better performance for the discrete-time case. This holds for both the cases plotted, namely heterogeneous and homogeneous traffic.

As in the continuous time case, we compared the effect on the gain of making different assumptions about the traffic within the network (see section 7.2). The graph shows that, again, ignoring the effect of packet dropping on down-stream nodes leads to significant performance differences only for extremely tight local deadlines, i.e., unrealistically large discarding probabilities. The net-equilibrium traffic model leads to results that fall between the heterogeneous and homogeneous cases.

K	λ_i	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	gain
2	0.717	$2.454\cdot10^{-01}$	$9.152\cdot10^{-09}$	$1.213\cdot10^{-08}$	$2.454\cdot10^{-01}$	6.11
3	0.749	$1.571\cdot10^{-01}$	$-2.340\cdot10^{-17}$	$-2.776\cdot10^{-17}$	$1.571\cdot10^{-01}$	3.91
4	0.767	$1.029\cdot10^{-01}$	$6.166\cdot10^{-09}$	$6.873\cdot10^{-09}$	$1.029\cdot10^{-01}$	2.56
5	0.778	$6.819\cdot10^{-02}$	$6.318\cdot10^{-05}$	$6.780\cdot10^{-05}$	$6.825\cdot10^{-02}$	1.70
6	0.786	$4.596\cdot10^{-02}$	$8.306\cdot10^{-04}$	$8.706\cdot10^{-04}$	$4.679\cdot10^{-02}$	1.16
7	0.790	$3.064\cdot10^{-02}$	$3.250\cdot10^{-03}$	$3.353\cdot10^{-03}$	$3.389\cdot10^{-02}$	0.84
8	0.794	$2.041\cdot10^{-02}$	$7.421\cdot10^{-03}$	$7.576\cdot10^{-03}$	$2.783\cdot10^{-02}$	0.69
9	0.795	$1.346\cdot10^{-02}$	$1.247\cdot10^{-02}$	$1.264\cdot10^{-02}$	$2.593\cdot10^{-02}$	0.65
10	0.797	$8.934\cdot10^{-03}$	$1.788\cdot10^{-02}$	$1.804\cdot10^{-02}$	$2.682\cdot10^{-02}$	0.67
11	0.798	$5.901\cdot10^{-03}$	$2.284\cdot10^{-02}$	$2.298\cdot10^{-02}$	$2.874\cdot10^{-02}$	0.72
12	0.799	$3.881\cdot10^{-03}$	$2.706\cdot10^{-02}$	$2.717\cdot10^{-02}$	$3.094\cdot10^{-02}$	0.77
13	0.799	$2.545\cdot10^{-03}$	$3.048\cdot10^{-02}$	$3.056\cdot10^{-02}$	$3.303\cdot10^{-02}$	0.82
14	0.800	$1.691\cdot10^{-03}$	$3.374\cdot10^{-02}$	$3.380\cdot10^{-02}$	$3.543\cdot10^{-02}$	0.88
15	0.800	$1.099\cdot10^{-03}$	$3.561\cdot10^{-02}$	$3.565\cdot10^{-02}$	$3.671\cdot10^{-02}$	0.91
16	0.800	$7.142\cdot10^{-04}$	$3.701\cdot10^{-02}$	$3.704\cdot10^{-02}$	$3.773\cdot10^{-02}$	0.94
17	0.800	$4.642\cdot10^{-04}$	$3.803\cdot10^{-02}$	$3.805\cdot10^{-02}$	$3.849\cdot10^{-02}$	0.96
18	0.800	$3.017\cdot10^{-04}$	$3.875\cdot10^{-02}$	$3.876\cdot10^{-02}$	$3.905\cdot10^{-02}$	0.97
19	0.800	$1.961\cdot10^{-04}$	$3.924\cdot10^{-02}$	$3.925\cdot10^{-02}$	$3.943\cdot10^{-02}$	0.98
20	0.800	$1.274\cdot10^{-04}$	$3.957\cdot10^{-02}$	$3.957\cdot10^{-02}$	$3.970\cdot10^{-02}$	0.99
21	0.800	$8.283\cdot10^{-05}$	$3.978\cdot10^{-02}$	$3.979\cdot10^{-02}$	$3.987\cdot10^{-02}$	0.99
22	0.800	$5.383\cdot10^{-05}$	$3.992\cdot10^{-02}$	$3.993\cdot10^{-02}$	$3.998\cdot10^{-02}$	0.99

Table 8.2: Tandem $D^{[M]}/D/1/K$ system with $M = 5$ stages, $\lambda = 0.8$, $d = 20$, Poisson-distributed uncontrolled loss of 4.018%; net-equilibrium traffic

K	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	gain
2	$3.339\cdot10^{-01}$	$-1.549\cdot10^{-08}$	$-2.326\cdot10^{-08}$	$3.339\cdot10^{-01}$	8.31
4	$1.321\cdot10^{-01}$	$6.837\cdot10^{-09}$	$7.877\cdot10^{-09}$	$1.321\cdot10^{-01}$	3.29
6	$5.421\cdot10^{-02}$	$1.299\cdot10^{-03}$	$1.373\cdot10^{-03}$	$5.551\cdot10^{-02}$	1.38
8	$2.262\cdot10^{-02}$	$9.058\cdot10^{-03}$	$9.267\cdot10^{-03}$	$3.168\cdot10^{-02}$	0.79
10	$9.507\cdot10^{-03}$	$1.964\cdot10^{-02}$	$1.983\cdot10^{-02}$	$2.915\cdot10^{-02}$	0.73
12	$4.007\cdot10^{-03}$	$2.816\cdot10^{-02}$	$2.828\cdot10^{-02}$	$3.217\cdot10^{-02}$	0.80
14	$1.691\cdot10^{-03}$	$3.374\cdot10^{-02}$	$3.380\cdot10^{-02}$	$3.543\cdot10^{-02}$	0.88
16	$7.142\cdot10^{-04}$	$3.701\cdot10^{-02}$	$3.704\cdot10^{-02}$	$3.773\cdot10^{-02}$	0.94
18	$3.017\cdot10^{-04}$	$3.875\cdot10^{-02}$	$3.876\cdot10^{-02}$	$3.905\cdot10^{-02}$	0.97
20	$1.274\cdot10^{-04}$	$3.957\cdot10^{-02}$	$3.957\cdot10^{-02}$	$3.970\cdot10^{-02}$	0.99

Table 8.3: Tandem $D^{[M]}/D/1/K$ system with $M = 5$ stages, $\lambda = 0.8$, $d = 20$, uncontrolled loss of 4.018%; homogeneous traffic

Fig. 8.2 depicts contour lines of equal gain for four different loss values ranging from $1 \cdot 10^{-5}$ to 0.05 parametrized by the length of the VC and the load. The same observations as made for Fig. 7.13 hold, namely that packet dropping performs best under high loads and with shorter VCs. Simulation results will be presented below in the context of a wider-ranging study.

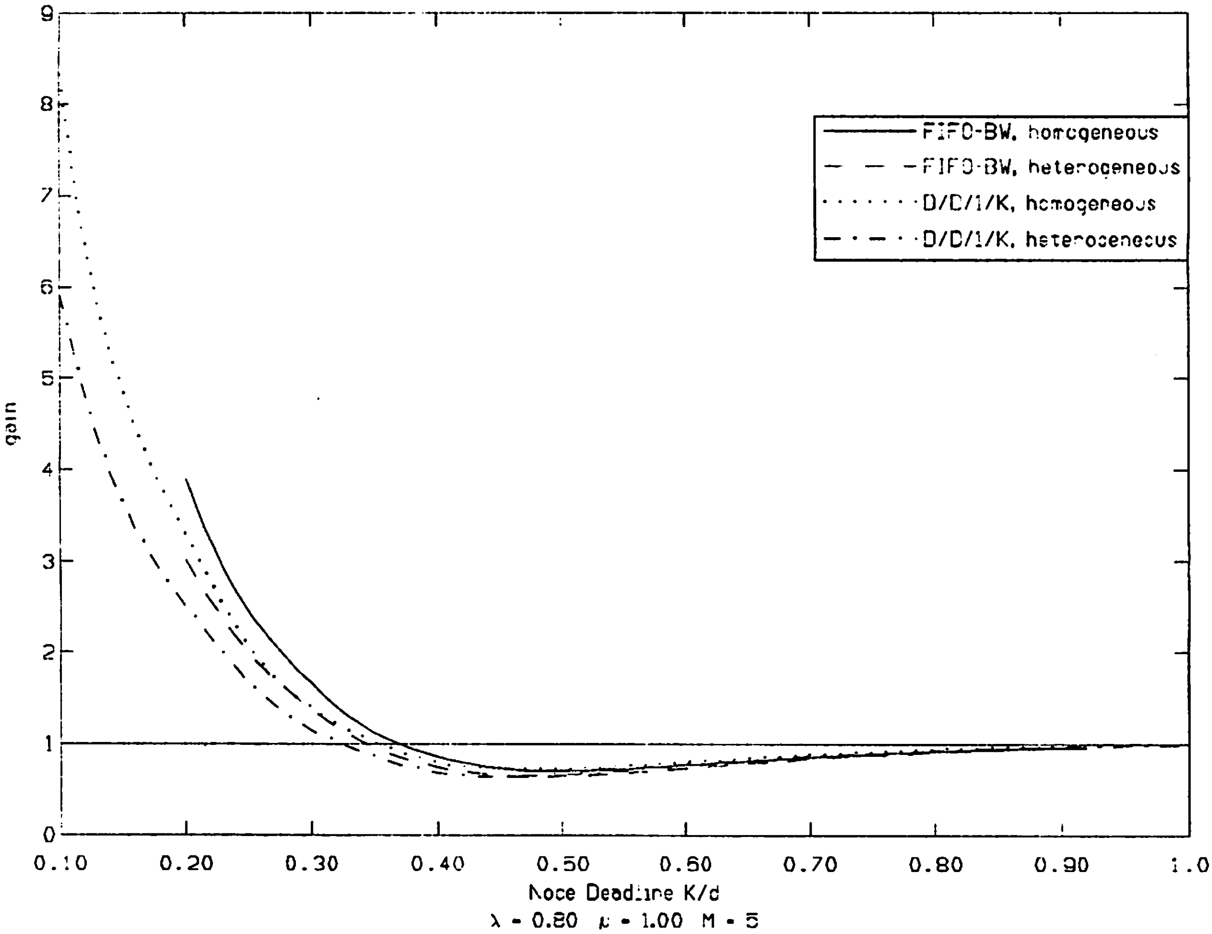

Figure 8.1: Comparison of relative gain for FIFO-BW and $D^{[M]}/D/1/K$ system with $M = 5$ stages and traffic $\lambda = 0.8$

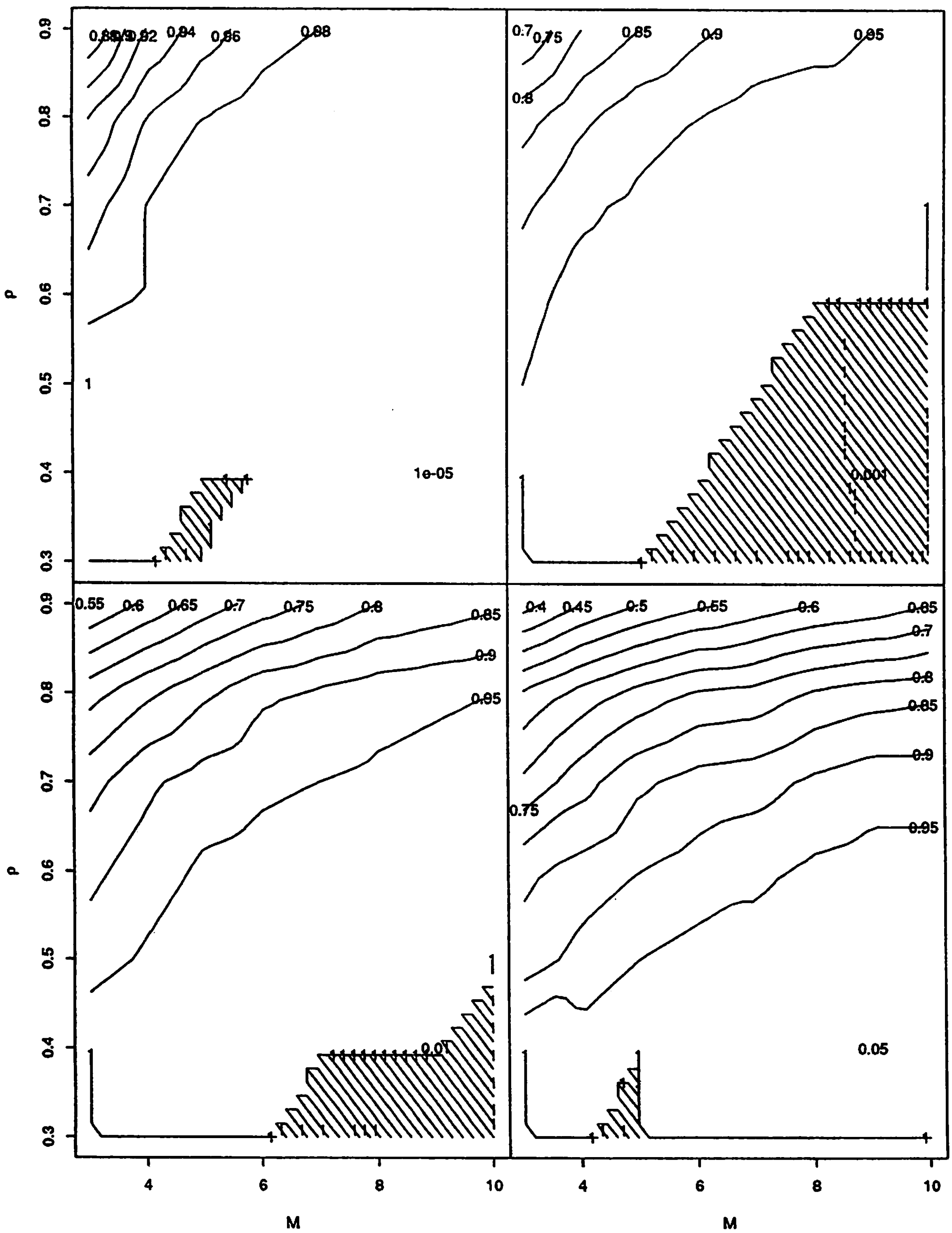

Figure 8.2: Contour plot of optimal ratio of controlled ($D^{[M]}/D/1/K$) to uncontrolled ($D^{[M]}/D/1$) loss for homogeneous traffic and deadlines; nominal losses of $1{\cdot}10^{-5}$, 0.001, 0.01 and 0.05

Chapter 9

Laxity-based Scheduling: A Simulation Study

9.1 Discarding and Service Policies

After having investigated FIFO queues with discarding in some detail, we now turn our attention to a number of other queue control and scheduling mechanisms for the slotted system covered in the previous chapter, comparing them to the simple discarding mechanism described there. We again consider a discrete-time model in which each packet requires one unit of time for transmission. Due to the relative complexity of these mechanisms, only simulation results are available. Independently, Kobza and Liu [87] simulated related laxity-based scheduling disciplines for a three-node continuous-time system with deterministic service. In their study, the laxity-based scheduler only operated in the final node of the virtual circuit. Here, we attempt to cover a wide range of parameter values for a more realistic network model. Also, the combined effect of scheduling and packet discarding will be investigated. A subset of the simulations results are also used to validate the discrete-time results obtained in the previous chapter.

In the analysis of the previous chapter, interfering traffic was accounted for by reducing the service rate. In a discrete-time simulation, interfering traffic must be generated explicitly as a straightforward implementation of a tandem queue with deterministic servers would lead to zero waiting times at all but the first queue (the so-called "pipelining" effect). We introduce interfering traffic in such a way that it contributes to our performance statistics. Specifically, we choose for each packet an arbitrary, but loop-free random path of length M through the network of N nodes for each packet. Since a node can receive packets from any other node, this model is representative of a highly-interconnected network with homogeneous links and uniform traffic.

In this section, we extend our focus from simple first-in, first-out service to a number of service disciplines, the last two of which are based on laxity[1]:

FIFO Packets are served in order of arrival.

LIFO The most recent local arrival is served first. This discipline was investigated since [88] indicated that for concave deadline distributions, LIFO yields the highest fraction of on-

[1]Laxity is the time remaining until the packet's deadline expires, i.e., until its time in the network exceeds the deadline

time packets among non-preemptive, non-deadline dependent scheduling disciplines. Thus, performance better than FIFO might be expected even for deterministic deadlines.

EDF Packets are served earliest-deadline-first. The laxity measure determining the order of service at a packet's ith hop is given by the ratio of laxity to hops remaining,

$$d_i = \frac{d + M - a}{M - i + 1}$$

where a is the age of the packet, that is, the time between its creation and the current time slot at the node. While waiting, the d_i value of a packet decreases with a rate that depends on the number of nodes left to travel. At an output link, the packet with the lowest value of d_i is always transmitted next. Ties between packets with the same laxity are broken in favor of the earliest arrival. A packet is eligible for consideration one slot after its arrival, reflecting the processing delay. This local time-based priority captures the system time remaining until extinction, scaled by the number of nodes that the packet still has to traverse. Thus, packets tend to be "compensated" by a higher priority for above-average delays suffered at earlier nodes. This policy is identical to the "Budgeted Residual-Life Dependent Discipline" (BURD) in [87].

EEF The earliest-extinction-first "seniority" discipline is similar to EDF, but based solely on laxity, without regard to the number of hops left to travel, i.e., $d_i = d + M - a$. Again, the packet with the lowest value of d_i is transmitted.

For all policies, we chose arbitrarily to let arrivals from other nodes (internal arrivals) enter the queue before arrivals from outside the network (external arrivals). Among internal arrivals, those coming from lower-numbered queues acquire the outgoing link first. This ordering should be equivalent to random service since the sequence of node numbers a packet traverses is random. Pilot experiments we performed indicate that the ordering between internal and external arrivals has virtually no effect.

The policies above are scheduling policies. Two *discarding* policies were studied. In *local-wait-based discarding*, a packet is dropped if its waiting time at a given node exceeds the threshold τ_i. For FIFO service, this case corresponds to the analysis presented in the first part of this chapter, with τ_i replacing the system size limit K. In *age-based discarding*, on the other hand, a packet is discarded at the ith hop of its path of length M if its age, that is, the time between entering the system and departing from the ith node in its path, exceeds τ_i. Among the many possible ways to set the τ_i's within the network, we explored τ_i's of the general form

$$\tau_i = \beta \left(\frac{i}{M} \right)^{\alpha} d,$$

with control parameters α and β. As before, d denotes the end-to-end system time deadline. The expression for τ_i is motivated by trying to distribute the permissible travel time d over M nodes. For $\alpha = 1$, the local deadline is proportional to an equal allotment of travel time among all nodes. For $\alpha < 1$, deadlines are looser for early nodes, reflecting the higher uncertainty about the end-to-end waiting time (i.e., we might expect that a packet early in its travel has better chances of "making up" time along the remainder of the VC).

Note that the service and discarding policy are orthogonal. For FIFO, the discarding decision can be made on entering the system as the waiting time within the node is known at that point; for the other disciplines, a packet that is eventually discarded will occupy buffer space until it is discarded.

9.2 Simulation Results and Discussion

In this section, the simulation results for the various combinations of discarding and scheduling policy will be presented and discussed. For results where explicit confidence intervals are shown, 10^6 packets contributed to the loss statistic. For the other experiments, the simulation was terminated when all confidence interval halfwidths were within 20% of the point estimate.

The first check occurs after 10^6 observations, with the number of samples between confidence interval checks increasing by a factor of 1.5 thereafter.

The confidence level was set at 90%. The confidence intervals were computed using the spectral method [89]. The first 2000 observations were discarded as transient data in all experiments.

Unless noted otherwise, external arrival occur in Poisson-distributed batches so that the total system load (including arrivals from other nodes) at each node without discarding equals $\lambda = 0.8$.

τ	FIFO			LIFO	EDF	EEF
	$N=5$	$N=50$	$N=90$	$N=50$	$N=50$	$N=50$
4	4.66±0.09	9.35±0.11	9.66±0.15	32.2±0.14	33.000±0.267	16.000±0.094
5	2.55±0.08	6.24±0.13	6.48±0.14	28.1±0.15	27.500±0.249	12.700±0.106
6	1.55±0.07	4.24±0.13	4.47±0.13	24.9±0.15	22.900±0.249	10.100±0.105
7	**1.28**±0.09	3.11±0.11	3.28±0.13	22.3±0.17	18.080±0.231	8.110±0.114
8	1.39±0.12	2.58±0.13	2.78±0.15	20.2±0.15	15.200±0.202	6.460±0.094
9	1.56±0.13	**2.47**±0.16	**2.68**±0.18	18.4±0.16	12.000±0.158	5.110±0.110
10	1.73±0.16	2.58±0.15	2.78±0.20	16.8±0.16	9.140±0.110	4.040±0.082
11	1.84±0.17	2.80±0.18	3.02±0.23	15.6±0.14	6.570±0.103	3.140±0.100
12	1.90±0.18	3.02±0.18	3.23±0.27	14.6±0.15	4.410±0.789	2.430±0.079
13	1.94±0.18	3.31±0.23	3.45±0.29	13.7±0.14	2.650±0.064	1.840±0.080
14	1.96±0.18	3.40±0.25	3.61±0.36	13.0±0.12	1.410±0.042	1.380±0.067
15	1.96±0.18	3.51±0.23	3.77±0.36	12.4±0.15	0.605±0.029	1.020±0.061
16	1.97±0.18	3.66±0.23	3.89±0.40	12.0±0.13	0.211±0.026	0.740±0.056
17	1.97±0.18	3.66±0.25	3.98±0.42	11.6±0.12	0.094±0.020	0.532±0.049
18	1.97±0.18	3.76±0.28	4.01±0.39	11.4±0.14	0.059±0.017	0.387±0.047
19	1.97±0.18	3.80±0.29	4.09±0.40	11.2±0.14	**0.047**±0.019	0.305±0.042
20	1.97±0.18	3.78±0.28	4.05±0.43	**11.1**±0.17	0.050±0.023	**0.296**±0.050
21	1.97±0.18	3.79±0.28	4.05±0.45	11.3±0.15	0.069±0.030	0.459±0.087
50	1.97±0.18	3.81±0.28	4.14±0.47	12.7±0.14	0.129±0.061	0.905±0.160

Table 9.1: Losses (in percent) for discarding based on local wait, $M = 5$, $\lambda = 0.8$, $d = 20$; net-equilibrium traffic model

We performed a first experiment, whose results are shown in the first three columns of Tab. 9.1 and Figures 9.1 and 9.2, to validate the analysis performed in the previous section. The parameters of our running example were used: a five-node VC with a deadline of $d = 20$. The size of the network, N, was varied between 5 and 90. We chose the performance predictions under the net-equilibrium traffic assumption since they most closely model the network situation. The table and

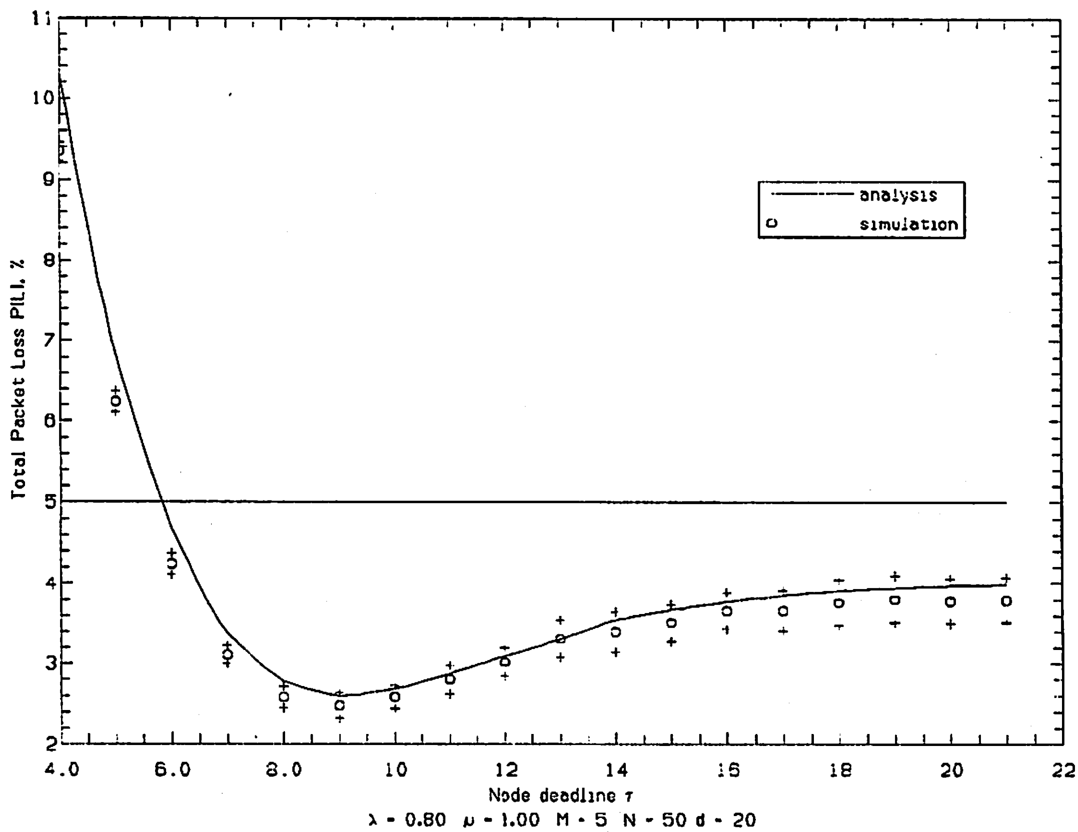

Figure 9.1: Comparison of simulation and analysis results ($N = 50$) for a FIFO system with discarding based on local wait

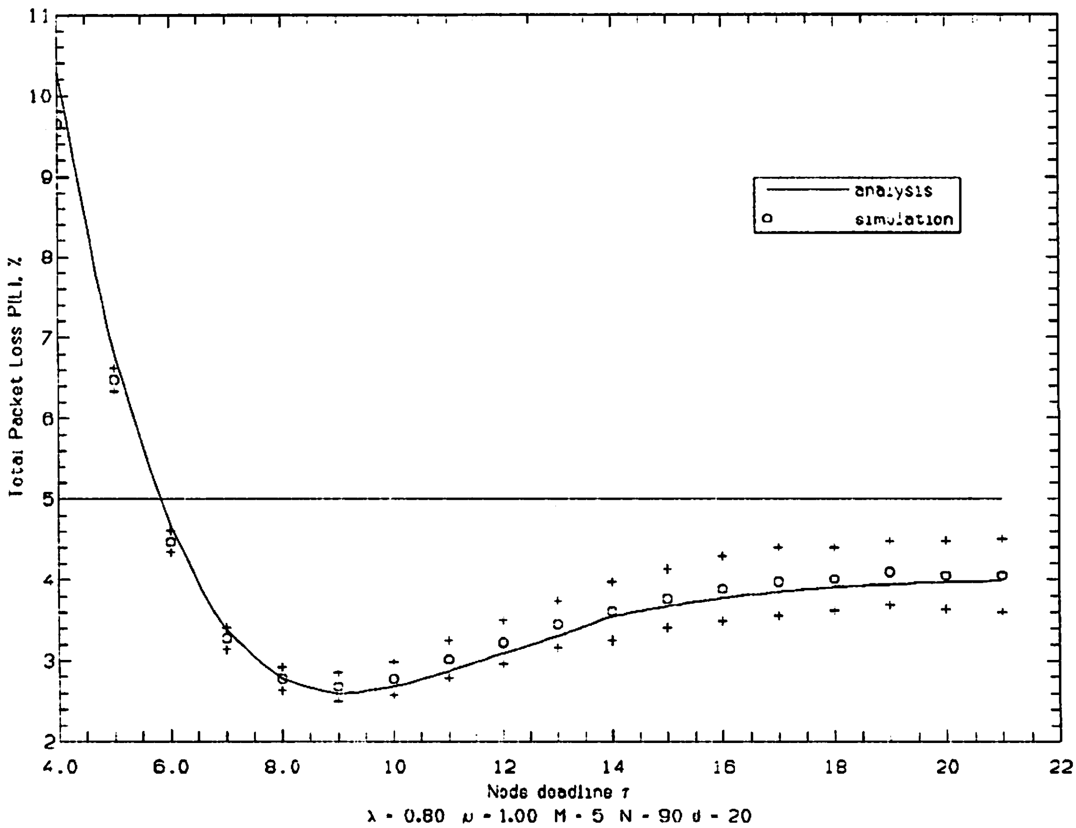

Figure 9.2: Comparison of simulation and analysis results ($N = 90$) for a FIFO system with discarding based on local wait

graphs[2] show that the analysis predicts the simulation results reasonably well, with the simulation loss value approaching the analytical results from below as N increases. For a small network, e.g., our $N = 5$ case, each queue sees only a small number of sources and the Poisson approximation for the input process does not hold well. Also, the correlation of the output process manifests itself on the succeeding queue as packets leaving a node have a non-negligible probability of proceeding to the same successor node. Fortunately, simulation and analysis agree on the optimal value for the discarding control parameter τ (called K in the analysis).

Let us now turn to the effect of scheduling and discarding on end-to-end loss, using the results for our running example, tabulated in Tab. 9.1 and Tab. 9.2 for local-wait-based and age-based discarding, respectively. In the tables, the best performance is shown in bold face. Table 9.2 also contains data for the extreme cases of no discarding (last row) and discarding of expired packets only (first row).

α	β	FIFO	LIFO	EDF	EEF
0.00	1.00†	1.99±0.13	**10.5**±0.12	**0.0317**±0.012	**0.21**±0.030
0.20	1.00	**1.92**±0.12	11.0±0.13	0.1140±0.023	1.36±0.074
0.33	1.00	1.98±0.10	11.5±0.15	0.2300±0.035	2.99±0.085
0.40	1.00	2.01±0.11	11.6±0.12	0.2880±0.040	3.83±0.091
0.50	1.00	2.30±0.11	12.1±0.14	0.4380±0.047	5.96±0.101
0.60	1.00	2.66±0.09	12.5±0.13	0.5600±0.049	7.38±0.098
0.66	1.00	3.23±0.10	12.7±0.12	0.6970±0.056	9.15±0.111
0.75	1.00	4.31±0.09	13.2±0.13	0.8480±0.060	11.20±0.097
1.00	1.00	6.26±0.10	14.0±0.13	1.1800±0.066	13.90±0.106
1.00	1.50	3.05±0.09	11.6±0.12	0.6210±0.051	9.05±0.108
1.00	2.00	2.41±0.12	11.4±0.14	0.3780±0.037	5.90±0.097
1.00	2.50	2.77±0.21	11.5±0.13	0.2400±0.032	3.75±0.083
1.00	3.00	3.14±0.19	11.8±0.14	0.1580±0.028	2.28±0.075
1.00	5.00	3.82±0.28	12.5±0.14	0.0744±0.036	0.33±0.062
1.00	50.00*	3.81±0.28	13.1±0.19	0.1290±0.061	0.91±0.160

Table 9.2: Losses (in percent) for age-based discarding, $M = 5$, $N = 50$, $\lambda = 0.8$, $d = 20$; † discard expired packets; * no discarding

The lowest packet loss fractions achieved for the various combinations of discarding and scheduling policies for the running example are summarized in Table 9.3. It is clear from this table that scheduling has a far greater effect on the loss than the discarding policy. EDF lowers the losses by about two orders of magnitude, from 3.8% for the uncontrolled system with FIFO scheduling to 0.03% for EDF scheduling with the optimally "tuned" discarding parameter. For the set of parameters studied, LIFO is by far the worst scheduling policy, with losses in excess of 10%.

The mean wait experienced at the ith hop of a packet (omitted here for reasons of space) clearly shows the effect of the scheduling disciplines. This mean wait is naturally identical for each hop in the case of FIFO and LIFO, it increases with i (as the packet approaches its destination) for EDF, while it decreases for EEF.

The relative benefits of discarding packets, the influence of the discarding policy and the parameter τ_i depend on the scheduling discipline. Judicious discarding can lower the losses by a factor of between 1.3 for LIFO, to 2 for FIFO and up to factors of 4 and 3 for EDF and EEF, respectively. In all cases, moreover, age-based discarding improves upon discarding based on local waiting times.

[2]The graphs indicate the confidence intervals as plus marks.

This should be expected since age-based discarding uses more information, reflecting the whole travel history, than local-wait based discarding. The difference between the two discarding policies is least pronounced for LIFO, with a factor of 1.05 improvement, while introducing age-based discarding with the other scheduling disciplines lowers losses by a factor of between 1.3 and 1.5 times compared to local-wait discarding. Since EDF and EEF scheduling require laxity information for scheduling, age-based discarding seems to recommend itself for these two policies, while for FIFO the choice is more of a tradeoff between performance and simplicity of implementation.

	Service policy			
Dropping	FIFO	LIFO	EDF	EEF
none	3.81	13.1	0.129	0.905
expired	1.99	10.5	0.032	0.210
age	1.92	10.5	0.032	0.210
local wait	2.47	11.1	0.047	0.296

Table 9.3: Packet losses (in percent) for different service and discarding policies; $M = 5$, $N = 50$, $\lambda = 0.8$, $d = 20$

For age-based discarding, discarding only expired packets seems to be the best policy (at least for the cases studied here), even though marginal improvements are sometimes possible for slightly tighter deadlines. (It is not clear why tighter deadlines work better for $N = 5$.) Discarding expired packets is a no-risk policy as it never discards packets that may make their deadline, requires no adjustment of parameters and does not depend on model assumptions with regard to traffic statistics.

For local-wait based discarding, the optimal deadline for FIFO differs markedly from that for the other scheduling disciplines. For FIFO, a deadline significantly below d (here, around $0.5d$) recommends itself, while for the others, discarding packets that have spend their entire allotment of d waiting at a single node performs best or very close to best.

After this rather detailed investigation of the five-node VC, the question of the performance of the policies studied under a wider range of parameters needs to be addressed. We limit ourselves to discarding expired packets for the reasons noted above as well as to keep the parameter space manageable. We maintain the node load at $\lambda = 0.8$ and the network size at $N = 50$, but vary the end-to-end deadline d to obtain the loss performance under different amounts of packet loss. Also, both the case of geometrically and that of Poisson-distributed external arrivals were simulated. The results are summarized in Table 9.4. Zero loss values indicate that no loss occurred during a simulation that processed 10^7 packets.

A number of conclusions can be drawn from the this table. For all parameter values simulated, the ranking established above, that is, EDF best, followed by EEF, FIFO and LIFO, persists. As might be expected, the differences between service disciplines become in general more pronounced as the VC length increases, although the difference between EDF and EEF actually decreases in going from $M = 5$ to $M = 10$. Thus, contrary to the claims in [87], single-node behavior is not a good predictor for the performance gain in longer VCs. Also, with one exception, for the same VC length, the difference between FIFO and the policies EDF and EEF increases as the loss rates decrease.

M	d	FIFO		LIFO		EDF		EEF	
		Poisson	geo.	Poisson	geo.	Poisson	geo.	Poisson	geo.
1	7	0.720	3.77	6.72	11.60	0.720	3.77	0.720	3.77
	10	0.200	1.76	4.84	8.83	0.200	1.76	0.200	1.76
	16	0.0150	0.469	2.88	5.78	0.0150	0.469	0.0150	0.469
2	10	1.72	6.46	9.45	14.40	0.501	3.36	0.645	3.52
	15	0.342	2.80	6.17	10.30	0.0515	1.11	0.0768	1.23
	21	0.0396	1.03	4.02	7.39	0.00364	0.287	0.00578	0.334
5	20	2.01	6.63	10.06	14.00	0.0304	1.21	0.208	1.94
	25	0.617	3.79	7.92	11.10	0.00289	0.413	0.0248	0.802
	33	0.0624	1.52	5.92	8.13	0.00000	0.0600	0.00081	0.165
10	33	2.54	7.06	11.20	13.50	0.00106	0.439	0.137	1.53
	40	0.720	3.83	8.39	10.70	0.00080	0.0672	0.00860	0.536
	49	0.110	1.61	5.89	8.05	0.00000	0.00559	0.00051	0.0929

Table 9.4: Packet losses (in percent) with discarding of expired packets; Poisson and geometric arrivals, $N = 50$, $\lambda = 0.8$

EEF, while inferior to EDF, still reduces losses by about an order of magnitude compared to FIFO. As discussed in the next section, it may, however, be easier to implement than EDF.

Intuition suggests that the statistics of the external arrival process have less and less influence as the VC length increases. For example, Table 9.4 shows that the ratio of loss between geometric and Poisson arrivals decreases from about 5.2 to roughly 2.8 as the VC length increases from 1 to 10 (for closest deadline and FIFO service). However, for EDF, no clear trend is discernible, with ratios of two orders of magnitude even for the longest VC.

Appendix A

The Tandem M/M/1 Queue

In the chapters on the $M/M/1/K$ and FIFO-BW model, the tandem-$M/M/1$ queue will frequently be used for comparison. For ease of reference, this appendix summarizes results relating to the distribution of waiting and system time for concatenations of this most basic of queues. To gain a feel for the accuracy of approximation methods for points on the distribution, we will also investigate estimators based on moments, the Laplace transform or the moment-generating function.

A.1 Relations between μ, λ, ρ and ν

$$\begin{aligned}
\nu &= \mu - \lambda \\
\lambda(1-\rho) &= \rho\nu \\
\mu(1-\rho) &= \nu \\
\mu\rho &= \lambda \\
\mu + \nu &= 2\mu - \lambda \\
\lambda + \nu &= \mu
\end{aligned}$$

A.2 Waiting Time for an M/M/1 Queue

The waiting time of an M/M/1 queue has the distribution

$$W(t) = 1 - \rho e^{-\nu t}$$

By differentiating its moment-generating function

$$w^*(-s) = (1-\rho) + \frac{\nu\rho}{\nu - s}$$

n times as

$$\frac{dw^*(-s)}{ds} = \nu\rho\frac{n!}{(\nu - s)^n}$$

and evaluating the derivatives at $s = 0$, the nth moment is obtained as

$$E[w^n] = \frac{n!}{\nu^n}\rho \tag{A.1}$$

A.3 Approximating the Tail of the M/M/1 Waiting Time Distribution

Since a closed-form expression trivially provides points on waiting time distribution, it is instructive to see how well approximation methods of interest for other distributions work here.

A.3.1 Bienaymé's Inequality

Given the moments of the waiting time, the tail of the waiting time distribution can be bounded from above by the inequality of Bienaymé [71, p. 115], which is a generalized version of Chebyshev's inequality

$$P\{|x-a| \geq \varepsilon\} \leq \frac{E[|x-a|^n]}{\varepsilon^n}$$

For numerical accuracy, it is evaluated as

$$\exp(\ln(n!) + \ln(\rho) - n\ln(\nu\varepsilon))$$

Unfortunately, the bound cannot be tightened arbitrarily by increasing n, as the evaluation for $\rho = 0.8, \nu = 0.2, \varepsilon = 40$, and $a = 0$ shows (true value $2.6837 \cdot 10^{-4}$):

n	$E[w^n]$	$P\{x \geq \varepsilon\} \leq$
1	4	$1.0000 \cdot 10^{-1}$
2	40	$2.5000 \cdot 10^{-2}$
3	600	$9.3750 \cdot 10^{-3}$
4	12000	$4.6875 \cdot 10^{-3}$
5	300000	$2.9297 \cdot 10^{-3}$
6	$9.000 \cdot 10^{6}$	$2.1973 \cdot 10^{-3}$
7	$3.150 \cdot 10^{8}$	$1.9226 \cdot 10^{-3}$
8	$1.260 \cdot 10^{10}$	$1.9226 \cdot 10^{-3}$
9	$5.670 \cdot 10^{11}$	$2.1629 \cdot 10^{-3}$
10	$2.835 \cdot 10^{13}$	$2.7037 \cdot 10^{-3}$
11	$1.559 \cdot 10^{15}$	$3.7175 \cdot 10^{-3}$
12	$9.356 \cdot 10^{16}$	$5.5763 \cdot 10^{-3}$
13	$6.081 \cdot 10^{18}$	$9.0615 \cdot 10^{-3}$
14	$4.257 \cdot 10^{20}$	$1.5858 \cdot 10^{-2}$
15	$3.193 \cdot 10^{22}$	$2.9733 \cdot 10^{-2}$
16	$2.554 \cdot 10^{24}$	$5.9366 \cdot 10^{-2}$
17	$2.171 \cdot 10^{26}$	$1.2637 \cdot 10^{-1}$

See also [90].

A.3.2 The Chernoff Bound

Using the moment-generating function instead of the moments within the Chebyshev equation leads to the Chernoff bound, as described in [91, p. 126f] and [61, p. 391f]. With the moment-generating function for the waiting time, the bound is written as

$$P\{t \geq \varepsilon\} \leq e^{-s\varepsilon} w^*(-s) \tag{A.2}$$

for any value s where the mgf exists. The bound is tightest for the value s that solves the equation

$$\varepsilon = \frac{\log w^*(-s)}{ds} = \frac{dw^*(-s)}{ds}/w^*(-s).$$

For the mgf given, the bound is tightest for

$$s = \frac{\pm\sqrt{(\nu\rho t)^2 + 4\nu\rho t(1-\rho)} + \nu t(\rho - 2)}{2(\rho - 1)t}$$

For the values of ε, ρ and ν used above, $s = 0.175736$ and $s = 1.024264$. The latter solution falls outside the region of convergence of the moment-generating function, the former yields a bound of $P\{t \geq 40\} \leq 6.0157 \cdot 10^{-3}$, which is unfortunately looser than the moment-based bound shown earlier.

A.3.3 Approximate Inversion of Laplace Transform

Feller provides an asymptotic inversion formula for the Laplace transform, given its derivatives [79, p. 233]:

$$F(t) = \lim_{s \to \infty} \sum_{k=0}^{st} \frac{(-1)^k}{k!} s^k \mathcal{L}\{f\}^{(k)}(s) \tag{A.3}$$

Note that the zeroth derivative is included in the summation.

For the M/M/1 case, a closed-form expression can be easily derived.

$$\begin{aligned}
F(t) &\leftarrow \sum_{k=1}^{st} \frac{(-1)^k}{k!} s^k \rho\nu \frac{(-1)^k k!}{(s+\nu)^{k+1}} + (1-\rho) + \frac{\rho\nu}{s+\nu} \\
&= \frac{\rho\nu}{s+\nu}\left(\frac{(s/(s+v))^{st+1} - 1}{s/(s+\nu) - 1}\right) + (1-\rho) \\
&= 1 - \rho\left(\frac{s}{s+\nu}\right)^{st+1} \\
&\to 1 - \rho e^{-\nu t}
\end{aligned}$$

The expression converges reasonably fast for the test case presented above:

s	$P\{x \geq \varepsilon\} \approx$	error
1	$4.5359 \cdot 10^{-4}$	0.408
2	$3.5504 \cdot 10^{-4}$	0.244
5	$3.0157 \cdot 10^{-4}$	0.011
10	$2.8472 \cdot 10^{-4}$	0.057
20	$2.7648 \cdot 10^{-4}$	0.029
50	$2.7160 \cdot 10^{-4}$	0.012
100	$2.6998 \cdot 10^{-4}$	0.006
500	$2.6869 \cdot 10^{-4}$	0.001
1000	$2.6853 \cdot 10^{-4}$	0.001
5000	$2.6840 \cdot 10^{-4}$	

A.4 Waiting Time Pdf for Tandem-M/M/1 Queue

A.4.1 Homogeneous System

This section presents an expression for the density function of the waiting time in a tandem $M/M/1$ queueing system with M stages and identical service times and arrival rates.

In the Laplace domain, we have with $\nu = \mu - \lambda$ and $r = 1 - \rho$

$$\begin{aligned} w_1^*(s) * w_2^*(s) * \ldots * w_M^*(s) &= \left[r + \frac{\rho\nu}{s+\nu}\right]^M \\ &= \sum_{j=0}^{M} \binom{M}{j} r^{M-j} \left[\frac{\rho\nu}{s+\nu}\right]^j \\ &= \sum_{j=0}^{M} \frac{M!}{j!(M-j)!} r^{M-j} \left[\frac{\rho\nu}{s+\nu}\right]^j \end{aligned}$$

The inverse is computed readily [92] as

$$w_M(t) = e^{-\nu t}\left[M! \sum_{j=1}^{M} \frac{(\rho\nu)^j r^{M-j}}{j!(M-j)!(j-1)!} t^{j-1}\right] + r^M \delta(t)$$

For M =1, 2 and 3, this specializes to the standard results

$$\begin{aligned} w_1(t) &= \rho\nu e^{-\nu t} + r\delta(t) \\ w_2(t) &= e^{-\nu t}\left[2\rho\nu r + (\rho\nu)^2 t\right] + r^2\delta(t) \\ w_3(t) &= e^{-\nu t}\left[3\rho\nu r^2 + 3(\rho\nu)^2 r t + \frac{1}{2}(\rho\nu)^3 t^2\right] + r^3\delta(t) \end{aligned}$$

A.4.2 Inhomogeneous System

For all $\nu_i \neq \nu_j, i \neq j$, we can apply the very simple partial fraction expansion ([92, p. 693, item 11]

$$\mathcal{L}^{-1}\frac{1}{D(s)} = \sum_{k=1}^{M} \frac{1}{D'(s_k)} e^{s_k t} = \sum_{k=1}^{M} \frac{1}{\prod_{i\neq k}(\nu_i - \nu_k)} e^{-\nu_k t}$$

to the convolution expression

$$w_1^*(s) * w_2^*(s) * \ldots * w_M^*(s) = \prod_{i=1}^{M}\left[r_i + \frac{\rho_i\nu_i}{s+\nu_i}\right] = \sum_{k=0}^{2^M-1}\prod_{i=1}^{M} r_i^{l_{ki}} \left(\frac{\rho_i\nu_i}{s+\nu_i}\right)^{1-l_{ki}}$$

l_{ki} is the ith digit in the binary representation of k. (Needless to say, an actual implementation of the equation would not use powers to pick terms in the product.)

After some algebra, the closed-form expression can be written out as

$$w_M(t) = \sum_{k=1}^{2^M-1}\left\{\left[\prod_{j=1}^{M} r_j^{1-l_{kj}}(\rho_j\nu_j)^{l_{kj}}\right] \sum_{j\in l_k} \frac{e^{-\nu_j t}}{\prod_{i\in l_k, i\neq j} \nu_i - \nu_j}\right\} + \prod_{j=1}^{M} r_j\delta(t). \qquad (A.4)$$

Note that the lower index of the outer sum has changed, resulting in the delta function term. For brevity, $i \in l_k$ denotes all indices i such that l_{ki} is 1.

For $M = 2$, the result specializes to

$$w_2(t) = r_1 r_2 \delta(t) + r_1 \rho_2 \nu_2 e^{-\nu_2 t} + r_2 \rho_1 \nu_1 e^{-\nu_1 t} + \frac{\rho_1 \rho_2 \nu_1 \nu_2}{\nu_2 - \nu_1}(e^{-\nu_1 t} - e^{-\nu_2 t})$$

The condition $D(s_k) = 0, D'(s_k) \neq 0$, that is, no multiple poles, is satisfied for a strictly nonhomogeneous network in the sense mentioned above.

A.5 Waiting Time Cdf for Tandem-M/M/1 Queue

A.5.1 Homogeneous System

To obtain the distribution, we could invert the above result, multiplied by $1/s$. It seems easier to use the integration formula [93, p. 584]

$$\begin{aligned}
&\int_0^t e^{-\nu t} t^m \, dt \\
&= e^{-\nu t} \sum_{k=0}^{m} (-1)^k \frac{m!}{(m-k)!(-\nu)^{k+1}} t^{m-k} \Bigg|_0^t \\
&= -\frac{e^{-\nu t}}{\nu^{m+1}} \sum_{k=0}^{m} (k+1)\cdots m \, (\nu t)^k \Bigg|_0^t \\
&= \frac{m!}{\nu^{m+1}} - \frac{e^{-\nu t}}{\nu^{m+1}} \sum_{k=0}^{m} (k+1)\cdots m \, (\nu t)^k
\end{aligned}$$

With this, the waiting time cdf becomes

$$\begin{aligned}
W_M(t) &= \sum_{j=1}^{M} \binom{M}{j} \rho^j r^{M-j} \left[1 - \frac{e^{-\nu t}}{(j-1)!} \sum_{k=0}^{j-1} (k+1)\cdots(j-1) \, (\nu t)^k \right] + r^M \\
&= \sum_{j=1}^{M} \binom{M}{j} \rho^j r^{M-j} \left[1 - \frac{e^{-\nu t}}{(j-1)!} \sum_{k=0}^{j-1} (k+1)_{j-1-k;} (\nu t)^k \right] + r^M
\end{aligned}$$

for $t \geq 0$, zero otherwise. Noting that

$$\sum_{j=1}^{M} \binom{M}{j} \rho^j r^{M-j} + r^M = (\rho + r)^M = 1,$$

we find that $\lim_{t\to\infty} W_M(t) = 1$, as required. Also, for $t = 0$, the bracketed term becomes zero, so that $W_M(0) = r^M$, giving the correct probability that all M queues are empty. For $M = 1$, the standard equation $W(t) = 1 - \rho e^{-\nu t}$ evolves.

For $M = 1$, the quantile of the distribution can be computed as

$$t = \frac{1}{\mu r} \log_e \left[\frac{\rho}{1-w}\right]$$

For higher values of M, only numerical zero-finding helps, with the t-value for $M = 1$ as lower and $M \cdot t$ as an upper bound. Some sample values of the quantile for given M and loss p are tabulated in Table A.1.

p	M	$\rho = 0.3$ $\lambda = 0.3$	$\rho = 0.4$ $\lambda = 0.4$	$\rho = 0.5$ $\lambda = 0.5$	$\rho = 0.6$ $\lambda = 0.6$	$\rho = 0.7$ $\lambda = 0.7$	$\rho = 0.8$ $\lambda = 0.8$	$\rho = 0.8$ $\lambda = 1.6$	$\rho = 0.9$ $\lambda = 0.9$
$5 \cdot 10^{-2}$	1	2.5597	3.4657	4.6052	6.2123	8.7969	13.8629	6.9315	28.9037
	2	3.8767	5.2117	6.9556	9.4636	13.5343	21.5441	10.7721	45.3556
	3	4.9148	6.6462	8.9345	12.2449	17.6345	28.2557	14.1279	59.8512
	4	5.8364	7.9419	10.7414	14.8042	21.4310	34.5036	17.2518	73.4094
	5	6.6901	9.1544	12.4435	17.2275	25.0410	40.4660	20.2330	86.3897
	10	10.4587	14.5887	20.1556	28.3014	41.6552	68.0767	34.0383	146.8335
$2 \cdot 10^{-2}$	1	3.8686	4.9929	6.4378	8.5030	11.8512	18.4444	9.2222	38.0666
	2	5.3516	6.9688	9.0937	12.1602	17.1496	26.9840	13.4920	56.2517
	3	6.5242	8.5815	11.3042	15.2465	21.6706	34.3402	17.1701	72.0506
	4	7.5602	10.0269	13.3047	18.0609	25.8193	41.1280	20.5640	86.7014
	5	8.5147	11.3707	15.1768	20.7080	29.7384	47.5647	23.7824	100.6422
	10	12.6770	17.3175	23.5519	32.6557	47.5611	77.0318	38.5159	164.8478
$1 \cdot 10^{-2}$	1	4.8589	6.1481	7.8240	10.2359	14.1617	21.9101	10.9551	44.9981
	2	6.4545	8.2767	10.6799	14.1561	19.8213	31.0006	15.5003	64.2936
	3	7.7166	10.0069	13.0422	17.4417	24.6166	38.7761	19.3881	80.9393
	4	8.8284	11.5511	15.1704	20.4236	28.9962	45.9173	22.9587	96.3044
	5	9.8497	12.9819	17.1546	23.2182	33.1186	52.6656	26.3328	110.8754
	10	14.2732	19.2683	25.9687	35.7438	51.7397	83.3577	41.6789	177.5621
$1 \cdot 10^{-3}$	1	8.1483	9.9858	12.4292	15.9923	21.8369	33.4231	16.7115	68.0239
	2	10.0601	12.5320	15.8225	20.6123	28.4517	43.9646	21.9823	90.2394
	3	11.5704	14.5844	18.5984	24.4377	33.9866	52.8677	26.4338	109.1584
	4	12.8921	16.4006	21.0761	27.8770	38.9943	60.9682	30.4841	126.4602
	5	14.0985	18.0712	23.3690	31.0759	43.6731	68.5672	34.2836	142.7500
	10	19.2480	25.2997	33.3982	45.1979	64.4955	102.6322	51.3161	216.2601
$1 \cdot 10^{-5}$	1	14.7271	17.6611	21.6396	27.5052	37.1875	56.4489	28.2245	114.0757
	2	17.1075	20.8044	25.8750	33.0915	45.1111	68.9705	34.4853	140.2671
	3	18.9856	23.3268	29.1571	37.6885	51.6966	79.4686	39.7343	162.3952
	4	20.6204	25.5446	32.1464	41.7924	57.6122	88.9507	44.4754	182.4802
	5	22.1043	27.5719	34.8951	45.5851	63.1039	97.7885	48.8942	201.2678
	10	28.3504	36.2165	46.7421	62.0849	87.1930	136.8450	68.4225	284.8585
$1 \cdot 10^{-7}$	1	21.3059	25.3363	30.8499	39.0182	52.5381	79.4748	39.3774	160.1273
	2	24.0332	28.9107	35.5301	45.2858	61.3798	93.3816	46.6908	189.0968
	3	26.1889	31.7825	39.3408	50.4457	68.7280	105.0341	52.5170	213.5438
	4	28.0629	34.3034	42.7128	55.0434	75.3150	115.5344	57.7672	235.6765
	5	29.7601	36.6025	45.8060	59.2822	81.4145	125.2958	62.6479	250.3232
	10	36.8586	46.3407	59.0489	77.5988	107.9900	168.1409	84.0705	347.5527

Table A.1: Quantiles for tandem M/M/1 waiting time

A.5.2 Nonhomogeneous System

Here, the integration is almost trivial, as there are no $t^n e^t$-style terms to contend with. The $e^{-\nu_j t}$ term in the expression for the density is simply replaced by

$$\frac{1}{\nu_j}(1 - e^{-\nu_j t})$$

and the delta term ($\prod_{j=1}^{M} r_j$) becomes an additive constant.

p	M	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
$5.0 \cdot 10^{-2}$	1	4.2796	4.9929	5.9915	7.4893	9.9858	14.9787	29.9573
	2	6.7769	7.9064	9.4877	11.8597	15.8129	23.7193	47.4386
	3	8.9940	10.4930	12.5916	15.7395	20.9860	31.4790	62.9579
	4	11.0767	12.9228	15.5073	19.3841	25.8455	38.7683	77.5366
	5	13.0765	15.2559	18.3070	22.8838	30.5117	45.7676	91.5352
	10	22.4360	26.1754	31.4104	39.2630	52.3507	78.5261	157.0522
$2.0 \cdot 10^{-2}$	1	5.5886	6.5200	7.8240	9.7801	13.0401	19.5601	39.1202
	2	8.3342	9.7232	11.6678	14.5848	19.4464	29.1696	58.3392
	3	10.7380	12.5277	15.0332	18.7915	25.0553	37.5830	75.1660
	4	12.9773	15.1402	18.1682	22.7103	30.2804	45.4206	90.8412
	5	15.1148	17.6340	21.1608	26.4510	35.2679	52.9019	105.8038
	10	25.0140	29.1830	35.0196	43.7745	58.3660	87.5491	175.0981
$1.0 \cdot 10^{-2}$	1	6.5788	7.6753	9.2103	11.5129	15.3506	23.0259	46.0517
	2	9.4834	11.0639	13.2767	16.5959	22.1278	33.1918	66.3835
	3	12.0085	14.0099	16.8119	21.0149	28.0198	42.0297	84.0595
	4	14.3502	16.7419	20.0902	25.1128	33.4837	50.2256	100.4512
	5	16.5780	19.3410	23.2093	29.0116	38.6821	58.0231	116.0463
	10	26.8330	31.3052	37.5662	46.9578	62.6104	93.9156	187.8312
$1.0 \cdot 10^{-3}$	1	9.8682	11.5129	13.8155	17.2694	23.0259	34.5388	69.0776
	2	13.1906	15.3890	18.4668	23.0835	30.7780	46.1671	92.3341
	3	16.0412	18.7148	22.4577	28.0722	37.4296	56.1444	112.2887
	4	18.6603	21.7704	26.1245	32.6556	43.5408	65.3112	130.6224
	5	21.1345	24.6569	29.5883	36.9854	49.3138	73.9707	147.9415
	10	32.3677	37.7623	45.3147	56.6434	75.5246	113.2869	226.5737

Table A.2: Quantiles for tandem M/M/1 system time

A.6 System Time Cdf

It is well known that the end-to-end system time in such a tandem queue is Erlang-distributed according to

$$S_{\mathrm{VC}}(t) = 1 - e^{-\nu t} \sum_{j=0}^{M-1} \frac{(\nu t)^j}{j!} \tag{A.5}$$

where $\nu = \mu(1-\rho) = \mu - \lambda$ [60, eq. (5.53), p. 65]. For $M = 1$, this reduces to the familiar $M/M/1$ case

$$S(t) = 1 - e^{-\nu t}$$

The corresponding quantile for $M = 1$ is therefore given by

$$t = -\frac{\ln(1-s)}{\nu}$$

Some sample values of the quantile for given M and loss p are tabulated in Table A.2.

The average delay is

$$E[S_{\mathrm{VC}}] = \frac{M}{\nu} \tag{A.6}$$

In the Laplace domain, we know that [61, eq. (5.117)]

$$s_{\dot{V}C}(s) = \frac{\nu}{s+\nu} \tag{A.7}$$

A.7 Transient Period for Tandem-M/M/1 System

From a diffusion approximation for a G/G/1 queue and graphical inspection, [94] [95, p. 293] provides an estimate of the transient period. (The point of stationarity is assumed when the state distributions for different initial values do not differ signficantly, for reasonable initial values.) In general, the transient period T_0 of a G/G/1 queue has a duration

$$T_{0,G/G/1} = \tau_0 \frac{E[S](C_s^2 + C_a^2\rho)}{(1-\rho)^2}$$

where C_s and C_a are the coefficients of variation (mean over standard deviation) of the service and interarrival time. From graphical inspection and two initial states, $\tau = 5$ was chosen. For exponential service and interarrival time, $C_s = C_a = 1$. For a homogeneous M-stage system, $C_s = M/\sqrt{M\sigma_s^2}$ and $E[S] = M\mu$. Thus, for normalized $\mu = 1$, we estimate the transient period as

$$T_{0,M/M/1} = \tau_0 \frac{M(M/\sqrt{M} + \rho)}{(1-\rho)^2}$$

	ρ						
M	0.30	0.40	0.50	0.60	0.70	0.80	0.90
1	13	19	30	50	94	225	950
2	35	50	77	126	235	554	2314
3	62	89	134	219	405	950	3948
4	94	133	200	325	600	1400	5800
5	129	183	274	443	816	1898	7840
6	168	237	354	572	1050	2437	10048
7	210	296	440	710	1301	3015	12410
8	255	359	533	857	1568	3628	14914
9	303	425	630	1013	1850	4275	17550
10	353	495	732	1176	2146	4953	20311

Table A.3: Estimate for duration of transient period for tandem-M/M/1 system, in units of service time

A.8 Confidence Intervals for M/M/1 Queue

A.8.1 Mean Interarrival and Service Time

With the standard estimators (see Eq. A.8), the estimates for $1/\mu$ and $1/\lambda$ are Erlang-distributed, with order equal to the number of observations. For example, the 90% equal-tailed confidence interval around $\lambda = 0.8$ and $\mu = 1$ covers 1.19...1.32 and 0.95...1.05 for $n = 1000$.

A.8.2 Traffic Intensity and Queue Length

If ρ is not known exactly, it has to be estimated from a number of observations. [96] (also cited in [97, p. 289]) provides confidence intervals on ρ and thus immediately for any monotonically increasing function of ρ. ρ lies in the interval

$$\hat{\rho}F(1-\alpha/2; 2a, 2d) \ldots \hat{\rho}/F(1-\alpha/2; 2a, 2d)$$

with confidence level $100(1-\alpha)\%$, where $\hat{\rho}$ is the estimate for ρ and a, d are the number of arrivals and departures, respectively. Here, $F(p, \nu_1, \nu_2)$ stands for the quantile of Fisher's F-distribution with probability p and (ν_1, ν_2) degrees of freedom. For example, for 60 arrivals and departures and a nominal $\hat{\rho} = 0.5$, ρ should fall between 0.715 and 0.350 in 95% ($\alpha = 0.05$) of the experiments. Finally, two examples for 90% ($\alpha = 0.1$) confidence intervals and a point estimate of $\hat{\rho} = 0.8$. For $n = k = 1000$, the interval for ρ spans 0.7433 to 0.8611; thus, the expected number of customers has a confidence interval of 2.89 to 6.20. For 100,000 arrivals and departures, ρ falls between 0.794137 and 0.805906 in 90% of the experiments; therefore, the 0.90 confidence interval on the mean number of customers ($= \rho/(1-\rho)$) in the system spans the range 3.8576 to 4.1521. The range of ρ's can be used in any monotonically increasing function of ρ to obtain confidence intervals on performance measures.

A.8.3 Expected System Time

Since the system time is not a monotonic function of ρ only, the previous approach cannot be used directly. The estimate for the expected system time S is $1/(\hat{\mu} - \hat{\lambda})$. Since the standard estimators for the traffic and service parameters are

$$1/\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

$$1/\hat{\mu} = \frac{1}{k}\sum_{i=1}^{k} b_i,$$

where a_i is the interarrival time between the $i-1$th and ith customer and b_i is the service time of the ith customer, $1/\hat{\lambda}$ and $1/\hat{\mu}$ are Erlang-n and Erlang-k distributed, with densities

$$1/\hat{\lambda} \sim f(x) = \frac{(n\lambda)^n}{(n-1)!} x^{n-1} e^{-n\lambda x}$$

and

$$1/\hat{\mu} \sim g(x) = \frac{(k\mu)^k}{(k-1)!} x^{k-1} e^{-k\lambda x},$$

respectively. Therefore, $\hat{\lambda}$ and $\hat{\mu}$ have the densities $f'(x) = f(1/x)/x^2$ and $g'(x) = g(1/x)/x^2$. The density of the difference $\hat{\mu} - \hat{\lambda}$ is given, assuming independence of the observations, by the convolution of their densities, $h(x) = f(x) * g(x)$. The distribution of the system time is now computed by double integration as

$$S(t) = \int_0^t \int_{1/x}^{\infty} f'(y - 1/x) g(y)\, dy\, dx$$

Since the integration term decays rapidly to zero, the indefinite integral can be replaced by a definite integral over the range from $1/x$ to, say, 10. $S(t)$ was evaluated numerically using the IMSL routine `DTWODQ` in double precision. Due to its factorial-exponential form, $f(x)$ has to be evaluated logarithmically as

$$\exp\left[n\ln(x\lambda x) - \ln x - \ln(n-1)! - n\lambda x)\right]$$

and *mutatis mutandis* for $g(x)$. Even so, for very large n and k, say 100,000, no numerical answer could be obtained. Table A.4 tabulates some point of the distribution of $S(t)$ for different values of $n = k$ and $\hat{\lambda} = 0.8$, $\hat{\mu} = 1$. The nominal system time is 5.

t	$n = 200$	$n = 1000$
2	0.0009	0.0000
3	0.0750	0.0006
4	0.2919	0.1101
5	0.5000	0.5000
6	0.6440	0.7956
7	0.7371	0.9221
8	0.7977	0.9689
9	0.8386	0.9865
10	0.8671	0.9936
11	0.8877	0.9967

Table A.4: $P[S < t]$ for M/M/1 queue with $\lambda = 0.8$, $\mu = 1$

Any interval that covers a $1 - \alpha$ of the area under the density function can be considered a confidence interval. Ideally, the tightest such interval is desired. For ease of computation, however, it is recommended [98, p. 208] to pick equal tails. For example, given a 90% confidence interval, the lower bound is $\arg(P[S < t]) = 0.05$ and the upper bound is $\arg(P[S < t]) = 0.95$. Note that the bound will in general not be symmetrical around the point estimate.

Appendix B

Some Useful Results

This appendix contains results in algebra, elementary calculus and general queueing theory that we found useful in calculations and derivations.

B.1 Some Laplace Transform pairs

$$\frac{\mu^k t^{k-1} e^{-\mu t}}{(k-1)!} = \left(\frac{\mu}{s+\mu}\right)^k \tag{B.1}$$

B.2 Integration of $x^n e^x$

$$\int x^n e^{ax}\,dx = e^{ax}\left[\frac{x^n}{a} + \sum_{k=1}^{n}(-1)^k c_k\right] = e^{ax}\left[\frac{x^n}{a} + \sum_{k=1}^{n}(-1)^k \frac{n(n-1)\cdots(n-k+1)}{a^{k+1}} x^{n-k}\right] \tag{B.2}$$

The coefficient c_k within the summation can be computed recursively as

$$c_k = -\frac{n-k+1}{ax} c_{k-1}$$

For $n = 0$ and $n = 1$, the results are simply

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

and

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integrals of this type lead to the incomplete gamma function $\gamma(n,b)$, with

$$\gamma(n,b) = \int_0^b x^{n-1}e^{-x}\,dx, \text{ for } n > 0$$

and the normalized incomplete gamma function $P(n,b)$ (see [57, p. 160f] for notation)

$$P(n,b) = \frac{\gamma(n,b)}{\Gamma(n)}.$$

Here, n does not have to be an integer. For n integral, $\Gamma(n) = (n-1)!$. The *Numerical Recipes* routine `GAMMP` computes $P(n, b)$.

A simpler equation results for the definite integral from t to infinity, assuming $a > 0$

$$\int_t^\infty \frac{a(ax)^n e^{-ax}}{n!}\, dx = \sum_{k=0}^{n} \frac{(at)^k}{k!} e^{-at} \tag{B.3}$$

which follows by induction from the recursive relation

$$\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx.$$

With these definitions, the cdf of the Erlang density[1] of nth order and mean $1/\mu$

$$E_n = \frac{(n\mu)^n}{(n-1)!} x^{n-1} e^{-n\mu x}$$

becomes simply $P(n, n\mu x)$. Hence, the sum of n exponentially distributed RVs, each with mean $1/\mu$ has the cdf $P(n, \mu x)$.

B.3 Scaled and Shifted Integration Variable

For a scaled and shifted integration variable $ct - a$,

$$\int_{t_1}^{t_2} f(ct-a)\, dt = \frac{1}{c} \int_{ct_1 - a}^{ct_2 - a} f(t)\, dt \tag{B.4}$$

holds.

B.4 Reversing the Order of Double Summations

$$\sum_{k=1}^{m} \sum_{j=\beta}^{k} a_{jk} = \sum_{j=\beta}^{m} \sum_{k=j}^{m} a_{jk}$$

$$\sum_{k=1}^{\infty} a_k \sum_{j=k}^{\infty} b_j = \sum_{j=1}^{\infty} b_j \sum_{k=1}^{j} a_k$$

B.5 The Generalized Leibniz Product Rule

From the Leibniz product rule for the nth derivative of a product of two functions (see, e.g., [99, p. 321]), we can easily generalize to a product of M functions:

$$\left[\prod_{i=1}^{M} f_i\right]^{(n)} = \sum_{k_1+k_2+\ldots+k_M=n} \binom{n}{k_1, k_2, \ldots, k_M} \prod_{i=1}^{M} f_i^{(k_i)} \tag{B.5}$$

[1] The Erlang density is the integer-parameter special case of the gamma distribution.

where the polynomial coefficient is defined as

$$\binom{n}{k_1, k_2, \ldots, k_M} = \frac{n!}{k_1! k_2! \cdots k_M!}$$

For the first derivative, the generalized product rule emerges

$$\left[\prod_{i=1}^{M} f_i\right]' = \sum_{k=1}^{M} f_1 \cdot f_2 \cdots f_{k-1} f_k' f_{k+1} \cdots f_M$$

with a total of M terms.

B.6 Expansion of a Product of Two-Term Sums

$$\prod_{i=1}^{M}(a+b_i) = \sum_{k=0}^{M} a^k \sum_{\sum_{l_i}=M-k} \prod_{i=1}^{M} b_i^{l_i}, \text{ where } l_i \in \{0,1\}$$

The inner sum iterates over all $l_i, l_i \in \{0,1\}$, such that the sum of the l_i equals $M-k$. In other words, each term in the inner sum consists of the product of $M-k$ of the b_i. The inner sum consists of all $\binom{n}{M-k}$ possible such arrangements.

$$\prod_{i=1}^{M}(a_{i1} + a_{i2} + \ldots + a_{ir}) = \sum_{l_{ik}} \prod_{i,k} a_{ik}^{l_{ik}}, \text{ where } l_{ik} \in \{0,1\} \tag{B.6}$$

where the sum is over all permutations with repetition of M elements out of r, i.e., it has r^M terms. The l_{ik} are easily generated by treating them as digits of a base-r number.

B.7 The Moments of the Exponential Distribution

The first three moments of a distribution give a good indication of its shape. Since the moment-generating function of the exponential distribution with parameter μ is

$$f^*(-s) = \Theta(s) = \frac{\mu}{\mu - s},$$

the n-th derivative with respect to s is shown to be

$$\Theta^{(n)}(s) = \frac{\mu n!}{(\mu - s)^{n+1}}.$$

Therefore, the n-th moment is

$$M_n = \Theta^{(n)}(0) = \frac{n!}{\mu^n}. \tag{B.7}$$

Thus, for $\mu = 1$, the moments are 1, 2 and 6, while for $\mu = 0.8$, they come out to be 1.25, 3.125 and 11.72.

B.8 The Cdf of the Sum of Two Independent, Positive RVs

If the RVs $\mathbf{x}$ and $\mathbf{y}$ are independent and positive RVs, the distribution of their sum z is given by

$$F(z) = \int_0^\infty \int_0^{z-y} f_2(y)F_1(z-y)\,dy$$

or

$$F(z) = \int_0^\infty \int_0^{z-y} f_1(y)F_2(z-y)\,dy$$

B.9 The Pdf of the Sum of n Independent, Positive RVs

The sum $s_n = \sum_{i=1}^n x_i$, with x_i independent and $f_i(x) = 0$ for $x \notin [0,\infty)$, is distributed with the density function

$$f(s_n) = \int_0^{x_n} \cdots \int_0^{x_2} f_n(x_n - x_{n-1}) f_{n-1}(x_{n-1} - x_{n-2}) \cdots f_1(x_1)\,dx_1 dx_2 \cdots dx_{n-1}$$

as can be shown by repeatedly applying the standard results for the sum of two RVs.

B.10 The Sum of Two Positive RVs with Point Masses at the Origin

Let $X_{12} = X_1 + X_2$, where $f_1(x) + a_1\delta(x)$ and $f_2(y) + a_2\delta(y)$ are the densities of x_1 and x_2, respectively. $f_i(x)$ is zero outside the interval $[0, \tau_i]$, where τ_i may be infinity. Then,

$$\begin{aligned}
f_{12}(y) &= \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} [f_1(x) + a_1\delta(x)][f_2(y-x) + a_2\delta(y-x)]\,dx \\
&= \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} f_1(x)f_2(y-x)\,dx + a_2 \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} f_1(x)\delta(y-x)\,dx + a_1 \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} \delta(x)f_2(y-x)\,dx \\
&\quad + a_1a_2 \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} \delta(x)\delta(y-x)\,dx \\
&= \int_{0\wedge(y-\tau_2)}^{y\wedge\tau_1} f_1(x)f_2(y-x)\,dx + a_2 f_1(y) + a_1 f_2(y) + a_1a_2\delta(y) \qquad \text{(B.8)}
\end{aligned}$$

Here, $a \wedge b \triangleq \min(a,b)$.

B.11 The Moments of the Sum of Two Independent RVs

Since the moment-generating function of the sum of two independent RVs, X and Y, is the product of their respective MGFs, we obtain

$$E[(X+Y)^r] = \frac{\partial^r}{\partial s^r} E[e^{sx}]E[e^{sy}]\Big|_{s=0} = \sum_{k=0}^{r} \binom{r}{k} E[e^{sx}]^{(k)} E[e^{sy}]^{(r-k)}\Big|_{s=0} = \sum_{k=0}^{r} \binom{r}{k} E[x^k]E[y^{r-k}]$$

B.12 A Geometric Sum of Exponential RVs

It is easy to show that a sum with N exponentially distributed random variables with parameter μ, where N is geometrically distributed with parameter p, is again exponentially distributed with parameter $p\mu$. The characteristic function $\Phi(\omega)$ of the sum S of a random number of random variables X is given by [100, p. 276] $\Phi_S(\omega) = G_N(\Phi_X(\omega))$, where $G_N(z)$ is the generating function of the discrete random variable N. Here, $G_N(z) = pz/(1-qz)$, with $q = 1-p$, and $\Phi_X(\omega;\mu) = \mu/(\mu - j\omega)$. Thus,

$$\Phi_S(\omega) = \frac{p\frac{\mu}{\mu-j\omega}}{1-q\frac{\mu}{\mu-j\omega}} = \frac{p\mu}{j\mu\omega-(1-p)\mu} = \frac{p\mu}{p\mu-j\omega} = \Phi_x(\omega;p\mu)$$

This result is a consequence of the memorylessness of both distributions and is useful in the simulation of voice processes.

B.13 Nth Derivatives

It is easy to show by induction that

$$\frac{\partial^r}{\partial s^r}\frac{1}{\nu-s} = \frac{r!}{(\nu-s)^{r+1}}. \tag{B.9}$$

In general,

$$\frac{\partial^r}{\partial s^r} f(s) = (-1)^r \frac{\partial^r}{\partial s^r} f(-s)$$

Somewhat more tedious is the derivation of the fact that

$$\frac{\partial^r}{\partial s^r}\frac{e^{\tau s}}{\nu-s} = r!e^{\tau s}\sum_{i=1}^{r+1}\frac{\tau^{r+1-i}}{(\nu-s)^i(r+1-i)!} \tag{B.10}$$

The induction step proceeds by differentiating the right hand side of the previous equation with respect to s:

$$\begin{aligned}
&\frac{\partial^{r+1}}{\partial s^{r+1}}\frac{e^{\tau s}}{\nu-s}\\
&= r!e^{\tau s}\left[\sum_{i=1}^{r+1}\frac{i\tau^{r+1-i}}{(\nu-s)^{i+1}(r+1-i)!} + \tau\sum_{i=1}^{r+1}\frac{i\tau^{r+1-i}}{(\nu-s)^i(r+1-i)!}\right]\\
&= r!e^{\tau s}\left[\sum_{i=2}^{r+2}\frac{(i-1)\tau^{r+2-i}}{(\nu-s)^i(r+2-i)!} + \tau\sum_{i=1}^{r+1}\frac{i\tau^{r+2-i}}{(\nu-s)^i(r+1-i)!}\right]\\
&= r!e^{\tau s}\left[\sum_{i=2}^{r+2}\frac{[(i-1)+(r+2-i)]\tau^{r+2-i}}{(\nu-s)^i(r+2-i)!} + \frac{r+1}{(\nu-s)^{r+2}} + \frac{\tau^{r+1}}{(\nu-s)r!}\right]\\
&= r!e^{\tau s}\left[\sum_{i=1}^{r+2}\frac{\tau^{r+2-i}}{(\nu-s)^i(r+2-i)!}\right]
\end{aligned}$$

B.14 Finite Sums and Series

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$$

$$\sum_{k=0}^{\infty} \frac{kx^k}{k!} = xe^x$$

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$\sum_{k=0}^{n} k\binom{n}{k} = 2^{n-1}n$$

$$\sum_{k=0}^{n} k\rho^k = \frac{\rho + (n\rho - n - 1)\rho^{n+1}}{(1-\rho)^2} \to \frac{\rho}{(1-\rho)^2}$$

B.15 Relation between Arrival, Departure and Time-Average Probabilities

In any queueing system that changes state by positive or negative unit step values (thus excluding systems with batch arrivals and departures), the state probabilities just prior to arrivals (seen by an arriving customer) are equal to those left behind by a departure [61, p. 175] [62, p. 359].

In a queueing system with Poisson arrivals, the state probabilities seen by an arrival equal those observed at an arbitrary instant. (Poisson Arrivals See Time Averages, PASTA.) [61, p. 117] [62, p. 264] [101, p. 293f, Section 4.3]. Note that PAsTA holds only if the probability measures converge as $t \to \infty$.

In a queueing system obeying a first-come, first-serve discipline, the unfinished work is equal to the virtual waiting time, the waiting time of a customer if it arrived at a given time [61, p. 206].

Appendix C

Elementary Combinatorics

C.1 Permutations

Here, order within each selection matters. Bronstein calls permutations only those with $n = k$ and refers to others as variations.

C.1.1 Permutation without Repetition

From a set of n distinct elements, we count the number of ways we pick k and arrange these.

There are

$$_nP_k = \frac{n!}{(n-k)!}$$

permutations of n elements taken k at a time, without repetitions.

C.1.2 Permutation with Repetition

Here, we may pick each element more than once to arrive at a subset of k.

There are

$$_n\tilde{P}_k = n^k$$

permutations of n elements taken k at a time, allowing repetitions.

C.1.3 Permutation of Groups

Given l groups of identical elements with a total of $\sum_i^l k_i = k$ members, there are

$$_n\tilde{P}_{k_1,k_2,\ldots} = \frac{n!}{k_1! \cdot k_2! \cdots k_l!} = \binom{n}{k_1, k_2, \ldots, k_l}$$

permutations with n total elements where the element of group i occurs k_i times. For $l = 2$, we have $_nC_{k_1}$. Example: There are $\binom{n}{k}$ n-bit binary numbers with k ones (or zeros).

C.2 Combinations

Different orderings are not distinguished. Visualize as picking k items from a lineup of n items, noting the position indices without regard to the order of picking.

C.2.1 Combinations without Repetition

There are

$$_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

combinations of n elements taken k at a time, without repetitions.

C.2.2 Combinations with Repetition

There are

$$_n\tilde{C}_k = \binom{n+k-1}{k}$$

combinations of n elements taken k at a time, allowing repetition.

C.3 Examples

Let $n = 4, k = 2, k_1 = 3, k_2 = 1$.

$_4C_2$		*ab*	*ac*	*ad*			*bc*	*bd*				*cd*				*dd*
$_4\tilde{C}_2$	*aa*	*ab*	*ac*	*ad*		*bb*	*bc*	*bd*			*cc*	*cd*				*dd*
$_4P_2$		*ab*	*ac*	*ad*	*ba*		*bc*	*bd*	*ca*	*cb*		*cd*	*da*	*db*	*dc*	
$_4\tilde{P}_2$	*aa*	*ab*	*ac*	*ad*	*ba*	*bb*	*bc*	*bd*	*ca*	*cb*	*cc*	*cd*	*da*	*db*	*dc*	*dd*
$_4\tilde{P}_{2,2}$	*aabb*	*abab*	*abba*	*baba*	*bbaa*	*abba*	*baab*									

Appendix D

FIFO-BW: Loss Tables

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	1.94	$1.077\cdot10^{-1}$	$0.000\cdot10^{-0}$	$0.000\cdot10^{-0}$	$1.077\cdot10^{-1}$
	2.13	$9.406\cdot10^{-2}$	$2.091\cdot10^{-4}$	$2.308\cdot10^{-4}$	$9.427\cdot10^{-2}$
	2.33	$8.216\cdot10^{-2}$	$7.660\cdot10^{-4}$	$8.345\cdot10^{-4}$	$8.293\cdot10^{-2}$
	2.52	$7.176\cdot10^{-2}$	$1.583\cdot10^{-3}$	$1.706\cdot10^{-3}$	$7.335\cdot10^{-2}$
	2.71	$6.267\cdot10^{-2}$	$2.596\cdot10^{-3}$	$2.770\cdot10^{-3}$	$6.527\cdot10^{-2}$
	2.91	$5.474\cdot10^{-2}$	$3.756\cdot10^{-3}$	$3.974\cdot10^{-3}$	$5.849\cdot10^{-2}$
	3.10	$4.783\cdot10^{-2}$	$5.019\cdot10^{-3}$	$5.272\cdot10^{-3}$	$5.285\cdot10^{-2}$
	3.29	$4.177\cdot10^{-2}$	$6.372\cdot10^{-3}$	$6.650\cdot10^{-3}$	$4.814\cdot10^{-2}$
	3.49	$3.647\cdot10^{-2}$	$7.788\cdot10^{-3}$	$8.083\cdot10^{-3}$	$4.426\cdot10^{-2}$
	3.68	$3.185\cdot10^{-2}$	$9.251\cdot10^{-3}$	$9.556\cdot10^{-3}$	$4.110\cdot10^{-2}$
	3.88	$2.781\cdot10^{-2}$	$1.076\cdot10^{-2}$	$1.106\cdot10^{-2}$	$3.856\cdot10^{-2}$
0.50	3.48	$8.977\cdot10^{-2}$	$1.330\cdot10^{-10}$	$1.461\cdot10^{-10}$	$8.977\cdot10^{-2}$
	3.83	$7.518\cdot10^{-2}$	$4.506\cdot10^{-4}$	$4.873\cdot10^{-4}$	$7.563\cdot10^{-2}$
	4.17	$6.302\cdot10^{-2}$	$1.596\cdot10^{-3}$	$1.704\cdot10^{-3}$	$6.462\cdot10^{-2}$
	4.52	$5.283\cdot10^{-2}$	$3.218\cdot10^{-3}$	$3.398\cdot10^{-3}$	$5.605\cdot10^{-2}$
	4.87	$4.430\cdot10^{-2}$	$5.163\cdot10^{-3}$	$5.403\cdot10^{-3}$	$4.946\cdot10^{-2}$
	5.22	$3.716\cdot10^{-2}$	$7.329\cdot10^{-3}$	$7.612\cdot10^{-3}$	$4.449\cdot10^{-2}$
	5.56	$3.118\cdot10^{-2}$	$9.645\cdot10^{-3}$	$9.955\cdot10^{-3}$	$4.083\cdot10^{-2}$
	5.91	$2.618\cdot10^{-2}$	$1.206\cdot10^{-2}$	$1.238\cdot10^{-2}$	$3.824\cdot10^{-2}$
	6.26	$2.198\cdot10^{-2}$	$1.455\cdot10^{-2}$	$1.488\cdot10^{-2}$	$3.653\cdot10^{-2}$
	6.61	$1.845\cdot10^{-2}$	$1.709\cdot10^{-2}$	$1.741\cdot10^{-2}$	$3.554\cdot10^{-2}$
	6.96	$1.549\cdot10^{-2}$	$1.967\cdot10^{-2}$	$1.998\cdot10^{-2}$	$3.517\cdot10^{-2}$
0.80	10.77	$3.968\cdot10^{-2}$	$-2.665\cdot10^{-17}$	$-2.776\cdot10^{-17}$	$3.968\cdot10^{-2}$
	11.85	$3.157\cdot10^{-2}$	$7.867\cdot10^{-4}$	$8.123\cdot10^{-4}$	$3.236\cdot10^{-2}$
	12.93	$2.518\cdot10^{-2}$	$2.699\cdot10^{-3}$	$2.769\cdot10^{-3}$	$2.788\cdot10^{-2}$
	14.00	$2.013\cdot10^{-2}$	$5.286\cdot10^{-3}$	$5.395\cdot10^{-3}$	$2.541\cdot10^{-2}$
	15.08	$1.612\cdot10^{-2}$	$8.275\cdot10^{-3}$	$8.410\cdot10^{-3}$	$2.439\cdot10^{-2}$
	16.16	$1.292\cdot10^{-2}$	$1.151\cdot10^{-2}$	$1.166\cdot10^{-2}$	$2.443\cdot10^{-2}$
	17.23	$1.037\cdot10^{-2}$	$1.489\cdot10^{-2}$	$1.505\cdot10^{-2}$	$2.527\cdot10^{-2}$
	18.31	$8.334\cdot10^{-3}$	$1.837\cdot10^{-2}$	$1.853\cdot10^{-2}$	$2.671\cdot10^{-2}$
	19.39	$6.699\cdot10^{-3}$	$2.191\cdot10^{-2}$	$2.206\cdot10^{-2}$	$2.861\cdot10^{-2}$
	20.47	$5.389\cdot10^{-3}$	$2.549\cdot10^{-2}$	$2.563\cdot10^{-2}$	$3.088\cdot10^{-2}$
0.90	22.68	$2.024\cdot10^{-2}$	$6.483\cdot10^{-12}$	$6.616\cdot10^{-12}$	$2.024\cdot10^{-2}$
	24.95	$1.586\cdot10^{-2}$	$8.871\cdot10^{-4}$	$9.014\cdot10^{-4}$	$1.674\cdot10^{-2}$
	27.21	$1.247\cdot10^{-2}$	$3.011\cdot10^{-3}$	$3.049\cdot10^{-3}$	$1.548\cdot10^{-2}$
	29.48	$9.833\cdot10^{-3}$	$5.845\cdot10^{-3}$	$5.903\cdot10^{-3}$	$1.568\cdot10^{-2}$
	31.75	$7.772\cdot10^{-3}$	$9.091\cdot10^{-3}$	$9.162\cdot10^{-3}$	$1.686\cdot10^{-2}$
	34.02	$6.154\cdot10^{-3}$	$1.258\cdot10^{-2}$	$1.266\cdot10^{-2}$	$1.873\cdot10^{-2}$
	36.29	$4.879\cdot10^{-3}$	$1.621\cdot10^{-2}$	$1.629\cdot10^{-2}$	$2.109\cdot10^{-2}$
	38.55	$3.873\cdot10^{-3}$	$1.993\cdot10^{-2}$	$2.000\cdot10^{-2}$	$2.380\cdot10^{-2}$
	40.82	$3.077\cdot10^{-3}$	$2.370\cdot10^{-2}$	$2.377\cdot10^{-2}$	$2.678\cdot10^{-2}$
	43.09	$2.446\cdot10^{-3}$	$2.751\cdot10^{-2}$	$2.757\cdot10^{-2}$	$2.995\cdot10^{-2}$

Table D.1: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 2$, uncontrolled loss of $5\cdot10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	1.64	$1.922\cdot10^{-1}$	$-2.242\cdot10^{-16}$	$-2.776\cdot10^{-16}$	$1.922\cdot10^{-1}$
	1.97	$1.541\cdot10^{-1}$	$4.278\cdot10^{-5}$	$5.058\cdot10^{-5}$	$1.542\cdot10^{-1}$
	2.29	$1.233\cdot10^{-1}$	$2.914\cdot10^{-4}$	$3.324\cdot10^{-4}$	$1.236\cdot10^{-1}$
	2.62	$9.862\cdot10^{-2}$	$9.974\cdot10^{-4}$	$1.106\cdot10^{-3}$	$9.962\cdot10^{-2}$
	2.95	$7.871\cdot10^{-2}$	$2.838\cdot10^{-3}$	$3.081\cdot10^{-3}$	$8.155\cdot10^{-2}$
	3.28	$6.277\cdot10^{-2}$	$5.445\cdot10^{-3}$	$5.810\cdot10^{-3}$	$6.822\cdot10^{-2}$
	3.60	$5.006\cdot10^{-2}$	$8.466\cdot10^{-3}$	$8.912\cdot10^{-3}$	$5.852\cdot10^{-2}$
	3.93	$3.987\cdot10^{-2}$	$1.171\cdot10^{-2}$	$1.219\cdot10^{-2}$	$5.158\cdot10^{-2}$
	4.26	$3.176\cdot10^{-2}$	$1.502\cdot10^{-2}$	$1.551\cdot10^{-2}$	$4.678\cdot10^{-2}$
	4.59	$2.528\cdot10^{-2}$	$1.834\cdot10^{-2}$	$1.882\cdot10^{-2}$	$4.362\cdot10^{-2}$
	4.92	$2.011\cdot10^{-2}$	$2.162\cdot10^{-2}$	$2.206\cdot10^{-2}$	$4.173\cdot10^{-2}$
0.50	2.98	$1.688\cdot10^{-1}$	$-2.307\cdot10^{-16}$	$-2.776\cdot10^{-16}$	$1.688\cdot10^{-1}$
	3.57	$1.254\cdot10^{-1}$	$1.574\cdot10^{-4}$	$1.800\cdot10^{-4}$	$1.256\cdot10^{-1}$
	4.17	$9.321\cdot10^{-2}$	$1.012\cdot10^{-3}$	$1.116\cdot10^{-3}$	$9.423\cdot10^{-2}$
	4.77	$6.921\cdot10^{-2}$	$3.003\cdot10^{-3}$	$3.227\cdot10^{-3}$	$7.222\cdot10^{-2}$
	5.36	$5.139\cdot10^{-2}$	$6.656\cdot10^{-3}$	$7.016\cdot10^{-3}$	$5.804\cdot10^{-2}$
	5.96	$3.817\cdot10^{-2}$	$1.112\cdot10^{-2}$	$1.156\cdot10^{-2}$	$4.929\cdot10^{-2}$
	6.55	$2.833\cdot10^{-2}$	$1.585\cdot10^{-2}$	$1.632\cdot10^{-2}$	$4.419\cdot10^{-2}$
	7.15	$2.103\cdot10^{-2}$	$2.055\cdot10^{-2}$	$2.099\cdot10^{-2}$	$4.158\cdot10^{-2}$
	7.74	$1.562\cdot10^{-2}$	$2.504\cdot10^{-2}$	$2.544\cdot10^{-2}$	$4.066\cdot10^{-2}$
	8.34	$1.159\cdot10^{-2}$	$2.926\cdot10^{-2}$	$2.961\cdot10^{-2}$	$4.086\cdot10^{-2}$
	8.94	$8.607\cdot10^{-3}$	$3.317\cdot10^{-2}$	$3.346\cdot10^{-2}$	$4.178\cdot10^{-2}$
0.80	9.42	$7.867\cdot10^{-2}$	$6.393\cdot10^{-14}$	$6.939\cdot10^{-14}$	$7.867\cdot10^{-2}$
	11.30	$5.270\cdot10^{-2}$	$4.104\cdot10^{-4}$	$4.332\cdot10^{-4}$	$5.311\cdot10^{-2}$
	13.19	$3.557\cdot10^{-2}$	$2.464\cdot10^{-3}$	$2.555\cdot10^{-3}$	$3.803\cdot10^{-2}$
	15.07	$2.413\cdot10^{-2}$	$6.626\cdot10^{-3}$	$6.790\cdot10^{-3}$	$3.076\cdot10^{-2}$
	16.95	$1.644\cdot10^{-2}$	$1.243\cdot10^{-2}$	$1.263\cdot10^{-2}$	$2.886\cdot10^{-2}$
	18.84	$1.122\cdot10^{-2}$	$1.857\cdot10^{-2}$	$1.879\cdot10^{-2}$	$2.979\cdot10^{-2}$
	20.72	$7.669\cdot10^{-3}$	$2.446\cdot10^{-2}$	$2.465\cdot10^{-2}$	$3.213\cdot10^{-2}$
	22.60	$5.249\cdot10^{-3}$	$2.982\cdot10^{-2}$	$2.997\cdot10^{-2}$	$3.507\cdot10^{-2}$
	24.49	$3.596\cdot10^{-3}$	$3.453\cdot10^{-2}$	$3.466\cdot10^{-2}$	$3.813\cdot10^{-2}$
	26.37	$2.464\cdot10^{-3}$	$3.856\cdot10^{-2}$	$3.866\cdot10^{-2}$	$4.103\cdot10^{-2}$
0.90	19.95	$4.071\cdot10^{-2}$	$1.331\cdot10^{-16}$	$1.388\cdot10^{-16}$	$4.071\cdot10^{-2}$
	23.94	$2.637\cdot10^{-2}$	$5.017\cdot10^{-4}$	$5.153\cdot10^{-4}$	$2.687\cdot10^{-2}$
	27.93	$1.729\cdot10^{-2}$	$2.950\cdot10^{-3}$	$3.002\cdot10^{-3}$	$2.024\cdot10^{-2}$
	31.92	$1.143\cdot10^{-2}$	$7.763\cdot10^{-3}$	$7.853\cdot10^{-3}$	$1.919\cdot10^{-2}$
	35.91	$7.594\cdot10^{-3}$	$1.409\cdot10^{-2}$	$1.420\cdot10^{-2}$	$2.169\cdot10^{-2}$
	39.90	$5.062\cdot10^{-3}$	$2.058\cdot10^{-2}$	$2.068\cdot10^{-2}$	$2.564\cdot10^{-2}$
	43.89	$3.381\cdot10^{-3}$	$2.663\cdot10^{-2}$	$2.672\cdot10^{-2}$	$3.002\cdot10^{-2}$
	47.88	$2.262\cdot10^{-3}$	$3.203\cdot10^{-2}$	$3.210\cdot10^{-2}$	$3.429\cdot10^{-2}$
	51.87	$1.515\cdot10^{-3}$	$3.666\cdot10^{-2}$	$3.672\cdot10^{-2}$	$3.817\cdot10^{-2}$
	55.86	$1.015\cdot10^{-3}$	$4.049\cdot10^{-2}$	$4.053\cdot10^{-2}$	$4.150\cdot10^{-2}$
	59.85	$6.805\cdot10^{-4}$	$4.349\cdot10^{-2}$	$4.352\cdot10^{-2}$	$4.417\cdot10^{-2}$

Table D.2: Loss components in FIFO-BW system with homogeneous traffic and deadlines, $M = 3$, uncontrolled loss of $5\cdot10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	1.34	$3.598\cdot10^{-1}$	$1.777\cdot10^{-16}$	$2.776\cdot10^{-16}$	$3.598\cdot10^{-1}$
	1.87	$2.583\cdot10^{-1}$	$4.728\cdot10^{-6}$	$6.375\cdot10^{-6}$	$2.583\cdot10^{-1}$
	2.41	$1.828\cdot10^{-1}$	$2.209\cdot10^{-4}$	$2.704\cdot10^{-4}$	$1.831\cdot10^{-1}$
	2.94	$1.281\cdot10^{-1}$	$1.606\cdot10^{-3}$	$1.842\cdot10^{-3}$	$1.298\cdot10^{-1}$
	3.48	$8.931\cdot10^{-2}$	$4.806\cdot10^{-3}$	$5.277\cdot10^{-3}$	$9.411\cdot10^{-2}$
	4.01	$6.198\cdot10^{-2}$	$1.004\cdot10^{-2}$	$1.070\cdot10^{-2}$	$7.201\cdot10^{-2}$
	4.55	$4.289\cdot10^{-2}$	$1.604\cdot10^{-2}$	$1.676\cdot10^{-2}$	$5.892\cdot10^{-2}$
	5.08	$2.962\cdot10^{-2}$	$2.194\cdot10^{-2}$	$2.261\cdot10^{-2}$	$5.155\cdot10^{-2}$
	5.62	$2.041\cdot10^{-2}$	$2.737\cdot10^{-2}$	$2.794\cdot10^{-2}$	$4.778\cdot10^{-2}$
	6.16	$1.406\cdot10^{-2}$	$3.218\cdot10^{-2}$	$3.264\cdot10^{-2}$	$4.625\cdot10^{-2}$
	6.69	$9.684\cdot10^{-3}$	$3.635\cdot10^{-2}$	$3.671\cdot10^{-2}$	$4.604\cdot10^{-2}$
0.50	2.49	$3.323\cdot10^{-1}$	$2.780\cdot10^{-16}$	$4.163\cdot10^{-16}$	$3.323\cdot10^{-1}$
	3.48	$2.090\cdot10^{-1}$	$4.669\cdot10^{-5}$	$5.903\cdot10^{-5}$	$2.090\cdot10^{-1}$
	4.48	$1.294\cdot10^{-1}$	$1.180\cdot10^{-3}$	$1.356\cdot10^{-3}$	$1.306\cdot10^{-1}$
	5.47	$7.958\cdot10^{-2}$	$5.413\cdot10^{-3}$	$5.881\cdot10^{-3}$	$8.500\cdot10^{-2}$
	6.47	$4.869\cdot10^{-2}$	$1.240\cdot10^{-2}$	$1.303\cdot10^{-2}$	$6.108\cdot10^{-2}$
	7.47	$2.972\cdot10^{-2}$	$2.042\cdot10^{-2}$	$2.104\cdot10^{-2}$	$5.014\cdot10^{-2}$
	8.46	$1.811\cdot10^{-2}$	$2.779\cdot10^{-2}$	$2.830\cdot10^{-2}$	$4.590\cdot10^{-2}$
	9.46	$1.103\cdot10^{-2}$	$3.392\cdot10^{-2}$	$3.430\cdot10^{-2}$	$4.495\cdot10^{-2}$
	10.45	$6.706\cdot10^{-3}$	$3.878\cdot10^{-2}$	$3.904\cdot10^{-2}$	$4.548\cdot10^{-2}$
	11.45	$4.080\cdot10^{-3}$	$4.246\cdot10^{-2}$	$4.264\cdot10^{-2}$	$4.654\cdot10^{-2}$
	12.44	$2.482\cdot10^{-3}$	$4.516\cdot10^{-2}$	$4.527\cdot10^{-2}$	$4.764\cdot10^{-2}$
0.80	8.09	$1.689\cdot10^{-1}$	$-9.228\cdot10^{-16}$	$-1.110\cdot10^{-15}$	$1.689\cdot10^{-1}$
	11.33	$8.578\cdot10^{-2}$	$2.671\cdot10^{-4}$	$2.921\cdot10^{-4}$	$8.604\cdot10^{-2}$
	14.57	$4.418\cdot10^{-2}$	$4.012\cdot10^{-3}$	$4.197\cdot10^{-3}$	$4.820\cdot10^{-2}$
	17.81	$2.294\cdot10^{-2}$	$1.277\cdot10^{-2}$	$1.307\cdot10^{-2}$	$3.571\cdot10^{-2}$
	21.04	$1.195\cdot10^{-2}$	$2.300\cdot10^{-2}$	$2.328\cdot10^{-2}$	$3.495\cdot10^{-2}$
	24.28	$6.241\cdot10^{-3}$	$3.181\cdot10^{-2}$	$3.201\cdot10^{-2}$	$3.805\cdot10^{-2}$
	27.52	$3.263\cdot10^{-3}$	$3.839\cdot10^{-2}$	$3.852\cdot10^{-2}$	$4.166\cdot10^{-2}$
	30.75	$1.707\cdot10^{-3}$	$4.296\cdot10^{-2}$	$4.304\cdot10^{-2}$	$4.467\cdot10^{-2}$
	33.99	$8.929\cdot10^{-4}$	$4.595\cdot10^{-2}$	$4.599\cdot10^{-2}$	$4.684\cdot10^{-2}$
	37.23	$4.672\cdot10^{-4}$	$4.778\cdot10^{-2}$	$4.780\cdot10^{-2}$	$4.825\cdot10^{-2}$
0.90	17.28	$8.997\cdot10^{-2}$	$-5.052\cdot10^{-16}$	$-5.551\cdot10^{-16}$	$8.997\cdot10^{-2}$
	24.19	$4.243\cdot10^{-2}$	$3.841\cdot10^{-4}$	$4.011\cdot10^{-4}$	$4.282\cdot10^{-2}$
	31.10	$2.065\cdot10^{-2}$	$5.143\cdot10^{-3}$	$5.251\cdot10^{-3}$	$2.579\cdot10^{-2}$
	38.01	$1.020\cdot10^{-2}$	$1.511\cdot10^{-2}$	$1.527\cdot10^{-2}$	$2.531\cdot10^{-2}$
	44.92	$5.073\cdot10^{-3}$	$2.581\cdot10^{-2}$	$2.595\cdot10^{-2}$	$3.089\cdot10^{-2}$
	51.83	$2.533\cdot10^{-3}$	$3.444\cdot10^{-2}$	$3.453\cdot10^{-2}$	$3.697\cdot10^{-2}$
	58.75	$1.267\cdot10^{-3}$	$4.056\cdot10^{-2}$	$4.061\cdot10^{-2}$	$4.183\cdot10^{-2}$
	65.66	$6.342\cdot10^{-4}$	$4.459\cdot10^{-2}$	$4.462\cdot10^{-2}$	$4.522\cdot10^{-2}$
	72.57	$3.176\cdot10^{-4}$	$4.707\cdot10^{-2}$	$4.708\cdot10^{-2}$	$4.739\cdot10^{-2}$
	79.48	$1.591\cdot10^{-4}$	$4.849\cdot10^{-2}$	$4.849\cdot10^{-2}$	$4.865\cdot10^{-2}$
	86.39	$7.969\cdot10^{-5}$	$4.924\cdot10^{-2}$	$4.924\cdot10^{-2}$	$4.932\cdot10^{-2}$

Table D.3: Loss components in FIFO-BW system with homogeneous traffic and deadlines for $M = 5$ and uncontrolled loss of $5\cdot10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	3.23	$4.380{\cdot}10^{-2}$	$0.000{\cdot}10^{-0}$	$0.000{\cdot}10^{-0}$	$4.380{\cdot}10^{-2}$
	3.55	$3.495{\cdot}10^{-2}$	$8.802{\cdot}10^{-5}$	$9.121{\cdot}10^{-5}$	$3.504{\cdot}10^{-2}$
	3.87	$2.789{\cdot}10^{-2}$	$3.071{\cdot}10^{-4}$	$3.159{\cdot}10^{-4}$	$2.819{\cdot}10^{-2}$
	4.19	$2.226{\cdot}10^{-2}$	$6.075{\cdot}10^{-4}$	$6.214{\cdot}10^{-4}$	$2.287{\cdot}10^{-2}$
	4.52	$1.776{\cdot}10^{-2}$	$9.609{\cdot}10^{-4}$	$9.783{\cdot}10^{-4}$	$1.872{\cdot}10^{-2}$
	4.84	$1.417{\cdot}10^{-2}$	$1.347{\cdot}10^{-3}$	$1.366{\cdot}10^{-3}$	$1.552{\cdot}10^{-2}$
	5.16	$1.130{\cdot}10^{-2}$	$1.754{\cdot}10^{-3}$	$1.774{\cdot}10^{-3}$	$1.306{\cdot}10^{-2}$
	5.49	$9.023{\cdot}10^{-3}$	$2.173{\cdot}10^{-3}$	$2.193{\cdot}10^{-3}$	$1.120{\cdot}10^{-2}$
	5.81	$7.197{\cdot}10^{-3}$	$2.602{\cdot}10^{-3}$	$2.620{\cdot}10^{-3}$	$9.799{\cdot}10^{-3}$
	6.13	$5.741{\cdot}10^{-3}$	$3.035{\cdot}10^{-3}$	$3.053{\cdot}10^{-3}$	$8.776{\cdot}10^{-3}$
	6.45	$4.583{\cdot}10^{-3}$	$3.471{\cdot}10^{-3}$	$3.487{\cdot}10^{-3}$	$8.054{\cdot}10^{-3}$
0.50	5.34	$3.493{\cdot}10^{-2}$	$2.942{\cdot}10^{-11}$	$3.049{\cdot}10^{-11}$	$3.493{\cdot}10^{-2}$
	5.87	$2.669{\cdot}10^{-2}$	$1.481{\cdot}10^{-4}$	$1.522{\cdot}10^{-4}$	$2.684{\cdot}10^{-2}$
	6.41	$2.040{\cdot}10^{-2}$	$5.039{\cdot}10^{-4}$	$5.144{\cdot}10^{-4}$	$2.091{\cdot}10^{-2}$
	6.94	$1.560{\cdot}10^{-2}$	$9.786{\cdot}10^{-4}$	$9.942{\cdot}10^{-4}$	$1.658{\cdot}10^{-2}$
	7.48	$1.194{\cdot}10^{-2}$	$1.522{\cdot}10^{-3}$	$1.540{\cdot}10^{-3}$	$1.346{\cdot}10^{-2}$
	8.01	$9.133{\cdot}10^{-3}$	$2.105{\cdot}10^{-3}$	$2.124{\cdot}10^{-3}$	$1.124{\cdot}10^{-2}$
	8.54	$6.989{\cdot}10^{-3}$	$2.710{\cdot}10^{-3}$	$2.729{\cdot}10^{-3}$	$9.699{\cdot}10^{-3}$
	9.08	$5.349{\cdot}10^{-3}$	$3.329{\cdot}10^{-3}$	$3.347{\cdot}10^{-3}$	$8.679{\cdot}10^{-3}$
	9.61	$4.094{\cdot}10^{-3}$	$3.956{\cdot}10^{-3}$	$3.973{\cdot}10^{-3}$	$8.051{\cdot}10^{-3}$
	10.15	$3.134{\cdot}10^{-3}$	$4.588{\cdot}10^{-3}$	$4.602{\cdot}10^{-3}$	$7.722{\cdot}10^{-3}$
	10.68	$2.399{\cdot}10^{-3}$	$5.222{\cdot}10^{-3}$	$5.235{\cdot}10^{-3}$	$7.622{\cdot}10^{-3}$
0.80	15.50	$1.479{\cdot}10^{-2}$	$0.000{\cdot}10^{-0}$	$0.000{\cdot}10^{-0}$	$1.479{\cdot}10^{-2}$
	17.05	$1.077{\cdot}10^{-2}$	$2.140{\cdot}10^{-4}$	$2.163{\cdot}10^{-4}$	$1.099{\cdot}10^{-2}$
	18.60	$7.862{\cdot}10^{-3}$	$7.092{\cdot}10^{-4}$	$7.148{\cdot}10^{-4}$	$8.571{\cdot}10^{-3}$
	20.15	$5.745{\cdot}10^{-3}$	$1.349{\cdot}10^{-3}$	$1.357{\cdot}10^{-3}$	$7.094{\cdot}10^{-3}$
	21.70	$4.202{\cdot}10^{-3}$	$2.065{\cdot}10^{-3}$	$2.074{\cdot}10^{-3}$	$6.267{\cdot}10^{-3}$
	23.25	$3.076{\cdot}10^{-3}$	$2.820{\cdot}10^{-3}$	$2.829{\cdot}10^{-3}$	$5.896{\cdot}10^{-3}$
	24.80	$2.253{\cdot}10^{-3}$	$3.596{\cdot}10^{-3}$	$3.604{\cdot}10^{-3}$	$5.849{\cdot}10^{-3}$
	26.35	$1.650{\cdot}10^{-3}$	$4.384{\cdot}10^{-3}$	$4.391{\cdot}10^{-3}$	$6.034{\cdot}10^{-3}$
	27.90	$1.210{\cdot}10^{-3}$	$5.177{\cdot}10^{-3}$	$5.183{\cdot}10^{-3}$	$6.387{\cdot}10^{-3}$
	29.45	$8.866{\cdot}10^{-4}$	$5.974{\cdot}10^{-3}$	$5.980{\cdot}10^{-3}$	$6.861{\cdot}10^{-3}$
	31.00	$6.500{\cdot}10^{-4}$	$6.774{\cdot}10^{-3}$	$6.778{\cdot}10^{-3}$	$7.424{\cdot}10^{-3}$
0.90	32.15	$7.459{\cdot}10^{-3}$	$1.004{\cdot}10^{-11}$	$1.011{\cdot}10^{-11}$	$7.459{\cdot}10^{-3}$
	35.36	$5.362{\cdot}10^{-3}$	$2.312{\cdot}10^{-4}$	$2.324{\cdot}10^{-4}$	$5.593{\cdot}10^{-3}$
	38.58	$3.864{\cdot}10^{-3}$	$7.606{\cdot}10^{-4}$	$7.636{\cdot}10^{-4}$	$4.624{\cdot}10^{-3}$
	41.79	$2.789{\cdot}10^{-3}$	$1.439{\cdot}10^{-3}$	$1.443{\cdot}10^{-3}$	$4.228{\cdot}10^{-3}$
	45.01	$2.016{\cdot}10^{-3}$	$2.193{\cdot}10^{-3}$	$2.198{\cdot}10^{-3}$	$4.209{\cdot}10^{-3}$
	48.22	$1.458{\cdot}10^{-3}$	$2.986{\cdot}10^{-3}$	$2.990{\cdot}10^{-3}$	$4.444{\cdot}10^{-3}$
	51.43	$1.055{\cdot}10^{-3}$	$3.798{\cdot}10^{-3}$	$3.802{\cdot}10^{-3}$	$4.854{\cdot}10^{-3}$
	54.65	$7.643{\cdot}10^{-4}$	$4.621{\cdot}10^{-3}$	$4.625{\cdot}10^{-3}$	$5.385{\cdot}10^{-3}$
	57.86	$5.537{\cdot}10^{-4}$	$5.450{\cdot}10^{-3}$	$5.453{\cdot}10^{-3}$	$6.004{\cdot}10^{-3}$
	61.08	$4.012{\cdot}10^{-4}$	$6.282{\cdot}10^{-3}$	$6.285{\cdot}10^{-3}$	$6.683{\cdot}10^{-3}$
	64.29	$2.908{\cdot}10^{-4}$	$7.117{\cdot}10^{-3}$	$7.119{\cdot}10^{-3}$	$7.407{\cdot}10^{-3}$

Table D.4: Loss components in FIFO-BW system with homogeneous traffic and deadlines for $M = 2$ and uncontrolled loss of $1{\cdot}10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	2.57	$1.020\cdot10^{-1}$	$0.000\cdot10^{-0}$	$0.000\cdot10^{-0}$	$1.020\cdot10^{-1}$
	3.09	$7.157\cdot10^{-2}$	$2.061\cdot10^{-5}$	$2.220\cdot10^{-5}$	$7.159\cdot10^{-2}$
	3.60	$5.016\cdot10^{-2}$	$1.317\cdot10^{-4}$	$1.386\cdot10^{-4}$	$5.029\cdot10^{-2}$
	4.12	$3.508\cdot10^{-2}$	$4.151\cdot10^{-4}$	$4.302\cdot10^{-4}$	$3.550\cdot10^{-2}$
	4.63	$2.453\cdot10^{-2}$	$1.040\cdot10^{-3}$	$1.066\cdot10^{-3}$	$2.557\cdot10^{-2}$
	5.14	$1.714\cdot10^{-2}$	$1.836\cdot10^{-3}$	$1.868\cdot10^{-3}$	$1.898\cdot10^{-2}$
	5.66	$1.197\cdot10^{-2}$	$2.693\cdot10^{-3}$	$2.726\cdot10^{-3}$	$1.466\cdot10^{-2}$
	6.17	$8.356\cdot10^{-3}$	$3.552\cdot10^{-3}$	$3.582\cdot10^{-3}$	$1.191\cdot10^{-2}$
	6.69	$5.830\cdot10^{-3}$	$4.391\cdot10^{-3}$	$4.417\cdot10^{-3}$	$1.022\cdot10^{-2}$
	7.20	$4.070\cdot10^{-3}$	$5.193\cdot10^{-3}$	$5.214\cdot10^{-3}$	$9.262\cdot10^{-3}$
	7.72	$2.839\cdot10^{-3}$	$5.956\cdot10^{-3}$	$5.972\cdot10^{-3}$	$8.794\cdot10^{-3}$
0.50	4.35	$8.529\cdot10^{-2}$	$-2.539\cdot10^{-16}$	$-2.776\cdot10^{-16}$	$8.529\cdot10^{-2}$
	5.22	$5.522\cdot10^{-2}$	$5.350\cdot10^{-5}$	$5.663\cdot10^{-5}$	$5.527\cdot10^{-2}$
	6.09	$3.577\cdot10^{-2}$	$3.279\cdot10^{-4}$	$3.400\cdot10^{-4}$	$3.609\cdot10^{-2}$
	6.96	$2.315\cdot10^{-2}$	$9.289\cdot10^{-4}$	$9.509\cdot10^{-4}$	$2.408\cdot10^{-2}$
	7.82	$1.499\cdot10^{-2}$	$1.921\cdot10^{-3}$	$1.950\cdot10^{-3}$	$1.691\cdot10^{-2}$
	8.70	$9.704\cdot10^{-3}$	$3.043\cdot10^{-3}$	$3.073\cdot10^{-3}$	$1.275\cdot10^{-2}$
	9.56	$6.284\cdot10^{-3}$	$4.156\cdot10^{-3}$	$4.182\cdot10^{-3}$	$1.044\cdot10^{-2}$
	10.43	$4.068\cdot10^{-3}$	$5.206\cdot10^{-3}$	$5.227\cdot10^{-3}$	$9.274\cdot10^{-3}$
	11.30	$2.634\cdot10^{-3}$	$6.169\cdot10^{-3}$	$6.185\cdot10^{-3}$	$8.803\cdot10^{-3}$
	12.17	$1.705\cdot10^{-3}$	$7.036\cdot10^{-3}$	$7.048\cdot10^{-3}$	$8.741\cdot10^{-3}$
	13.04	$1.104\cdot10^{-3}$	$7.802\cdot10^{-3}$	$7.811\cdot10^{-3}$	$8.906\cdot10^{-3}$
0.80	12.93	$3.755\cdot10^{-2}$	$0.000\cdot10^{-0}$	$0.000\cdot10^{-0}$	$3.755\cdot10^{-2}$
	15.51	$2.206\cdot10^{-2}$	$1.053\cdot10^{-4}$	$1.076\cdot10^{-4}$	$2.216\cdot10^{-2}$
	18.10	$1.303\cdot10^{-2}$	$6.158\cdot10^{-4}$	$6.240\cdot10^{-4}$	$1.365\cdot10^{-2}$
	20.68	$7.731\cdot10^{-3}$	$1.626\cdot10^{-3}$	$1.639\cdot10^{-3}$	$9.358\cdot10^{-3}$
	23.27	$4.596\cdot10^{-3}$	$2.970\cdot10^{-3}$	$2.984\cdot10^{-3}$	$7.566\cdot10^{-3}$
	25.85	$2.736\cdot10^{-3}$	$4.329\cdot10^{-3}$	$4.341\cdot10^{-3}$	$7.065\cdot10^{-3}$
	28.44	$1.629\cdot10^{-3}$	$5.581\cdot10^{-3}$	$5.590\cdot10^{-3}$	$7.210\cdot10^{-3}$
	31.02	$9.710\cdot10^{-4}$	$6.682\cdot10^{-3}$	$6.689\cdot10^{-3}$	$7.653\cdot10^{-3}$
	33.61	$5.788\cdot10^{-4}$	$7.619\cdot10^{-3}$	$7.624\cdot10^{-3}$	$8.198\cdot10^{-3}$
	36.19	$3.450\cdot10^{-4}$	$8.386\cdot10^{-3}$	$8.389\cdot10^{-3}$	$8.731\cdot10^{-3}$
	38.78	$2.057\cdot10^{-4}$	$8.979\cdot10^{-3}$	$8.981\cdot10^{-3}$	$9.185\cdot10^{-3}$
0.90	26.98	$1.911\cdot10^{-2}$	$4.084\cdot10^{-16}$	$4.163\cdot10^{-16}$	$1.911\cdot10^{-2}$
	32.38	$1.091\cdot10^{-2}$	$1.210\cdot10^{-4}$	$1.224\cdot10^{-4}$	$1.103\cdot10^{-2}$
	37.77	$6.283\cdot10^{-3}$	$6.980\cdot10^{-4}$	$7.025\cdot10^{-4}$	$6.981\cdot10^{-3}$
	43.17	$3.637\cdot10^{-3}$	$1.817\cdot10^{-3}$	$1.824\cdot10^{-3}$	$5.455\cdot10^{-3}$
	48.56	$2.112\cdot10^{-3}$	$3.240\cdot10^{-3}$	$3.247\cdot10^{-3}$	$5.352\cdot10^{-3}$
	53.96	$1.228\cdot10^{-3}$	$4.643\cdot10^{-3}$	$4.649\cdot10^{-3}$	$5.872\cdot10^{-3}$
	59.36	$7.152\cdot10^{-4}$	$5.913\cdot10^{-3}$	$5.917\cdot10^{-3}$	$6.628\cdot10^{-3}$
	64.75	$4.166\cdot10^{-4}$	$7.011\cdot10^{-3}$	$7.014\cdot10^{-3}$	$7.427\cdot10^{-3}$
	70.15	$2.428\cdot10^{-4}$	$7.925\cdot10^{-3}$	$7.927\cdot10^{-3}$	$8.168\cdot10^{-3}$
	75.54	$1.415\cdot10^{-4}$	$8.649\cdot10^{-3}$	$8.651\cdot10^{-3}$	$8.791\cdot10^{-3}$
	80.94	$8.248\cdot10^{-5}$	$9.182\cdot10^{-3}$	$9.183\cdot10^{-3}$	$9.265\cdot10^{-3}$

Table D.5: Loss components in FIFO-BW system with homogeneous traffic and deadlines for $M = 3$ and uncontrolled loss of $1\cdot10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	1.97	$2.428 \cdot 10^{-1}$	$-2.102 \cdot 10^{-16}$	$-2.776 \cdot 10^{-16}$	$2.428 \cdot 10^{-1}$
	2.76	$1.451 \cdot 10^{-1}$	$2.832 \cdot 10^{-6}$	$3.313 \cdot 10^{-6}$	$1.451 \cdot 10^{-1}$
	3.55	$8.533 \cdot 10^{-2}$	$1.033 \cdot 10^{-4}$	$1.129 \cdot 10^{-4}$	$8.543 \cdot 10^{-2}$
	4.33	$4.974 \cdot 10^{-2}$	$6.164 \cdot 10^{-4}$	$6.486 \cdot 10^{-4}$	$5.036 \cdot 10^{-2}$
	5.12	$2.885 \cdot 10^{-2}$	$1.637 \cdot 10^{-3}$	$1.686 \cdot 10^{-3}$	$3.048 \cdot 10^{-2}$
	5.91	$1.668 \cdot 10^{-2}$	$3.059 \cdot 10^{-3}$	$3.111 \cdot 10^{-3}$	$1.974 \cdot 10^{-2}$
	6.70	$9.630 \cdot 10^{-3}$	$4.500 \cdot 10^{-3}$	$4.543 \cdot 10^{-3}$	$1.413 \cdot 10^{-2}$
	7.49	$5.554 \cdot 10^{-3}$	$5.785 \cdot 10^{-3}$	$5.817 \cdot 10^{-3}$	$1.134 \cdot 10^{-2}$
	8.27	$3.202 \cdot 10^{-3}$	$6.873 \cdot 10^{-3}$	$6.896 \cdot 10^{-3}$	$1.008 \cdot 10^{-2}$
	9.06	$1.845 \cdot 10^{-3}$	$7.769 \cdot 10^{-3}$	$7.783 \cdot 10^{-3}$	$9.614 \cdot 10^{-3}$
	9.85	$1.063 \cdot 10^{-3}$	$8.486 \cdot 10^{-3}$	$8.495 \cdot 10^{-3}$	$9.549 \cdot 10^{-3}$
0.50	3.43	$2.143 \cdot 10^{-1}$	$-2.181 \cdot 10^{-16}$	$-2.776 \cdot 10^{-16}$	$2.143 \cdot 10^{-1}$
	4.80	$1.106 \cdot 10^{-1}$	$1.725 \cdot 10^{-5}$	$1.940 \cdot 10^{-5}$	$1.106 \cdot 10^{-1}$
	6.18	$5.633 \cdot 10^{-2}$	$3.778 \cdot 10^{-4}$	$4.004 \cdot 10^{-4}$	$5.671 \cdot 10^{-2}$
	7.55	$2.853 \cdot 10^{-2}$	$1.553 \cdot 10^{-3}$	$1.598 \cdot 10^{-3}$	$3.009 \cdot 10^{-2}$
	8.92	$1.441 \cdot 10^{-2}$	$3.284 \cdot 10^{-3}$	$3.332 \cdot 10^{-3}$	$1.769 \cdot 10^{-2}$
	10.29	$7.264 \cdot 10^{-3}$	$5.071 \cdot 10^{-3}$	$5.108 \cdot 10^{-3}$	$1.233 \cdot 10^{-2}$
	11.67	$3.661 \cdot 10^{-3}$	$6.567 \cdot 10^{-3}$	$6.591 \cdot 10^{-3}$	$1.023 \cdot 10^{-2}$
	13.04	$1.843 \cdot 10^{-3}$	$7.719 \cdot 10^{-3}$	$7.733 \cdot 10^{-3}$	$9.562 \cdot 10^{-3}$
	14.41	$9.284 \cdot 10^{-4}$	$8.560 \cdot 10^{-3}$	$8.568 \cdot 10^{-3}$	$9.489 \cdot 10^{-3}$
	15.78	$4.676 \cdot 10^{-4}$	$9.146 \cdot 10^{-3}$	$9.151 \cdot 10^{-3}$	$9.614 \cdot 10^{-3}$
	17.15	$2.354 \cdot 10^{-4}$	$9.532 \cdot 10^{-3}$	$9.534 \cdot 10^{-3}$	$9.767 \cdot 10^{-3}$
0.80	10.53	$1.012 \cdot 10^{-1}$	$-2.495 \cdot 10^{-16}$	$-2.776 \cdot 10^{-16}$	$1.012 \cdot 10^{-1}$
	14.75	$4.261 \cdot 10^{-2}$	$6.526 \cdot 10^{-5}$	$6.817 \cdot 10^{-5}$	$4.268 \cdot 10^{-2}$
	18.96	$1.817 \cdot 10^{-2}$	$9.544 \cdot 10^{-4}$	$9.721 \cdot 10^{-4}$	$1.913 \cdot 10^{-2}$
	23.17	$7.792 \cdot 10^{-3}$	$2.943 \cdot 10^{-3}$	$2.966 \cdot 10^{-3}$	$1.074 \cdot 10^{-2}$
	27.39	$3.349 \cdot 10^{-3}$	$5.142 \cdot 10^{-3}$	$5.159 \cdot 10^{-3}$	$8.491 \cdot 10^{-3}$
	31.60	$1.441 \cdot 10^{-3}$	$6.924 \cdot 10^{-3}$	$6.934 \cdot 10^{-3}$	$8.365 \cdot 10^{-3}$
	35.81	$6.202 \cdot 10^{-4}$	$8.178 \cdot 10^{-3}$	$8.183 \cdot 10^{-3}$	$8.799 \cdot 10^{-3}$
	40.03	$2.670 \cdot 10^{-4}$	$8.994 \cdot 10^{-3}$	$8.997 \cdot 10^{-3}$	$9.261 \cdot 10^{-3}$
	44.24	$1.150 \cdot 10^{-4}$	$9.487 \cdot 10^{-3}$	$9.488 \cdot 10^{-3}$	$9.602 \cdot 10^{-3}$
	48.45	$4.950 \cdot 10^{-5}$	$9.759 \cdot 10^{-3}$	$9.760 \cdot 10^{-3}$	$9.809 \cdot 10^{-3}$
	52.67	$2.131 \cdot 10^{-5}$	$9.894 \cdot 10^{-3}$	$9.895 \cdot 10^{-3}$	$9.916 \cdot 10^{-3}$
0.90	22.17	$5.259 \cdot 10^{-2}$	$0.000 \cdot 10^{-0}$	$0.000 \cdot 10^{-0}$	$5.259 \cdot 10^{-2}$
	31.04	$2.077 \cdot 10^{-2}$	$8.571 \cdot 10^{-5}$	$8.753 \cdot 10^{-5}$	$2.085 \cdot 10^{-2}$
	39.92	$8.410 \cdot 10^{-3}$	$1.150 \cdot 10^{-3}$	$1.160 \cdot 10^{-3}$	$9.560 \cdot 10^{-3}$
	48.79	$3.440 \cdot 10^{-3}$	$3.336 \cdot 10^{-3}$	$3.347 \cdot 10^{-3}$	$6.776 \cdot 10^{-3}$
	57.66	$1.413 \cdot 10^{-3}$	$5.593 \cdot 10^{-3}$	$5.601 \cdot 10^{-3}$	$7.006 \cdot 10^{-3}$
	66.53	$5.813 \cdot 10^{-4}$	$7.326 \cdot 10^{-3}$	$7.330 \cdot 10^{-3}$	$7.907 \cdot 10^{-3}$
	75.39	$2.393 \cdot 10^{-4}$	$8.492 \cdot 10^{-3}$	$8.494 \cdot 10^{-3}$	$8.731 \cdot 10^{-3}$
	84.27	$9.856 \cdot 10^{-5}$	$9.214 \cdot 10^{-3}$	$9.215 \cdot 10^{-3}$	$9.313 \cdot 10^{-3}$
	93.13	$4.059 \cdot 10^{-5}$	$9.625 \cdot 10^{-3}$	$9.626 \cdot 10^{-3}$	$9.666 \cdot 10^{-3}$
	102.01	$1.672 \cdot 10^{-5}$	$9.836 \cdot 10^{-3}$	$9.836 \cdot 10^{-3}$	$9.853 \cdot 10^{-3}$
	110.88	$6.886 \cdot 10^{-6}$	$9.932 \cdot 10^{-3}$	$9.932 \cdot 10^{-3}$	$9.939 \cdot 10^{-3}$

Table D.6: Loss components in FIFO-BW system with homogeneous traffic and deadlines for $M = 5$ and uncontrolled loss of $1 \cdot 10^{-2}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	5.03	$1.241 \cdot 10^{-2}$	$-2.741 \cdot 10^{-16}$	$-2.776 \cdot 10^{-16}$	$1.241 \cdot 10^{-2}$
	5.53	$8.731 \cdot 10^{-3}$	$1.569 \cdot 10^{-5}$	$1.583 \cdot 10^{-5}$	$8.746 \cdot 10^{-3}$
	6.04	$6.140 \cdot 10^{-3}$	$5.151 \cdot 10^{-5}$	$5.182 \cdot 10^{-5}$	$6.192 \cdot 10^{-3}$
	6.54	$4.318 \cdot 10^{-3}$	$9.723 \cdot 10^{-5}$	$9.766 \cdot 10^{-5}$	$4.415 \cdot 10^{-3}$
	7.04	$3.037 \cdot 10^{-3}$	$1.478 \cdot 10^{-4}$	$1.483 \cdot 10^{-4}$	$3.184 \cdot 10^{-3}$
	7.55	$2.135 \cdot 10^{-3}$	$2.009 \cdot 10^{-4}$	$2.013 \cdot 10^{-4}$	$2.336 \cdot 10^{-3}$
	8.05	$1.502 \cdot 10^{-3}$	$2.551 \cdot 10^{-4}$	$2.555 \cdot 10^{-4}$	$1.757 \cdot 10^{-3}$
	8.55	$1.056 \cdot 10^{-3}$	$3.099 \cdot 10^{-4}$	$3.102 \cdot 10^{-4}$	$1.366 \cdot 10^{-3}$
	9.05	$7.426 \cdot 10^{-4}$	$3.650 \cdot 10^{-4}$	$3.653 \cdot 10^{-4}$	$1.108 \cdot 10^{-3}$
	9.56	$5.222 \cdot 10^{-4}$	$4.202 \cdot 10^{-4}$	$4.205 \cdot 10^{-4}$	$9.425 \cdot 10^{-4}$
	10.06	$3.672 \cdot 10^{-4}$	$4.756 \cdot 10^{-4}$	$4.757 \cdot 10^{-4}$	$8.428 \cdot 10^{-4}$
0.50	7.91	$9.597 \cdot 10^{-3}$	$1.374 \cdot 10^{-16}$	$1.388 \cdot 10^{-16}$	$9.597 \cdot 10^{-3}$
	8.70	$6.457 \cdot 10^{-3}$	$2.253 \cdot 10^{-5}$	$2.268 \cdot 10^{-5}$	$6.480 \cdot 10^{-3}$
	9.49	$4.344 \cdot 10^{-3}$	$7.255 \cdot 10^{-5}$	$7.286 \cdot 10^{-5}$	$4.416 \cdot 10^{-3}$
	10.28	$2.924 \cdot 10^{-3}$	$1.348 \cdot 10^{-4}$	$1.352 \cdot 10^{-4}$	$3.058 \cdot 10^{-3}$
	11.08	$1.968 \cdot 10^{-3}$	$2.027 \cdot 10^{-4}$	$2.031 \cdot 10^{-4}$	$2.171 \cdot 10^{-3}$
	11.87	$1.325 \cdot 10^{-3}$	$2.730 \cdot 10^{-4}$	$2.734 \cdot 10^{-4}$	$1.598 \cdot 10^{-3}$
	12.66	$8.921 \cdot 10^{-4}$	$3.445 \cdot 10^{-4}$	$3.448 \cdot 10^{-4}$	$1.237 \cdot 10^{-3}$
	13.45	$6.007 \cdot 10^{-4}$	$4.165 \cdot 10^{-4}$	$4.167 \cdot 10^{-4}$	$1.017 \cdot 10^{-3}$
	14.24	$4.044 \cdot 10^{-4}$	$4.887 \cdot 10^{-4}$	$4.889 \cdot 10^{-4}$	$8.932 \cdot 10^{-4}$
	15.03	$2.723 \cdot 10^{-4}$	$5.611 \cdot 10^{-4}$	$5.612 \cdot 10^{-4}$	$8.334 \cdot 10^{-4}$
	15.82	$1.833 \cdot 10^{-4}$	$6.336 \cdot 10^{-4}$	$6.338 \cdot 10^{-4}$	$8.169 \cdot 10^{-4}$
0.80	21.98	$3.970 \cdot 10^{-3}$	$1.382 \cdot 10^{-16}$	$1.388 \cdot 10^{-16}$	$3.970 \cdot 10^{-3}$
	24.18	$2.551 \cdot 10^{-3}$	$2.890 \cdot 10^{-5}$	$2.898 \cdot 10^{-5}$	$2.580 \cdot 10^{-3}$
	26.38	$1.641 \cdot 10^{-3}$	$9.104 \cdot 10^{-5}$	$9.119 \cdot 10^{-5}$	$1.732 \cdot 10^{-3}$
	28.58	$1.056 \cdot 10^{-3}$	$1.668 \cdot 10^{-4}$	$1.670 \cdot 10^{-4}$	$1.223 \cdot 10^{-3}$
	30.77	$6.801 \cdot 10^{-4}$	$2.481 \cdot 10^{-4}$	$2.483 \cdot 10^{-4}$	$9.282 \cdot 10^{-4}$
	32.97	$4.380 \cdot 10^{-4}$	$3.317 \cdot 10^{-4}$	$3.319 \cdot 10^{-4}$	$7.697 \cdot 10^{-4}$
	35.17	$2.821 \cdot 10^{-4}$	$4.164 \cdot 10^{-4}$	$4.165 \cdot 10^{-4}$	$6.985 \cdot 10^{-4}$
	37.37	$1.817 \cdot 10^{-4}$	$5.014 \cdot 10^{-4}$	$5.015 \cdot 10^{-4}$	$6.831 \cdot 10^{-4}$
	39.57	$1.171 \cdot 10^{-4}$	$5.866 \cdot 10^{-4}$	$5.867 \cdot 10^{-4}$	$7.037 \cdot 10^{-4}$
	41.77	$7.542 \cdot 10^{-5}$	$6.719 \cdot 10^{-4}$	$6.720 \cdot 10^{-4}$	$7.473 \cdot 10^{-4}$
	43.96	$4.858 \cdot 10^{-5}$	$7.573 \cdot 10^{-4}$	$7.573 \cdot 10^{-4}$	$8.059 \cdot 10^{-4}$
0.90	45.12	$1.992 \cdot 10^{-3}$	$1.936 \cdot 10^{-12}$	$1.940 \cdot 10^{-12}$	$1.992 \cdot 10^{-3}$
	49.63	$1.265 \cdot 10^{-3}$	$3.041 \cdot 10^{-5}$	$3.045 \cdot 10^{-5}$	$1.295 \cdot 10^{-3}$
	54.14	$8.041 \cdot 10^{-4}$	$9.529 \cdot 10^{-5}$	$9.537 \cdot 10^{-5}$	$8.994 \cdot 10^{-4}$
	58.66	$5.115 \cdot 10^{-4}$	$1.739 \cdot 10^{-4}$	$1.740 \cdot 10^{-4}$	$6.854 \cdot 10^{-4}$
	63.17	$3.255 \cdot 10^{-4}$	$2.581 \cdot 10^{-4}$	$2.582 \cdot 10^{-4}$	$5.836 \cdot 10^{-4}$
	67.68	$2.072 \cdot 10^{-4}$	$3.445 \cdot 10^{-4}$	$3.446 \cdot 10^{-4}$	$5.517 \cdot 10^{-4}$
	72.19	$1.319 \cdot 10^{-4}$	$4.318 \cdot 10^{-4}$	$4.318 \cdot 10^{-4}$	$5.637 \cdot 10^{-4}$
	76.70	$8.400 \cdot 10^{-5}$	$5.195 \cdot 10^{-4}$	$5.195 \cdot 10^{-4}$	$6.035 \cdot 10^{-4}$
	81.21	$5.349 \cdot 10^{-5}$	$6.073 \cdot 10^{-4}$	$6.074 \cdot 10^{-4}$	$6.608 \cdot 10^{-4}$
	85.73	$3.406 \cdot 10^{-5}$	$6.953 \cdot 10^{-4}$	$6.953 \cdot 10^{-4}$	$7.293 \cdot 10^{-4}$
	90.24	$2.169 \cdot 10^{-5}$	$7.833 \cdot 10^{-4}$	$7.833 \cdot 10^{-4}$	$8.050 \cdot 10^{-4}$

Table D.7: Loss components in FIFO-BW system, $M = 2$, homogeneous traffic and deadlines, uncontrolled loss of $1 \cdot 10^{-3}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	3.86	$4.200\cdot10^{-2}$	$7.977\cdot10^{-16}$	$8.327\cdot10^{-16}$	$4.200\cdot10^{-2}$
	4.63	$2.457\cdot10^{-2}$	$4.094\cdot10^{-6}$	$4.197\cdot10^{-6}$	$2.457\cdot10^{-2}$
	5.40	$1.434\cdot10^{-2}$	$2.452\cdot10^{-5}$	$2.488\cdot10^{-5}$	$1.436\cdot10^{-2}$
	6.17	$8.368\cdot10^{-3}$	$7.187\cdot10^{-5}$	$7.248\cdot10^{-5}$	$8.440\cdot10^{-3}$
	6.94	$4.881\cdot10^{-3}$	$1.623\cdot10^{-4}$	$1.631\cdot10^{-4}$	$5.043\cdot10^{-3}$
	7.71	$2.844\cdot10^{-3}$	$2.664\cdot10^{-4}$	$2.672\cdot10^{-4}$	$3.111\cdot10^{-3}$
	8.49	$1.658\cdot10^{-3}$	$3.702\cdot10^{-4}$	$3.708\cdot10^{-4}$	$2.029\cdot10^{-3}$
	9.26	$9.669\cdot10^{-4}$	$4.693\cdot10^{-4}$	$4.697\cdot10^{-4}$	$1.436\cdot10^{-3}$
	10.03	$5.633\cdot10^{-4}$	$5.621\cdot10^{-4}$	$5.624\cdot10^{-4}$	$1.125\cdot10^{-3}$
	10.80	$3.284\cdot10^{-4}$	$6.479\cdot10^{-4}$	$6.481\cdot10^{-4}$	$9.763\cdot10^{-4}$
	11.57	$1.914\cdot10^{-4}$	$7.267\cdot10^{-4}$	$7.268\cdot10^{-4}$	$9.181\cdot10^{-4}$
0.50	6.20	$3.380\cdot10^{-2}$	$-2.682\cdot10^{-16}$	$-2.776\cdot10^{-16}$	$3.380\cdot10^{-2}$
	7.44	$1.818\cdot10^{-2}$	$8.310\cdot10^{-6}$	$8.463\cdot10^{-6}$	$1.819\cdot10^{-2}$
	8.68	$9.782\cdot10^{-3}$	$4.820\cdot10^{-5}$	$4.867\cdot10^{-5}$	$9.830\cdot10^{-3}$
	9.92	$5.262\cdot10^{-3}$	$1.305\cdot10^{-4}$	$1.312\cdot10^{-4}$	$5.393\cdot10^{-3}$
	11.16	$2.831\cdot10^{-3}$	$2.539\cdot10^{-4}$	$2.546\cdot10^{-4}$	$3.085\cdot10^{-3}$
	12.40	$1.523\cdot10^{-3}$	$3.825\cdot10^{-4}$	$3.831\cdot10^{-4}$	$1.905\cdot10^{-3}$
	13.64	$8.192\cdot10^{-4}$	$5.033\cdot10^{-4}$	$5.037\cdot10^{-4}$	$1.323\cdot10^{-3}$
	14.88	$4.407\cdot10^{-4}$	$6.125\cdot10^{-4}$	$6.128\cdot10^{-4}$	$1.053\cdot10^{-3}$
	16.12	$2.371\cdot10^{-4}$	$7.090\cdot10^{-4}$	$7.092\cdot10^{-4}$	$9.461\cdot10^{-4}$
	17.36	$1.275\cdot10^{-4}$	$7.925\cdot10^{-4}$	$7.926\cdot10^{-4}$	$9.200\cdot10^{-4}$
	18.60	$6.864\cdot10^{-5}$	$8.628\cdot10^{-4}$	$8.629\cdot10^{-4}$	$9.314\cdot10^{-4}$
0.80	17.62	$1.435\cdot10^{-2}$	$2.736\cdot10^{-16}$	$2.776\cdot10^{-16}$	$1.435\cdot10^{-2}$
	21.15	$7.039\cdot10^{-3}$	$1.348\cdot10^{-5}$	$1.358\cdot10^{-5}$	$7.052\cdot10^{-3}$
	24.67	$3.465\cdot10^{-3}$	$7.572\cdot10^{-5}$	$7.598\cdot10^{-5}$	$3.541\cdot10^{-3}$
	28.20	$1.710\cdot10^{-3}$	$1.949\cdot10^{-4}$	$1.953\cdot10^{-4}$	$1.905\cdot10^{-3}$
	31.72	$8.440\cdot10^{-4}$	$3.449\cdot10^{-4}$	$3.452\cdot10^{-4}$	$1.189\cdot10^{-3}$
	35.25	$4.169\cdot10^{-4}$	$4.890\cdot10^{-4}$	$4.892\cdot10^{-4}$	$9.060\cdot10^{-4}$
	38.77	$2.060\cdot10^{-4}$	$6.171\cdot10^{-4}$	$6.172\cdot10^{-4}$	$8.230\cdot10^{-4}$
	42.29	$1.018\cdot10^{-4}$	$7.265\cdot10^{-4}$	$7.265\cdot10^{-4}$	$8.282\cdot10^{-4}$
	45.82	$5.029\cdot10^{-5}$	$8.166\cdot10^{-4}$	$8.167\cdot10^{-4}$	$8.669\cdot10^{-4}$
	49.34	$2.485\cdot10^{-5}$	$8.874\cdot10^{-4}$	$8.874\cdot10^{-4}$	$9.122\cdot10^{-4}$
	52.87	$1.228\cdot10^{-5}$	$9.386\cdot10^{-4}$	$9.387\cdot10^{-4}$	$9.509\cdot10^{-4}$
0.90	36.39	$7.235\cdot10^{-3}$	$4.133\cdot10^{-16}$	$4.163\cdot10^{-16}$	$7.235\cdot10^{-3}$
	43.66	$3.460\cdot10^{-3}$	$1.485\cdot10^{-5}$	$1.490\cdot10^{-5}$	$3.475\cdot10^{-3}$
	50.94	$1.663\cdot10^{-3}$	$8.273\cdot10^{-5}$	$8.287\cdot10^{-5}$	$1.746\cdot10^{-3}$
	58.22	$8.015\cdot10^{-4}$	$2.109\cdot10^{-4}$	$2.110\cdot10^{-4}$	$1.012\cdot10^{-3}$
	65.49	$3.867\cdot10^{-4}$	$3.664\cdot10^{-4}$	$3.665\cdot10^{-4}$	$7.531\cdot10^{-4}$
	72.77	$1.867\cdot10^{-4}$	$5.132\cdot10^{-4}$	$5.133\cdot10^{-4}$	$6.999\cdot10^{-4}$
	80.05	$9.014\cdot10^{-5}$	$6.420\cdot10^{-4}$	$6.421\cdot10^{-4}$	$7.322\cdot10^{-4}$
	87.33	$4.354\cdot10^{-5}$	$7.506\cdot10^{-4}$	$7.506\cdot10^{-4}$	$7.941\cdot10^{-4}$
	94.60	$2.103\cdot10^{-5}$	$8.384\cdot10^{-4}$	$8.384\cdot10^{-4}$	$8.594\cdot10^{-4}$
	101.88	$1.016\cdot10^{-5}$	$9.052\cdot10^{-4}$	$9.052\cdot10^{-4}$	$9.153\cdot10^{-4}$
	109.16	$4.906\cdot10^{-6}$	$9.510\cdot10^{-4}$	$9.510\cdot10^{-4}$	$9.559\cdot10^{-4}$

Table D.8: Loss components in FIFO-BW system, $M = 3$, homogeneous traffic and deadlines, uncontrolled loss of $1\cdot10^{-3}$

ρ	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$
0.30	2.82	$1.392 \cdot 10^{-1}$	$0.000 \cdot 10^{-0}$	$0.000 \cdot 10^{-0}$	$1.392 \cdot 10^{-1}$
	3.95	$6.485 \cdot 10^{-2}$	$6.972 \cdot 10^{-7}$	$7.456 \cdot 10^{-7}$	$6.485 \cdot 10^{-2}$
	5.07	$2.980 \cdot 10^{-2}$	$2.029 \cdot 10^{-5}$	$2.091 \cdot 10^{-5}$	$2.982 \cdot 10^{-2}$
	6.20	$1.360 \cdot 10^{-2}$	$1.022 \cdot 10^{-4}$	$1.036 \cdot 10^{-4}$	$1.370 \cdot 10^{-2}$
	7.33	$6.190 \cdot 10^{-3}$	$2.431 \cdot 10^{-4}$	$2.446 \cdot 10^{-4}$	$6.433 \cdot 10^{-3}$
	8.46	$2.813 \cdot 10^{-3}$	$4.127 \cdot 10^{-4}$	$4.139 \cdot 10^{-4}$	$3.226 \cdot 10^{-3}$
	9.59	$1.278 \cdot 10^{-3}$	$5.661 \cdot 10^{-4}$	$5.669 \cdot 10^{-4}$	$1.844 \cdot 10^{-3}$
	10.72	$5.803 \cdot 10^{-4}$	$6.920 \cdot 10^{-4}$	$6.924 \cdot 10^{-4}$	$1.272 \cdot 10^{-3}$
	11.84	$2.635 \cdot 10^{-4}$	$7.912 \cdot 10^{-4}$	$7.914 \cdot 10^{-4}$	$1.055 \cdot 10^{-3}$
	12.97	$1.196 \cdot 10^{-4}$	$8.669 \cdot 10^{-4}$	$8.670 \cdot 10^{-4}$	$9.865 \cdot 10^{-4}$
	14.10	$5.436 \cdot 10^{-5}$	$9.224 \cdot 10^{-4}$	$9.224 \cdot 10^{-4}$	$9.767 \cdot 10^{-4}$
0.50	4.67	$1.178 \cdot 10^{-1}$	$-2.449 \cdot 10^{-16}$	$-2.776 \cdot 10^{-16}$	$1.178 \cdot 10^{-1}$
	6.54	$4.698 \cdot 10^{-2}$	$2.909 \cdot 10^{-6}$	$3.053 \cdot 10^{-6}$	$4.699 \cdot 10^{-2}$
	8.41	$1.855 \cdot 10^{-2}$	$5.584 \cdot 10^{-5}$	$5.690 \cdot 10^{-5}$	$1.861 \cdot 10^{-2}$
	10.28	$7.304 \cdot 10^{-3}$	$2.072 \cdot 10^{-4}$	$2.087 \cdot 10^{-4}$	$7.511 \cdot 10^{-3}$
	12.15	$2.870 \cdot 10^{-3}$	$4.071 \cdot 10^{-4}$	$4.083 \cdot 10^{-4}$	$3.277 \cdot 10^{-3}$
	14.02	$1.128 \cdot 10^{-3}$	$5.931 \cdot 10^{-4}$	$5.937 \cdot 10^{-4}$	$1.721 \cdot 10^{-3}$
	15.89	$4.428 \cdot 10^{-4}$	$7.365 \cdot 10^{-4}$	$7.368 \cdot 10^{-4}$	$1.179 \cdot 10^{-3}$
	17.76	$1.739 \cdot 10^{-4}$	$8.392 \cdot 10^{-4}$	$8.394 \cdot 10^{-4}$	$1.013 \cdot 10^{-3}$
	19.63	$6.828 \cdot 10^{-5}$	$9.090 \cdot 10^{-4}$	$9.090 \cdot 10^{-4}$	$9.772 \cdot 10^{-4}$
	21.50	$2.682 \cdot 10^{-5}$	$9.532 \cdot 10^{-4}$	$9.533 \cdot 10^{-4}$	$9.801 \cdot 10^{-4}$
	23.37	$1.053 \cdot 10^{-5}$	$9.790 \cdot 10^{-4}$	$9.790 \cdot 10^{-4}$	$9.895 \cdot 10^{-4}$
0.80	13.71	$5.259 \cdot 10^{-2}$	$-5.259 \cdot 10^{-16}$	$-5.551 \cdot 10^{-16}$	$5.259 \cdot 10^{-2}$
	19.20	$1.732 \cdot 10^{-2}$	$8.037 \cdot 10^{-6}$	$8.179 \cdot 10^{-6}$	$1.733 \cdot 10^{-2}$
	24.68	$5.755 \cdot 10^{-3}$	$1.129 \cdot 10^{-4}$	$1.136 \cdot 10^{-4}$	$5.868 \cdot 10^{-3}$
	30.17	$1.918 \cdot 10^{-3}$	$3.343 \cdot 10^{-4}$	$3.349 \cdot 10^{-4}$	$2.252 \cdot 10^{-3}$
	35.66	$6.401 \cdot 10^{-4}$	$5.642 \cdot 10^{-4}$	$5.646 \cdot 10^{-4}$	$1.204 \cdot 10^{-3}$
	41.14	$2.137 \cdot 10^{-4}$	$7.393 \cdot 10^{-4}$	$7.394 \cdot 10^{-4}$	$9.530 \cdot 10^{-4}$
	46.63	$7.132 \cdot 10^{-5}$	$8.558 \cdot 10^{-4}$	$8.559 \cdot 10^{-4}$	$9.271 \cdot 10^{-4}$
	52.11	$2.381 \cdot 10^{-5}$	$9.273 \cdot 10^{-4}$	$9.274 \cdot 10^{-4}$	$9.511 \cdot 10^{-4}$
	57.60	$7.950 \cdot 10^{-6}$	$9.674 \cdot 10^{-4}$	$9.674 \cdot 10^{-4}$	$9.753 \cdot 10^{-4}$
	63.08	$2.654 \cdot 10^{-6}$	$9.872 \cdot 10^{-4}$	$9.872 \cdot 10^{-4}$	$9.899 \cdot 10^{-4}$
	68.57	$8.860 \cdot 10^{-7}$	$9.956 \cdot 10^{-4}$	$9.956 \cdot 10^{-4}$	$9.965 \cdot 10^{-4}$
0.90	28.55	$2.687 \cdot 10^{-2}$	$9.453 \cdot 10^{-16}$	$9.714 \cdot 10^{-16}$	$2.687 \cdot 10^{-2}$
	39.97	$8.364 \cdot 10^{-3}$	$9.859 \cdot 10^{-6}$	$9.942 \cdot 10^{-6}$	$8.373 \cdot 10^{-3}$
	51.39	$2.648 \cdot 10^{-3}$	$1.299 \cdot 10^{-4}$	$1.303 \cdot 10^{-4}$	$2.778 \cdot 10^{-3}$
	62.81	$8.432 \cdot 10^{-4}$	$3.669 \cdot 10^{-4}$	$3.672 \cdot 10^{-4}$	$1.210 \cdot 10^{-3}$
	74.23	$2.689 \cdot 10^{-4}$	$5.999 \cdot 10^{-4}$	$6.000 \cdot 10^{-4}$	$8.688 \cdot 10^{-4}$
	85.65	$8.581 \cdot 10^{-5}$	$7.695 \cdot 10^{-4}$	$7.695 \cdot 10^{-4}$	$8.553 \cdot 10^{-4}$
	97.07	$2.739 \cdot 10^{-5}$	$8.783 \cdot 10^{-4}$	$8.783 \cdot 10^{-4}$	$9.056 \cdot 10^{-4}$
	108.49	$8.741 \cdot 10^{-6}$	$9.421 \cdot 10^{-4}$	$9.421 \cdot 10^{-4}$	$9.509 \cdot 10^{-4}$
	119.91	$2.790 \cdot 10^{-6}$	$9.758 \cdot 10^{-4}$	$9.758 \cdot 10^{-4}$	$9.786 \cdot 10^{-4}$
	131.33	$8.905 \cdot 10^{-7}$	$9.913 \cdot 10^{-4}$	$9.913 \cdot 10^{-4}$	$9.922 \cdot 10^{-4}$
	142.75	$2.842 \cdot 10^{-7}$	$9.972 \cdot 10^{-4}$	$9.972 \cdot 10^{-4}$	$9.974 \cdot 10^{-4}$

Table D.9: Loss components in FIFO-BW system, $M = 5$, homogeneous traffic and deadlines, uncontrolled loss of $1 \cdot 10^{-3}$

M	ρ	$M/M/1$	τ	$P[D]$	$P[T]$	$P[T\mid A]$	$P[L]$	$P[D]/P[T]$	gain
2	0.30	$5.0\cdot10^{-2}$	3.88	$2.781\cdot10^{-2}$	$1.075\cdot10^{-2}$	$1.106\cdot10^{-2}$	$3.856\cdot10^{-2}$	$2.588\cdot10^{-0}$	0.771
		$1.0\cdot10^{-2}$	6.45	$4.581\cdot10^{-3}$	$3.472\cdot10^{-3}$	$3.488\cdot10^{-3}$	$8.053\cdot10^{-3}$	$1.320\cdot10^{-0}$	0.805
		$1.0\cdot10^{-3}$	10.06	$3.672\cdot10^{-4}$	$4.756\cdot10^{-4}$	$4.757\cdot10^{-4}$	$8.428\cdot10^{-4}$	$7.721\cdot10^{-1}$	0.843
		$1.0\cdot10^{-5}$	17.11	$2.645\cdot10^{-6}$	$6.221\cdot10^{-6}$	$6.221\cdot10^{-6}$	$8.866\cdot10^{-6}$	$4.252\cdot10^{-1}$	0.887
	0.40	$5.0\cdot10^{-2}$	5.21	$2.108\cdot10^{-2}$	$1.545\cdot10^{-2}$	$1.578\cdot10^{-2}$	$3.653\cdot10^{-2}$	$1.365\cdot10^{-0}$	0.731
		$1.0\cdot10^{-2}$	8.28	$3.347\cdot10^{-3}$	$4.441\cdot10^{-3}$	$4.456\cdot10^{-3}$	$7.788\cdot10^{-3}$	$7.536\cdot10^{-1}$	0.779
		$1.0\cdot10^{-3}$	12.53	$2.604\cdot10^{-4}$	$5.661\cdot10^{-4}$	$5.662\cdot10^{-4}$	$8.265\cdot10^{-4}$	$4.601\cdot10^{-1}$	0.827
		$1.0\cdot10^{-5}$	20.80	$1.820\cdot10^{-6}$	$6.967\cdot10^{-6}$	$6.967\cdot10^{-6}$	$8.787\cdot10^{-6}$	$2.613\cdot10^{-1}$	0.879
	0.50	$5.0\cdot10^{-2}$	6.96	$1.550\cdot10^{-2}$	$1.967\cdot10^{-2}$	$1.998\cdot10^{-2}$	$3.516\cdot10^{-2}$	$7.880\cdot10^{-1}$	0.703
		$1.0\cdot10^{-2}$	10.68	$2.400\cdot10^{-3}$	$5.222\cdot10^{-3}$	$5.235\cdot10^{-3}$	$7.622\cdot10^{-3}$	$4.595\cdot10^{-1}$	0.762
		$1.0\cdot10^{-3}$	15.82	$1.833\cdot10^{-4}$	$6.336\cdot10^{-4}$	$6.337\cdot10^{-4}$	$8.169\cdot10^{-4}$	$2.893\cdot10^{-1}$	0.817
		$1.0\cdot10^{-5}$	25.78	$1.258\cdot10^{-6}$	$7.483\cdot10^{-6}$	$7.483\cdot10^{-6}$	$8.742\cdot10^{-6}$	$1.682\cdot10^{-1}$	0.874
	0.60	$5.0\cdot10^{-2}$	8.61	$1.545\cdot10^{-2}$	$1.796\cdot10^{-2}$	$1.824\cdot10^{-2}$	$3.341\cdot10^{-2}$	$8.600\cdot10^{-1}$	0.668
		$1.0\cdot10^{-2}$	13.18	$2.470\cdot10^{-3}$	$4.879\cdot10^{-3}$	$4.891\cdot10^{-3}$	$7.349\cdot10^{-3}$	$5.063\cdot10^{-1}$	0.735
		$1.0\cdot10^{-3}$	19.60	$1.890\cdot10^{-4}$	$6.085\cdot10^{-4}$	$6.086\cdot10^{-4}$	$7.975\cdot10^{-4}$	$3.107\cdot10^{-1}$	0.798
		$1.0\cdot10^{-5}$	32.08	$1.285\cdot10^{-6}$	$7.338\cdot10^{-6}$	$7.338\cdot10^{-6}$	$8.623\cdot10^{-6}$	$1.752\cdot10^{-1}$	0.862
	0.80	$5.0\cdot10^{-2}$	15.56	$1.460\cdot10^{-2}$	$9.695\cdot10^{-3}$	$9.839\cdot10^{-3}$	$2.430\cdot10^{-2}$	$1.506\cdot10^{-0}$	0.486
		$1.0\cdot10^{-2}$	24.28	$2.500\cdot10^{-3}$	$3.335\cdot10^{-3}$	$3.344\cdot10^{-3}$	$5.835\cdot10^{-3}$	$7.494\cdot10^{-1}$	0.584
		$1.0\cdot10^{-3}$	37.06	$1.935\cdot10^{-4}$	$4.893\cdot10^{-4}$	$4.894\cdot10^{-4}$	$6.827\cdot10^{-4}$	$3.955\cdot10^{-1}$	0.683
		$1.0\cdot10^{-5}$	62.04	$1.308\cdot10^{-6}$	$6.552\cdot10^{-6}$	$6.552\cdot10^{-6}$	$7.860\cdot10^{-6}$	$1.996\cdot10^{-1}$	0.786
	0.90	$5.0\cdot10^{-2}$	27.94	$1.155\cdot10^{-2}$	$3.865\cdot10^{-3}$	$3.910\cdot10^{-3}$	$1.541\cdot10^{-2}$	$2.988\cdot10^{-0}$	0.308
		$1.0\cdot10^{-2}$	43.56	$2.333\cdot10^{-3}$	$1.847\cdot10^{-3}$	$1.851\cdot10^{-3}$	$4.180\cdot10^{-3}$	$1.263\cdot10^{-0}$	0.418
		$1.0\cdot10^{-3}$	68.40	$1.928\cdot10^{-4}$	$3.583\cdot10^{-4}$	$3.584\cdot10^{-4}$	$5.511\cdot10^{-4}$	$5.382\cdot10^{-1}$	0.551
		$1.0\cdot10^{-5}$	118.31	$1.310\cdot10^{-6}$	$5.663\cdot10^{-6}$	$5.663\cdot10^{-6}$	$6.973\cdot10^{-6}$	$2.313\cdot10^{-1}$	0.697
3	0.30	$5.0\cdot10^{-2}$	4.91	$2.012\cdot10^{-2}$	$2.162\cdot10^{-2}$	$2.206\cdot10^{-2}$	$4.173\cdot10^{-2}$	$9.306\cdot10^{-1}$	0.835
		$1.0\cdot10^{-2}$	7.72	$2.839\cdot10^{-3}$	$5.954\cdot10^{-3}$	$5.971\cdot10^{-3}$	$8.794\cdot10^{-3}$	$4.768\cdot10^{-1}$	0.879
		$1.0\cdot10^{-3}$	11.57	$1.914\cdot10^{-4}$	$7.267\cdot10^{-4}$	$7.268\cdot10^{-4}$	$9.181\cdot10^{-4}$	$2.633\cdot10^{-1}$	0.918
		$1.0\cdot10^{-5}$	18.99	$1.066\cdot10^{-6}$	$8.478\cdot10^{-6}$	$8.478\cdot10^{-6}$	$9.543\cdot10^{-6}$	$1.257\cdot10^{-1}$	0.954
	0.40	$5.0\cdot10^{-2}$	6.57	$1.392\cdot10^{-2}$	$2.757\cdot10^{-2}$	$2.795\cdot10^{-2}$	$4.148\cdot10^{-2}$	$5.049\cdot10^{-1}$	0.830
		$1.0\cdot10^{-2}$	9.88	$1.923\cdot10^{-3}$	$6.899\cdot10^{-3}$	$6.912\cdot10^{-3}$	$8.822\cdot10^{-3}$	$2.787\cdot10^{-1}$	0.882
		$1.0\cdot10^{-3}$	14.44	$1.242\cdot10^{-4}$	$7.995\cdot10^{-4}$	$7.996\cdot10^{-4}$	$9.237\cdot10^{-4}$	$1.554\cdot10^{-1}$	0.924
		$1.0\cdot10^{-5}$	23.19	$6.543\cdot10^{-7}$	$8.943\cdot10^{-6}$	$8.943\cdot10^{-6}$	$9.598\cdot10^{-6}$	$7.316\cdot10^{-2}$	0.960
	0.50	$5.0\cdot10^{-2}$	7.90	$1.442\cdot10^{-2}$	$2.620\cdot10^{-2}$	$2.659\cdot10^{-2}$	$4.062\cdot10^{-2}$	$5.503\cdot10^{-1}$	0.812
		$1.0\cdot10^{-2}$	11.90	$1.953\cdot10^{-3}$	$6.776\cdot10^{-3}$	$6.789\cdot10^{-3}$	$8.729\cdot10^{-3}$	$2.882\cdot10^{-1}$	0.873
		$1.0\cdot10^{-3}$	17.43	$1.229\cdot10^{-4}$	$7.971\cdot10^{-4}$	$7.972\cdot10^{-4}$	$9.200\cdot10^{-4}$	$1.542\cdot10^{-1}$	0.920
		$1.0\cdot10^{-5}$	27.99	$6.253\cdot10^{-7}$	$8.966\cdot10^{-6}$	$8.966\cdot10^{-6}$	$9.592\cdot10^{-6}$	$6.974\cdot10^{-2}$	0.959
	0.60	$5.0\cdot10^{-2}$	9.66	$1.512\cdot10^{-2}$	$2.334\cdot10^{-2}$	$2.370\cdot10^{-2}$	$3.846\cdot10^{-2}$	$6.480\cdot10^{-1}$	0.769
		$1.0\cdot10^{-2}$	14.65	$2.056\cdot10^{-3}$	$6.393\cdot10^{-3}$	$6.406\cdot10^{-3}$	$8.449\cdot10^{-3}$	$3.216\cdot10^{-1}$	0.845
		$1.0\cdot10^{-3}$	21.59	$1.278\cdot10^{-4}$	$7.761\cdot10^{-4}$	$7.762\cdot10^{-4}$	$9.039\cdot10^{-4}$	$1.647\cdot10^{-1}$	0.904
		$1.0\cdot10^{-5}$	34.84	$6.383\cdot10^{-7}$	$8.881\cdot10^{-6}$	$8.881\cdot10^{-6}$	$9.520\cdot10^{-6}$	$7.187\cdot10^{-2}$	0.952
	0.80	$5.0\cdot10^{-2}$	17.07	$1.604\cdot10^{-2}$	$1.282\cdot10^{-2}$	$1.303\cdot10^{-2}$	$2.886\cdot10^{-2}$	$1.251\cdot10^{-0}$	0.577
		$1.0\cdot10^{-2}$	26.30	$2.500\cdot10^{-3}$	$4.556\cdot10^{-3}$	$4.567\cdot10^{-3}$	$7.056\cdot10^{-3}$	$5.488\cdot10^{-1}$	0.706
		$1.0\cdot10^{-3}$	40.07	$1.587\cdot10^{-4}$	$6.597\cdot10^{-4}$	$6.598\cdot10^{-4}$	$8.184\cdot10^{-4}$	$2.406\cdot10^{-1}$	0.818
		$1.0\cdot10^{-5}$	66.66	$7.778\cdot10^{-7}$	$8.331\cdot10^{-6}$	$8.331\cdot10^{-6}$	$9.109\cdot10^{-6}$	$9.336\cdot10^{-2}$	0.911
	0.90	$5.0\cdot10^{-2}$	30.73	$1.292\cdot10^{-2}$	$6.087\cdot10^{-3}$	$6.167\cdot10^{-3}$	$1.901\cdot10^{-2}$	$2.123\cdot10^{-0}$	0.380
		$1.0\cdot10^{-2}$	46.30	$2.653\cdot10^{-3}$	$2.633\cdot10^{-3}$	$2.640\cdot10^{-3}$	$5.286\cdot10^{-3}$	$1.007\cdot10^{-0}$	0.529
		$1.0\cdot10^{-3}$	72.52	$1.915\cdot10^{-4}$	$5.084\cdot10^{-4}$	$5.085\cdot10^{-4}$	$6.999\cdot10^{-4}$	$3.766\cdot10^{-1}$	0.700
		$1.0\cdot10^{-5}$	125.57	$9.503\cdot10^{-7}$	$7.576\cdot10^{-6}$	$7.576\cdot10^{-6}$	$8.526\cdot10^{-6}$	$1.254\cdot10^{-1}$	0.853

Table D.10: Minimal losses and optimal homogeneous local deadline in FIFO-BW system with homogeneous traffic ($M = 2$ and $M = 3$)

M	ρ	$M/M/1$	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	$P[D]/P[T]$	gain
4	0.30	$5.0\cdot10^{-2}$	5.84	$1.407\cdot10^{-2}$	$3.016\cdot10^{-2}$	$3.059\cdot10^{-2}$	$4.423\cdot10^{-2}$	$4.666\cdot10^{-1}$	0.885
		$1.0\cdot10^{-2}$	8.83	$1.739\cdot10^{-3}$	$7.523\cdot10^{-3}$	$7.536\cdot10^{-3}$	$9.262\cdot10^{-3}$	$2.311\cdot10^{-1}$	0.926
		$1.0\cdot10^{-3}$	12.89	$1.012\cdot10^{-4}$	$8.555\cdot10^{-4}$	$8.556\cdot10^{-4}$	$9.567\cdot10^{-4}$	$1.182\cdot10^{-1}$	0.957
		$1.0\cdot10^{-5}$	20.62	$4.524\cdot10^{-7}$	$9.354\cdot10^{-6}$	$9.354\cdot10^{-6}$	$9.806\cdot10^{-6}$	$4.837\cdot10^{-2}$	0.981
	0.40	$5.0\cdot10^{-2}$	7.26	$1.229\cdot10^{-2}$	$3.200\cdot10^{-2}$	$3.239\cdot10^{-2}$	$4.428\cdot10^{-2}$	$3.841\cdot10^{-1}$	0.886
		$1.0\cdot10^{-2}$	10.80	$1.475\cdot10^{-3}$	$7.834\cdot10^{-3}$	$7.845\cdot10^{-3}$	$9.308\cdot10^{-3}$	$1.883\cdot10^{-1}$	0.931
		$1.0\cdot10^{-3}$	15.63	$8.133\cdot10^{-5}$	$8.805\cdot10^{-4}$	$8.806\cdot10^{-4}$	$9.618\cdot10^{-4}$	$9.237\cdot10^{-2}$	0.962
		$1.0\cdot10^{-5}$	24.77	$3.362\cdot10^{-7}$	$9.505\cdot10^{-6}$	$9.505\cdot10^{-6}$	$9.842\cdot10^{-6}$	$3.537\cdot10^{-2}$	0.984
	0.50	$5.0\cdot10^{-2}$	8.72	$1.273\cdot10^{-2}$	$3.055\cdot10^{-2}$	$3.095\cdot10^{-2}$	$4.329\cdot10^{-2}$	$4.167\cdot10^{-1}$	0.866
		$1.0\cdot10^{-2}$	13.01	$1.497\cdot10^{-3}$	$7.715\cdot10^{-3}$	$7.726\cdot10^{-3}$	$9.212\cdot10^{-3}$	$1.940\cdot10^{-1}$	0.921
		$1.0\cdot10^{-3}$	18.88	$7.935\cdot10^{-5}$	$8.788\cdot10^{-4}$	$8.789\cdot10^{-4}$	$9.582\cdot10^{-4}$	$9.029\cdot10^{-2}$	0.958
		$1.0\cdot10^{-5}$	29.95	$3.129\cdot10^{-7}$	$9.522\cdot10^{-6}$	$9.522\cdot10^{-6}$	$9.835\cdot10^{-6}$	$3.286\cdot10^{-2}$	0.983
	0.60	$5.0\cdot10^{-2}$	10.60	$1.383\cdot10^{-2}$	$2.746\cdot10^{-2}$	$2.784\cdot10^{-2}$	$4.128\cdot10^{-2}$	$5.036\cdot10^{-1}$	0.826
		$1.0\cdot10^{-2}$	15.94	$1.637\cdot10^{-3}$	$7.352\cdot10^{-3}$	$7.364\cdot10^{-3}$	$8.988\cdot10^{-3}$	$2.226\cdot10^{-1}$	0.899
		$1.0\cdot10^{-3}$	23.32	$8.530\cdot10^{-5}$	$8.623\cdot10^{-4}$	$8.623\cdot10^{-4}$	$9.476\cdot10^{-4}$	$9.892\cdot10^{-2}$	0.948
		$1.0\cdot10^{-5}$	37.24	$3.261\cdot10^{-7}$	$9.473\cdot10^{-6}$	$9.473\cdot10^{-6}$	$9.799\cdot10^{-6}$	$3.443\cdot10^{-2}$	0.980
	0.80	$5.0\cdot10^{-2}$	18.44	$1.617\cdot10^{-2}$	$1.586\cdot10^{-2}$	$1.612\cdot10^{-2}$	$3.203\cdot10^{-2}$	$1.020\cdot10^{-0}$	0.641
		$1.0\cdot10^{-2}$	28.14	$2.307\cdot10^{-3}$	$5.487\cdot10^{-3}$	$5.500\cdot10^{-3}$	$7.794\cdot10^{-3}$	$4.204\cdot10^{-1}$	0.779
		$1.0\cdot10^{-3}$	42.78	$1.231\cdot10^{-4}$	$7.633\cdot10^{-4}$	$7.634\cdot10^{-4}$	$8.864\cdot10^{-4}$	$1.613\cdot10^{-1}$	0.886
		$1.0\cdot10^{-5}$	70.74	$4.593\cdot10^{-7}$	$9.117\cdot10^{-6}$	$9.117\cdot10^{-6}$	$9.577\cdot10^{-6}$	$5.037\cdot10^{-2}$	0.958
	0.90	$5.0\cdot10^{-2}$	32.89	$1.377\cdot10^{-2}$	$8.164\cdot10^{-3}$	$8.278\cdot10^{-3}$	$2.194\cdot10^{-2}$	$1.687\cdot10^{-0}$	0.439
		$1.0\cdot10^{-2}$	49.01	$2.693\cdot10^{-3}$	$3.383\cdot10^{-3}$	$3.392\cdot10^{-3}$	$6.075\cdot10^{-3}$	$7.961\cdot10^{-1}$	0.608
		$1.0\cdot10^{-3}$	76.58	$1.701\cdot10^{-4}$	$6.176\cdot10^{-4}$	$6.177\cdot10^{-4}$	$7.877\cdot10^{-4}$	$2.754\cdot10^{-1}$	0.788
		$1.0\cdot10^{-5}$	132.37	$6.422\cdot10^{-7}$	$8.567\cdot10^{-6}$	$8.567\cdot10^{-6}$	$9.209\cdot10^{-6}$	$7.496\cdot10^{-2}$	0.921
5	0.30	$5.0\cdot10^{-2}$	6.53	$1.084\cdot10^{-2}$	$3.516\cdot10^{-2}$	$3.555\cdot10^{-2}$	$4.600\cdot10^{-2}$	$3.082\cdot10^{-1}$	0.920
		$1.0\cdot10^{-2}$	9.65	$1.225\cdot10^{-3}$	$8.317\cdot10^{-3}$	$8.327\cdot10^{-3}$	$9.542\cdot10^{-3}$	$1.473\cdot10^{-1}$	0.954
		$1.0\cdot10^{-3}$	13.89	$6.303\cdot10^{-5}$	$9.133\cdot10^{-4}$	$9.134\cdot10^{-4}$	$9.764\cdot10^{-4}$	$6.901\cdot10^{-2}$	0.976
		$1.0\cdot10^{-5}$	21.89	$2.321\cdot10^{-7}$	$9.681\cdot10^{-6}$	$9.681\cdot10^{-6}$	$9.913\cdot10^{-6}$	$2.398\cdot10^{-2}$	0.991
	0.40	$5.0\cdot10^{-2}$	7.90	$1.047\cdot10^{-2}$	$3.539\cdot10^{-2}$	$3.577\cdot10^{-2}$	$4.587\cdot10^{-2}$	$2.959\cdot10^{-1}$	0.917
		$1.0\cdot10^{-2}$	11.64	$1.115\cdot10^{-3}$	$8.442\cdot10^{-3}$	$8.451\cdot10^{-3}$	$9.556\cdot10^{-3}$	$1.320\cdot10^{-1}$	0.956
		$1.0\cdot10^{-3}$	16.71	$5.322\cdot10^{-5}$	$9.255\cdot10^{-4}$	$9.256\cdot10^{-4}$	$9.787\cdot10^{-4}$	$5.751\cdot10^{-2}$	0.979
		$1.0\cdot10^{-5}$	26.21	$1.781\cdot10^{-7}$	$9.751\cdot10^{-6}$	$9.751\cdot10^{-6}$	$9.929\cdot10^{-6}$	$1.826\cdot10^{-2}$	0.993
	0.50	$5.0\cdot10^{-2}$	9.45	$1.105\cdot10^{-2}$	$3.390\cdot10^{-2}$	$3.428\cdot10^{-2}$	$4.495\cdot10^{-2}$	$3.258\cdot10^{-1}$	0.899
		$1.0\cdot10^{-2}$	13.99	$1.144\cdot10^{-3}$	$8.333\cdot10^{-3}$	$8.343\cdot10^{-3}$	$9.478\cdot10^{-3}$	$1.373\cdot10^{-1}$	0.948
		$1.0\cdot10^{-3}$	20.17	$5.219\cdot10^{-5}$	$9.240\cdot10^{-4}$	$9.240\cdot10^{-4}$	$9.762\cdot10^{-4}$	$5.649\cdot10^{-2}$	0.976
		$1.0\cdot10^{-5}$	31.70	$1.637\cdot10^{-7}$	$9.761\cdot10^{-6}$	$9.761\cdot10^{-6}$	$9.925\cdot10^{-6}$	$1.677\cdot10^{-2}$	0.992
	0.60	$5.0\cdot10^{-2}$	11.42	$1.244\cdot10^{-2}$	$3.073\cdot10^{-2}$	$3.112\cdot10^{-2}$	$4.317\cdot10^{-2}$	$4.049\cdot10^{-1}$	0.863
		$1.0\cdot10^{-2}$	17.07	$1.298\cdot10^{-3}$	$8.007\cdot10^{-3}$	$8.017\cdot10^{-3}$	$9.304\cdot10^{-3}$	$1.621\cdot10^{-1}$	0.930
		$1.0\cdot10^{-3}$	24.86	$5.771\cdot10^{-5}$	$9.115\cdot10^{-4}$	$9.116\cdot10^{-4}$	$9.692\cdot10^{-4}$	$6.331\cdot10^{-2}$	0.969
		$1.0\cdot10^{-5}$	39.36	$1.743\cdot10^{-7}$	$9.733\cdot10^{-6}$	$9.733\cdot10^{-6}$	$9.907\cdot10^{-6}$	$1.791\cdot10^{-2}$	0.991
	0.80	$5.0\cdot10^{-2}$	19.65	$1.583\cdot10^{-2}$	$1.864\cdot10^{-2}$	$1.894\cdot10^{-2}$	$3.447\cdot10^{-2}$	$8.493\cdot10^{-1}$	0.689
		$1.0\cdot10^{-2}$	29.79	$2.070\cdot10^{-3}$	$6.226\cdot10^{-3}$	$6.239\cdot10^{-3}$	$8.296\cdot10^{-3}$	$3.324\cdot10^{-1}$	0.830
		$1.0\cdot10^{-3}$	45.21	$9.458\cdot10^{-5}$	$8.307\cdot10^{-4}$	$8.308\cdot10^{-4}$	$9.253\cdot10^{-4}$	$1.139\cdot10^{-1}$	0.925
		$1.0\cdot10^{-5}$	74.40	$2.759\cdot10^{-7}$	$9.507\cdot10^{-6}$	$9.507\cdot10^{-6}$	$9.782\cdot10^{-6}$	$2.902\cdot10^{-2}$	0.978
	0.90	$5.0\cdot10^{-2}$	34.57	$1.447\cdot10^{-2}$	$9.778\cdot10^{-3}$	$9.922\cdot10^{-3}$	$2.425\cdot10^{-2}$	$1.480\cdot10^{-0}$	0.485
		$1.0\cdot10^{-2}$	51.45	$2.632\cdot10^{-3}$	$4.049\cdot10^{-3}$	$4.059\cdot10^{-3}$	$6.680\cdot10^{-3}$	$6.500\cdot10^{-1}$	0.668
		$1.0\cdot10^{-3}$	80.34	$1.459\cdot10^{-4}$	$6.990\cdot10^{-4}$	$6.991\cdot10^{-4}$	$8.450\cdot10^{-4}$	$2.088\cdot10^{-1}$	0.845
		$1.0\cdot10^{-5}$	138.57	$4.315\cdot10^{-7}$	$9.118\cdot10^{-6}$	$9.118\cdot10^{-6}$	$9.549\cdot10^{-6}$	$4.733\cdot10^{-2}$	0.955

Table D.11: Minimal losses and optimal homogeneous local deadline in FIFO-BW system with homogeneous traffic ($M = 4$ and $M = 5$)

M	ρ	$M/M/1$	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	$P[D]/P[T]$	gain
6	0.30	$5.0{\cdot}10^{-2}$	7.02	$9.219{\cdot}10^{-3}$	$3.788{\cdot}10^{-2}$	$3.823{\cdot}10^{-2}$	$4.709{\cdot}10^{-2}$	$2.434{\cdot}10^{-1}$	0.942
		$1.0{\cdot}10^{-2}$	10.28	$9.448{\cdot}10^{-4}$	$8.754{\cdot}10^{-3}$	$8.762{\cdot}10^{-3}$	$9.698{\cdot}10^{-3}$	$1.079{\cdot}10^{-1}$	0.970
		$1.0{\cdot}10^{-3}$	14.68	$4.325{\cdot}10^{-5}$	$9.429{\cdot}10^{-4}$	$9.429{\cdot}10^{-4}$	$9.861{\cdot}10^{-4}$	$4.588{\cdot}10^{-2}$	0.986
		$1.0{\cdot}10^{-5}$	22.94	$1.340{\cdot}10^{-7}$	$9.823{\cdot}10^{-6}$	$9.823{\cdot}10^{-6}$	$9.957{\cdot}10^{-6}$	$1.365{\cdot}10^{-2}$	0.996
	0.40	$5.0{\cdot}10^{-2}$	8.48	$8.873{\cdot}10^{-3}$	$3.802{\cdot}10^{-2}$	$3.836{\cdot}10^{-2}$	$4.689{\cdot}10^{-2}$	$2.334{\cdot}10^{-1}$	0.938
		$1.0{\cdot}10^{-2}$	12.40	$8.469{\cdot}10^{-4}$	$8.853{\cdot}10^{-3}$	$8.861{\cdot}10^{-3}$	$9.700{\cdot}10^{-3}$	$9.566{\cdot}10^{-2}$	0.970
		$1.0{\cdot}10^{-3}$	17.68	$3.552{\cdot}10^{-5}$	$9.518{\cdot}10^{-4}$	$9.519{\cdot}10^{-4}$	$9.874{\cdot}10^{-4}$	$3.732{\cdot}10^{-2}$	0.987
		$1.0{\cdot}10^{-5}$	27.50	$9.840{\cdot}10^{-8}$	$9.867{\cdot}10^{-6}$	$9.867{\cdot}10^{-6}$	$9.965{\cdot}10^{-6}$	$9.973{\cdot}10^{-3}$	0.997
	0.50	$5.0{\cdot}10^{-2}$	10.11	$9.558{\cdot}10^{-3}$	$3.653{\cdot}10^{-2}$	$3.688{\cdot}10^{-2}$	$4.609{\cdot}10^{-2}$	$2.616{\cdot}10^{-1}$	0.922
		$1.0{\cdot}10^{-2}$	14.88	$8.825{\cdot}10^{-4}$	$8.757{\cdot}10^{-3}$	$8.765{\cdot}10^{-3}$	$9.640{\cdot}10^{-3}$	$1.008{\cdot}10^{-1}$	0.964
		$1.0{\cdot}10^{-3}$	21.33	$3.504{\cdot}10^{-5}$	$9.506{\cdot}10^{-4}$	$9.507{\cdot}10^{-4}$	$9.857{\cdot}10^{-4}$	$3.685{\cdot}10^{-2}$	0.986
		$1.0{\cdot}10^{-5}$	33.26	$8.982{\cdot}10^{-8}$	$9.873{\cdot}10^{-6}$	$9.873{\cdot}10^{-6}$	$9.963{\cdot}10^{-6}$	$9.097{\cdot}10^{-3}$	0.996
	0.60	$5.0{\cdot}10^{-2}$	12.16	$1.110{\cdot}10^{-2}$	$3.343{\cdot}10^{-2}$	$3.380{\cdot}10^{-2}$	$4.453{\cdot}10^{-2}$	$3.321{\cdot}10^{-1}$	0.891
		$1.0{\cdot}10^{-2}$	18.10	$1.032{\cdot}10^{-3}$	$8.473{\cdot}10^{-3}$	$8.482{\cdot}10^{-3}$	$9.505{\cdot}10^{-3}$	$1.217{\cdot}10^{-1}$	0.950
		$1.0{\cdot}10^{-3}$	26.25	$3.968{\cdot}10^{-5}$	$9.413{\cdot}10^{-4}$	$9.414{\cdot}10^{-4}$	$9.810{\cdot}10^{-4}$	$4.216{\cdot}10^{-2}$	0.981
		$1.0{\cdot}10^{-5}$	41.29	$9.654{\cdot}10^{-8}$	$9.857{\cdot}10^{-6}$	$9.857{\cdot}10^{-6}$	$9.954{\cdot}10^{-6}$	$9.794{\cdot}10^{-3}$	0.995
	0.80	$5.0{\cdot}10^{-2}$	20.68	$1.541{\cdot}10^{-2}$	$2.100{\cdot}10^{-2}$	$2.133{\cdot}10^{-2}$	$3.641{\cdot}10^{-2}$	$7.340{\cdot}10^{-1}$	0.728
		$1.0{\cdot}10^{-2}$	31.30	$1.837{\cdot}10^{-3}$	$6.819{\cdot}10^{-3}$	$6.831{\cdot}10^{-3}$	$8.656{\cdot}10^{-3}$	$2.694{\cdot}10^{-1}$	0.866
		$1.0{\cdot}10^{-3}$	47.45	$7.256{\cdot}10^{-5}$	$8.766{\cdot}10^{-4}$	$8.767{\cdot}10^{-4}$	$9.492{\cdot}10^{-4}$	$8.278{\cdot}10^{-2}$	0.949
		$1.0{\cdot}10^{-5}$	77.79	$1.681{\cdot}10^{-7}$	$9.714{\cdot}10^{-6}$	$9.714{\cdot}10^{-6}$	$9.882{\cdot}10^{-6}$	$1.730{\cdot}10^{-2}$	0.988
	0.90	$5.0{\cdot}10^{-2}$	36.04	$1.494{\cdot}10^{-2}$	$1.122{\cdot}10^{-2}$	$1.139{\cdot}10^{-2}$	$2.616{\cdot}10^{-2}$	$1.331{\cdot}10^{-0}$	0.523
		$1.0{\cdot}10^{-2}$	53.57	$2.553{\cdot}10^{-3}$	$4.603{\cdot}10^{-3}$	$4.615{\cdot}10^{-3}$	$7.156{\cdot}10^{-3}$	$5.547{\cdot}10^{-1}$	0.716
		$1.0{\cdot}10^{-3}$	83.79	$1.240{\cdot}10^{-4}$	$7.601{\cdot}10^{-4}$	$7.602{\cdot}10^{-4}$	$8.842{\cdot}10^{-4}$	$1.632{\cdot}10^{-1}$	0.884
		$1.0{\cdot}10^{-5}$	144.31	$2.918{\cdot}10^{-7}$	$9.440{\cdot}10^{-6}$	$9.440{\cdot}10^{-6}$	$9.732{\cdot}10^{-6}$	$3.091{\cdot}10^{-2}$	0.973

Table D.12: Minimal losses and optimal homogeneous local deadline in FIFO-BW system with homogeneous traffic ($M = 6$)

M	ρ	$M/M/1$	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	$P[D]/P[T]$	gain
8	0.30	$5.0 \cdot 10^{-2}$	7.91	$6.604 \cdot 10^{-3}$	$4.170 \cdot 10^{-2}$	$4.198 \cdot 10^{-2}$	$4.831 \cdot 10^{-2}$	$1.584 \cdot 10^{-1}$	0.966
		$1.0 \cdot 10^{-2}$	11.43	$5.643 \cdot 10^{-4}$	$9.288 \cdot 10^{-3}$	$9.293 \cdot 10^{-3}$	$9.852 \cdot 10^{-3}$	$6.075 \cdot 10^{-2}$	0.985
		$1.0 \cdot 10^{-3}$	16.12	$2.108 \cdot 10^{-5}$	$9.734 \cdot 10^{-4}$	$9.734 \cdot 10^{-4}$	$9.945 \cdot 10^{-4}$	$2.166 \cdot 10^{-2}$	0.994
		$1.0 \cdot 10^{-5}$	24.83	$4.753 \cdot 10^{-8}$	$9.940 \cdot 10^{-6}$	$9.940 \cdot 10^{-6}$	$9.987 \cdot 10^{-6}$	$4.782 \cdot 10^{-3}$	0.999
	0.40	$5.0 \cdot 10^{-2}$	9.50	$6.391 \cdot 10^{-3}$	$4.172 \cdot 10^{-2}$	$4.199 \cdot 10^{-2}$	$4.811 \cdot 10^{-2}$	$1.532 \cdot 10^{-1}$	0.962
		$1.0 \cdot 10^{-2}$	13.75	$5.013 \cdot 10^{-4}$	$9.349 \cdot 10^{-3}$	$9.354 \cdot 10^{-3}$	$9.850 \cdot 10^{-3}$	$5.363 \cdot 10^{-2}$	0.985
		$1.0 \cdot 10^{-3}$	19.42	$1.675 \cdot 10^{-5}$	$9.782 \cdot 10^{-4}$	$9.782 \cdot 10^{-4}$	$9.950 \cdot 10^{-4}$	$1.712 \cdot 10^{-2}$	0.995
		$1.0 \cdot 10^{-5}$	29.81	$3.274 \cdot 10^{-8}$	$9.957 \cdot 10^{-6}$	$9.957 \cdot 10^{-6}$	$9.990 \cdot 10^{-6}$	$3.288 \cdot 10^{-3}$	0.999
	0.50	$5.0 \cdot 10^{-2}$	11.26	$7.159 \cdot 10^{-3}$	$4.036 \cdot 10^{-2}$	$4.065 \cdot 10^{-2}$	$4.752 \cdot 10^{-2}$	$1.774 \cdot 10^{-1}$	0.950
		$1.0 \cdot 10^{-2}$	16.44	$5.380 \cdot 10^{-4}$	$9.276 \cdot 10^{-3}$	$9.281 \cdot 10^{-3}$	$9.814 \cdot 10^{-3}$	$5.800 \cdot 10^{-2}$	0.981
		$1.0 \cdot 10^{-3}$	23.39	$1.668 \cdot 10^{-5}$	$9.775 \cdot 10^{-4}$	$9.775 \cdot 10^{-4}$	$9.942 \cdot 10^{-4}$	$1.706 \cdot 10^{-2}$	0.994
		$1.0 \cdot 10^{-5}$	36.06	$2.962 \cdot 10^{-8}$	$9.960 \cdot 10^{-6}$	$9.960 \cdot 10^{-6}$	$9.990 \cdot 10^{-6}$	$2.974 \cdot 10^{-3}$	0.999
	0.60	$5.0 \cdot 10^{-2}$	13.45	$8.815 \cdot 10^{-3}$	$3.751 \cdot 10^{-2}$	$3.785 \cdot 10^{-2}$	$4.633 \cdot 10^{-2}$	$2.350 \cdot 10^{-1}$	0.927
		$1.0 \cdot 10^{-2}$	19.93	$6.633 \cdot 10^{-4}$	$9.068 \cdot 10^{-3}$	$9.074 \cdot 10^{-3}$	$9.731 \cdot 10^{-3}$	$7.315 \cdot 10^{-2}$	0.973
		$1.0 \cdot 10^{-3}$	28.71	$1.978 \cdot 10^{-5}$	$9.722 \cdot 10^{-4}$	$9.723 \cdot 10^{-4}$	$9.920 \cdot 10^{-4}$	$2.034 \cdot 10^{-2}$	0.992
		$1.0 \cdot 10^{-5}$	44.72	$3.270 \cdot 10^{-8}$	$9.954 \cdot 10^{-6}$	$9.954 \cdot 10^{-6}$	$9.987 \cdot 10^{-6}$	$3.285 \cdot 10^{-3}$	0.999
	0.80	$5.0 \cdot 10^{-2}$	22.45	$1.438 \cdot 10^{-2}$	$2.492 \cdot 10^{-2}$	$2.528 \cdot 10^{-2}$	$3.930 \cdot 10^{-2}$	$5.772 \cdot 10^{-1}$	0.786
		$1.0 \cdot 10^{-2}$	34.00	$1.427 \cdot 10^{-3}$	$7.700 \cdot 10^{-3}$	$7.711 \cdot 10^{-3}$	$9.126 \cdot 10^{-3}$	$1.853 \cdot 10^{-1}$	0.913
		$1.0 \cdot 10^{-3}$	51.48	$4.324 \cdot 10^{-5}$	$9.314 \cdot 10^{-4}$	$9.314 \cdot 10^{-4}$	$9.746 \cdot 10^{-4}$	$4.643 \cdot 10^{-2}$	0.975
		$1.0 \cdot 10^{-5}$	83.83	$6.691 \cdot 10^{-8}$	$9.894 \cdot 10^{-6}$	$9.894 \cdot 10^{-6}$	$9.961 \cdot 10^{-6}$	$6.763 \cdot 10^{-3}$	0.996
	0.90	$5.0 \cdot 10^{-2}$	38.51	$1.546 \cdot 10^{-2}$	$1.374 \cdot 10^{-2}$	$1.395 \cdot 10^{-2}$	$2.920 \cdot 10^{-2}$	$1.126 \cdot 10^{-0}$	0.584
		$1.0 \cdot 10^{-2}$	57.31	$2.340 \cdot 10^{-3}$	$5.513 \cdot 10^{-3}$	$5.526 \cdot 10^{-3}$	$7.853 \cdot 10^{-3}$	$4.244 \cdot 10^{-1}$	0.785
		$1.0 \cdot 10^{-3}$	90.10	$8.796 \cdot 10^{-5}$	$8.441 \cdot 10^{-4}$	$8.441 \cdot 10^{-4}$	$9.320 \cdot 10^{-4}$	$1.042 \cdot 10^{-1}$	0.932
		$1.0 \cdot 10^{-5}$	154.78	$1.366 \cdot 10^{-7}$	$9.759 \cdot 10^{-6}$	$9.759 \cdot 10^{-6}$	$9.895 \cdot 10^{-6}$	$1.399 \cdot 10^{-2}$	0.990

Table D.13: Minimal losses and optimal homogeneous local deadline in FIFO-BW system with homogeneous traffic ($M = 8$)

M	ρ	$M/M/1$	τ	$P[D]$	$P[T]$	$P[T\|A]$	$P[L]$	$P[D]/P[T]$	gain
10	0.30	$5.0\cdot10^{-2}$	8.70	$4.751\cdot10^{-3}$	$4.418\cdot10^{-2}$	$4.439\cdot10^{-2}$	$4.893\cdot10^{-2}$	$1.075\cdot10^{-1}$	0.979
		$1.0\cdot10^{-2}$	12.43	$3.491\cdot10^{-4}$	$9.571\cdot10^{-3}$	$9.575\cdot10^{-3}$	$9.920\cdot10^{-3}$	$3.648\cdot10^{-2}$	0.992
		$1.0\cdot10^{-3}$	17.39	$1.084\cdot10^{-5}$	$9.867\cdot10^{-4}$	$9.867\cdot10^{-4}$	$9.975\cdot10^{-4}$	$1.099\cdot10^{-2}$	0.998
		$1.0\cdot10^{-5}$	26.49	$1.859\cdot10^{-8}$	$9.977\cdot10^{-6}$	$9.977\cdot10^{-6}$	$9.996\cdot10^{-6}$	$1.864\cdot10^{-3}$	1.000
	0.40	$5.0\cdot10^{-2}$	10.40	$4.665\cdot10^{-3}$	$4.412\cdot10^{-2}$	$4.433\cdot10^{-2}$	$4.879\cdot10^{-2}$	$1.057\cdot10^{-1}$	0.976
		$1.0\cdot10^{-2}$	14.93	$3.089\cdot10^{-4}$	$9.610\cdot10^{-3}$	$9.613\cdot10^{-3}$	$9.919\cdot10^{-3}$	$3.214\cdot10^{-2}$	0.992
		$1.0\cdot10^{-3}$	20.93	$8.444\cdot10^{-6}$	$9.893\cdot10^{-4}$	$9.893\cdot10^{-4}$	$9.978\cdot10^{-4}$	$8.535\cdot10^{-3}$	0.998
		$1.0\cdot10^{-5}$	31.85	$1.204\cdot10^{-8}$	$9.985\cdot10^{-6}$	$9.985\cdot10^{-6}$	$9.997\cdot10^{-6}$	$1.206\cdot10^{-3}$	1.000
	0.50	$5.0\cdot10^{-2}$	12.27	$5.398\cdot10^{-3}$	$4.295\cdot10^{-2}$	$4.318\cdot10^{-2}$	$4.835\cdot10^{-2}$	$1.257\cdot10^{-1}$	0.967
		$1.0\cdot10^{-2}$	17.82	$3.378\cdot10^{-4}$	$9.559\cdot10^{-3}$	$9.562\cdot10^{-3}$	$9.897\cdot10^{-3}$	$3.533\cdot10^{-2}$	0.990
		$1.0\cdot10^{-3}$	25.19	$8.465\cdot10^{-6}$	$9.889\cdot10^{-4}$	$9.890\cdot10^{-4}$	$9.974\cdot10^{-4}$	$8.560\cdot10^{-3}$	0.997
		$1.0\cdot10^{-5}$	38.52	$1.079\cdot10^{-8}$	$9.986\cdot10^{-6}$	$9.986\cdot10^{-6}$	$9.997\cdot10^{-6}$	$1.081\cdot10^{-3}$	1.000
	0.60	$5.0\cdot10^{-2}$	14.59	$6.999\cdot10^{-3}$	$4.043\cdot10^{-2}$	$4.072\cdot10^{-2}$	$4.743\cdot10^{-2}$	$1.731\cdot10^{-1}$	0.949
		$1.0\cdot10^{-2}$	21.53	$4.373\cdot10^{-4}$	$9.407\cdot10^{-3}$	$9.411\cdot10^{-3}$	$9.844\cdot10^{-3}$	$4.649\cdot10^{-2}$	0.984
		$1.0\cdot10^{-3}$	30.88	$1.039\cdot10^{-5}$	$9.859\cdot10^{-4}$	$9.859\cdot10^{-4}$	$9.963\cdot10^{-4}$	$1.054\cdot10^{-2}$	0.996
		$1.0\cdot10^{-5}$	47.76	$1.213\cdot10^{-8}$	$9.984\cdot10^{-6}$	$9.984\cdot10^{-6}$	$9.996\cdot10^{-6}$	$1.215\cdot10^{-3}$	1.000
	0.80	$5.0\cdot10^{-2}$	23.94	$1.331\cdot10^{-2}$	$2.805\cdot10^{-2}$	$2.843\cdot10^{-2}$	$4.136\cdot10^{-2}$	$4.746\cdot10^{-1}$	0.827
		$1.0\cdot10^{-2}$	36.38	$1.107\cdot10^{-3}$	$8.302\cdot10^{-3}$	$8.311\cdot10^{-3}$	$9.408\cdot10^{-3}$	$1.333\cdot10^{-1}$	0.941
		$1.0\cdot10^{-3}$	55.06	$2.639\cdot10^{-5}$	$9.600\cdot10^{-4}$	$9.601\cdot10^{-4}$	$9.864\cdot10^{-4}$	$2.748\cdot10^{-2}$	0.986
		$1.0\cdot10^{-5}$	89.22	$2.850\cdot10^{-8}$	$9.957\cdot10^{-6}$	$9.957\cdot10^{-6}$	$9.985\cdot10^{-6}$	$2.862\cdot10^{-3}$	0.999
	0.90	$5.0\cdot10^{-2}$	40.58	$1.566\cdot10^{-2}$	$1.589\cdot10^{-2}$	$1.615\cdot10^{-2}$	$3.156\cdot10^{-2}$	$9.854\cdot10^{-1}$	0.631
		$1.0\cdot10^{-2}$	60.61	$2.102\cdot10^{-3}$	$6.233\cdot10^{-3}$	$6.247\cdot10^{-3}$	$8.335\cdot10^{-3}$	$3.372\cdot10^{-1}$	0.834
		$1.0\cdot10^{-3}$	95.77	$6.236\cdot10^{-5}$	$8.957\cdot10^{-4}$	$8.958\cdot10^{-4}$	$9.581\cdot10^{-4}$	$6.962\cdot10^{-2}$	0.958
		$1.0\cdot10^{-5}$	164.12	$6.708\cdot10^{-8}$	$9.888\cdot10^{-6}$	$9.888\cdot10^{-6}$	$9.955\cdot10^{-6}$	$6.784\cdot10^{-3}$	0.996

Table D.14: Minimal losses and optimal homogeneous local deadline in FIFO-BW system with homogeneous traffic ($M = 10$)

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	1.94	0.35	$0.00\cdot10^{+0}$	$1.04\cdot10^{-1}$
	2.13	0.39	$2.17\cdot10^{-4}$	$9.15\cdot10^{-2}$
	2.33	0.43	$7.90\cdot10^{-4}$	$8.07\cdot10^{-2}$
	2.52	0.47	$1.62\cdot10^{-3}$	$7.15\cdot10^{-2}$
	2.71	0.50	$2.65\cdot10^{-3}$	$6.38\cdot10^{-2}$
	2.91	0.53	$3.83\cdot10^{-3}$	$5.73\cdot10^{-2}$
	3.10	0.56	$5.11\cdot10^{-3}$	$5.18\cdot10^{-2}$
	3.30	0.59	$6.47\cdot10^{-3}$	$4.73\cdot10^{-2}$
	3.49	0.61	$7.89\cdot10^{-3}$	$4.35\cdot10^{-2}$
	3.68	0.63	$9.35\cdot10^{-3}$	$4.05\cdot10^{-2}$
	3.88	0.65	$1.09\cdot10^{-2}$	$3.80\cdot10^{-2}$
	∞	0.86	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.50	3.48	1.11	$0.00\cdot10^{+0}$	$8.60\cdot10^{-2}$
	3.83	1.20	$4.49\cdot10^{-4}$	$7.28\cdot10^{-2}$
	4.17	1.29	$1.60\cdot10^{-3}$	$6.24\cdot10^{-2}$
	4.52	1.37	$3.21\cdot10^{-3}$	$5.43\cdot10^{-2}$
	4.87	1.44	$5.15\cdot10^{-3}$	$4.80\cdot10^{-2}$
	5.22	1.50	$7.32\cdot10^{-3}$	$4.33\cdot10^{-2}$
	5.56	1.56	$9.63\cdot10^{-3}$	$3.98\cdot10^{-2}$
	5.91	1.61	$1.20\cdot10^{-2}$	$3.74\cdot10^{-2}$
	6.26	1.66	$1.45\cdot10^{-2}$	$3.58\cdot10^{-2}$
	6.61	1.70	$1.71\cdot10^{-2}$	$3.49\cdot10^{-2}$
	6.96	1.74	$1.96\cdot10^{-2}$	$3.46\cdot10^{-2}$
	∞	2.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.80	10.77	5.45	$6.94\cdot10^{-17}$	$3.73\cdot10^{-2}$
	11.85	5.79	$7.29\cdot10^{-4}$	$3.06\cdot10^{-2}$
	12.93	6.10	$2.54\cdot10^{-3}$	$2.64\cdot10^{-2}$
	14.00	6.37	$5.03\cdot10^{-3}$	$2.42\cdot10^{-2}$
	15.08	6.60	$7.94\cdot10^{-3}$	$2.33\cdot10^{-2}$
	16.16	6.80	$1.11\cdot10^{-2}$	$2.35\cdot10^{-2}$
	17.24	6.98	$1.45\cdot10^{-2}$	$2.44\cdot10^{-2}$
	18.31	7.13	$1.80\cdot10^{-2}$	$2.60\cdot10^{-2}$
	19.39	7.27	$2.15\cdot10^{-2}$	$2.80\cdot10^{-2}$
	20.47	7.38	$2.51\cdot10^{-2}$	$3.03\cdot10^{-2}$
	21.54	7.48	$2.88\cdot10^{-2}$	$3.29\cdot10^{-2}$
	∞	8.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.90	22.68	12.81	$0.00\cdot10^{+0}$	$1.88\cdot10^{-2}$
	24.95	13.55	$7.99\cdot10^{-4}$	$1.57\cdot10^{-2}$
	27.21	14.19	$2.77\cdot10^{-3}$	$1.45\cdot10^{-2}$
	29.48	14.76	$5.47\cdot10^{-3}$	$1.48\cdot10^{-2}$
	31.75	15.25	$8.62\cdot10^{-3}$	$1.60\cdot10^{-2}$
	34.02	15.67	$1.21\cdot10^{-2}$	$1.79\cdot10^{-2}$
	36.28	16.03	$1.57\cdot10^{-2}$	$2.03\cdot10^{-2}$
	38.55	16.34	$1.94\cdot10^{-2}$	$2.31\cdot10^{-2}$
	40.82	16.61	$2.32\cdot10^{-2}$	$2.61\cdot10^{-2}$
	43.09	16.84	$2.70\cdot10^{-2}$	$2.94\cdot10^{-2}$
	45.36	17.03	$3.14\cdot10^{-2}$	$3.33\cdot10^{-2}$
	∞	18.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$

Table D.15: Expected waiting time and loss for FIFO-BW with $M = 2$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 5%

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	1.64	0.42	$9.71 \cdot 10^{-17}$	$1.79 \cdot 10^{-1}$
	1.97	0.53	$4.15 \cdot 10^{-5}$	$1.45 \cdot 10^{-1}$
	2.29	0.62	$2.84 \cdot 10^{-4}$	$1.17 \cdot 10^{-1}$
	2.62	0.71	$9.85 \cdot 10^{-4}$	$9.52 \cdot 10^{-2}$
	2.95	0.79	$2.83 \cdot 10^{-3}$	$7.84 \cdot 10^{-2}$
	3.28	0.86	$5.44 \cdot 10^{-3}$	$6.59 \cdot 10^{-2}$
	3.60	0.93	$8.48 \cdot 10^{-3}$	$5.68 \cdot 10^{-2}$
	3.93	0.98	$1.17 \cdot 10^{-2}$	$5.02 \cdot 10^{-2}$
	4.26	1.03	$1.50 \cdot 10^{-2}$	$4.57 \cdot 10^{-2}$
	4.59	1.07	$1.84 \cdot 10^{-2}$	$4.28 \cdot 10^{-2}$
	4.91	1.10	$2.16 \cdot 10^{-2}$	$4.10 \cdot 10^{-2}$
	∞	1.29	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.50	2.98	1.39	$8.74 \cdot 10^{-16}$	$1.54 \cdot 10^{-1}$
	3.57	1.66	$1.42 \cdot 10^{-4}$	$1.16 \cdot 10^{-1}$
	4.17	1.89	$9.41 \cdot 10^{-4}$	$8.81 \cdot 10^{-2}$
	4.77	2.09	$2.85 \cdot 10^{-3}$	$6.81 \cdot 10^{-2}$
	5.36	2.27	$6.42 \cdot 10^{-3}$	$5.52 \cdot 10^{-2}$
	5.96	2.41	$1.08 \cdot 10^{-2}$	$4.72 \cdot 10^{-2}$
	6.55	2.53	$1.56 \cdot 10^{-2}$	$4.26 \cdot 10^{-2}$
	7.15	2.62	$2.03 \cdot 10^{-2}$	$4.04 \cdot 10^{-2}$
	7.74	2.70	$2.48 \cdot 10^{-2}$	$3.97 \cdot 10^{-2}$
	8.34	2.76	$2.90 \cdot 10^{-2}$	$4.01 \cdot 10^{-2}$
	8.93	2.81	$3.30 \cdot 10^{-2}$	$4.12 \cdot 10^{-2}$
	∞	3.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.80	9.42	7.23	$3.61 \cdot 10^{-16}$	$6.88 \cdot 10^{-2}$
	11.30	8.25	$3.21 \cdot 10^{-4}$	$4.74 \cdot 10^{-2}$
	13.19	9.10	$2.09 \cdot 10^{-3}$	$3.45 \cdot 10^{-2}$
	15.07	9.77	$5.92 \cdot 10^{-3}$	$2.82 \cdot 10^{-2}$
	16.95	10.31	$1.15 \cdot 10^{-2}$	$2.68 \cdot 10^{-2}$
	18.84	10.73	$1.76 \cdot 10^{-2}$	$2.82 \cdot 10^{-2}$
	20.72	11.05	$2.36 \cdot 10^{-2}$	$3.08 \cdot 10^{-2}$
	22.60	11.30	$2.91 \cdot 10^{-2}$	$3.40 \cdot 10^{-2}$
	24.49	11.48	$3.47 \cdot 10^{-2}$	$3.81 \cdot 10^{-2}$
	26.37	11.62	$3.87 \cdot 10^{-2}$	$4.10 \cdot 10^{-2}$
	28.26	11.72	$4.19 \cdot 10^{-2}$	$4.36 \cdot 10^{-2}$
	∞	12.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.90	19.95	17.23	$5.27 \cdot 10^{-16}$	$3.49 \cdot 10^{-2}$
	23.94	19.44	$3.71 \cdot 10^{-4}$	$2.36 \cdot 10^{-2}$
	27.93	21.23	$2.42 \cdot 10^{-3}$	$1.80 \cdot 10^{-2}$
	31.92	22.65	$6.81 \cdot 10^{-3}$	$1.73 \cdot 10^{-2}$
	35.91	23.76	$1.29 \cdot 10^{-2}$	$2.00 \cdot 10^{-2}$
	39.90	24.61	$1.94 \cdot 10^{-2}$	$2.41 \cdot 10^{-2}$
	43.89	25.25	$2.67 \cdot 10^{-2}$	$3.00 \cdot 10^{-2}$
	47.88	25.73	$3.21 \cdot 10^{-2}$	$3.43 \cdot 10^{-2}$
	51.87	26.08	$3.67 \cdot 10^{-2}$	$3.82 \cdot 10^{-2}$
	55.86	26.34	$4.05 \cdot 10^{-2}$	$4.15 \cdot 10^{-2}$
	59.85	26.53	$4.35 \cdot 10^{-2}$	$4.42 \cdot 10^{-2}$
	∞	27.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$

Table D.16: Expected waiting time and loss for FIFO-BW with $M = 3$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 5%

λ	τ	$E[w]$	$P[TA]$	$P[L]$
0.30	1.46	0.47	$0.00 \cdot 10^{+0}$	$2.50 \cdot 10^{-1}$
	1.90	0.65	$9.63 \cdot 10^{-6}$	$1.91 \cdot 10^{-1}$
	2.33	0.83	$2.19 \cdot 10^{-4}$	$1.46 \cdot 10^{-1}$
	2.77	0.99	$1.18 \cdot 10^{-3}$	$1.11 \cdot 10^{-1}$
	3.21	1.12	$3.46 \cdot 10^{-3}$	$8.57 \cdot 10^{-2}$
	3.65	1.23	$7.37 \cdot 10^{-3}$	$6.86 \cdot 10^{-2}$
	4.09	1.33	$1.20 \cdot 10^{-2}$	$5.75 \cdot 10^{-2}$
	4.52	1.41	$1.68 \cdot 10^{-2}$	$5.04 \cdot 10^{-2}$
	4.96	1.47	$2.15 \cdot 10^{-2}$	$4.63 \cdot 10^{-2}$
	5.40	1.52	$2.60 \cdot 10^{-2}$	$4.43 \cdot 10^{-2}$
	5.84	1.56	$3.01 \cdot 10^{-2}$	$4.36 \cdot 10^{-2}$
	∞	1.71	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.50	2.69	1.60	$1.07 \cdot 10^{-15}$	$2.20 \cdot 10^{-1}$
	3.49	2.11	$5.68 \cdot 10^{-5}$	$1.52 \cdot 10^{-1}$
	4.30	2.54	$8.74 \cdot 10^{-4}$	$1.06 \cdot 10^{-1}$
	5.10	2.89	$3.66 \cdot 10^{-3}$	$7.52 \cdot 10^{-2}$
	5.91	3.17	$8.64 \cdot 10^{-3}$	$5.72 \cdot 10^{-2}$
	6.71	3.38	$1.50 \cdot 10^{-2}$	$4.78 \cdot 10^{-2}$
	7.52	3.55	$2.15 \cdot 10^{-2}$	$4.35 \cdot 10^{-2}$
	8.32	3.67	$2.74 \cdot 10^{-2}$	$4.22 \cdot 10^{-2}$
	9.13	3.76	$3.27 \cdot 10^{-2}$	$4.25 \cdot 10^{-2}$
	9.94	3.83	$3.76 \cdot 10^{-2}$	$4.43 \cdot 10^{-2}$
	10.74	3.88	$4.11 \cdot 10^{-2}$	$4.56 \cdot 10^{-2}$
	∞	4.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.80	8.63	8.74	$0.00 \cdot 10^{+0}$	$1.00 \cdot 10^{-1}$
	11.21	10.53	$1.85 \cdot 10^{-4}$	$6.06 \cdot 10^{-2}$
	13.80	12.27	$2.35 \cdot 10^{-3}$	$3.90 \cdot 10^{-2}$
	16.39	13.42	$8.04 \cdot 10^{-3}$	$3.02 \cdot 10^{-2}$
	18.98	14.25	$1.61 \cdot 10^{-2}$	$2.95 \cdot 10^{-2}$
	21.56	14.83	$2.43 \cdot 10^{-2}$	$3.23 \cdot 10^{-2}$
	24.15	15.23	$3.26 \cdot 10^{-2}$	$3.76 \cdot 10^{-2}$
	26.74	15.50	$3.80 \cdot 10^{-2}$	$4.09 \cdot 10^{-2}$
	29.33	15.67	$4.20 \cdot 10^{-2}$	$4.38 \cdot 10^{-2}$
	31.92	15.79	$4.50 \cdot 10^{-2}$	$4.60 \cdot 10^{-2}$
	34.50	15.87	$4.69 \cdot 10^{-2}$	$4.75 \cdot 10^{-2}$
	∞	16.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$
0.90	18.35	21.05	$0.00 \cdot 10^{+0}$	$5.10 \cdot 10^{-2}$
	23.86	25.38	$2.28 \cdot 10^{-4}$	$2.97 \cdot 10^{-2}$
	29.36	28.66	$2.85 \cdot 10^{-3}$	$2.01 \cdot 10^{-2}$
	34.87	31.05	$9.39 \cdot 10^{-3}$	$1.95 \cdot 10^{-2}$
	40.38	32.73	$1.81 \cdot 10^{-2}$	$2.40 \cdot 10^{-2}$
	45.88	33.88	$2.83 \cdot 10^{-2}$	$3.18 \cdot 10^{-2}$
	51.39	34.65	$3.49 \cdot 10^{-2}$	$3.69 \cdot 10^{-2}$
	56.89	35.15	$4.00 \cdot 10^{-2}$	$4.12 \cdot 10^{-2}$
	62.40	35.47	$4.37 \cdot 10^{-2}$	$4.43 \cdot 10^{-2}$
	67.90	35.67	$4.62 \cdot 10^{-2}$	$4.66 \cdot 10^{-2}$
	73.41	35.80	$4.78 \cdot 10^{-2}$	$4.80 \cdot 10^{-2}$
	∞	36.00	$5.00 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$

Table D.17: Expected waiting time and loss for FIFO-BW with $M = 4$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 5%

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	1.34	0.50	$5.98\cdot10^{-14}$	$3.13\cdot10^{-1}$
	1.87	0.79	$3.52\cdot10^{-6}$	$2.30\cdot10^{-1}$
	2.41	1.05	$1.89\cdot10^{-4}$	$1.67\cdot10^{-1}$
	2.94	1.28	$1.47\cdot10^{-3}$	$1.21\cdot10^{-1}$
	3.48	1.48	$4.57\cdot10^{-3}$	$8.86\cdot10^{-2}$
	4.01	1.63	$9.75\cdot10^{-3}$	$6.85\cdot10^{-2}$
	4.55	1.76	$1.58\cdot10^{-2}$	$5.66\cdot10^{-2}$
	5.08	1.85	$2.17\cdot10^{-2}$	$5.00\cdot10^{-2}$
	5.62	1.93	$2.72\cdot10^{-2}$	$4.67\cdot10^{-2}$
	6.15	1.98	$3.26\cdot10^{-2}$	$4.62\cdot10^{-2}$
	6.69	2.03	$3.67\cdot10^{-2}$	$4.60\cdot10^{-2}$
	∞	2.14	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.50	2.49	1.77	$1.10\cdot10^{-14}$	$2.79\cdot10^{-1}$
	3.48	2.57	$3.06\cdot10^{-5}$	$1.81\cdot10^{-1}$
	4.48	3.23	$9.37\cdot10^{-4}$	$1.17\cdot10^{-1}$
	5.48	3.74	$4.76\cdot10^{-3}$	$7.76\cdot10^{-2}$
	6.47	4.12	$1.15\cdot10^{-2}$	$5.68\cdot10^{-2}$
	7.47	4.40	$1.95\cdot10^{-2}$	$4.74\cdot10^{-2}$
	8.46	4.60	$2.70\cdot10^{-2}$	$4.41\cdot10^{-2}$
	9.46	4.73	$3.34\cdot10^{-2}$	$4.38\cdot10^{-2}$
	10.45	4.82	$3.90\cdot10^{-2}$	$4.55\cdot10^{-2}$
	11.45	4.88	$4.26\cdot10^{-2}$	$4.65\cdot10^{-2}$
	12.44	5.92	$4.53\cdot10^{-2}$	$4.76\cdot10^{-2}$
	∞	5.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.80	8.09	10.06	$1.04\cdot10^{-13}$	$1.30\cdot10^{-1}$
	11.33	13.26	$1.38\cdot10^{-4}$	$7.02\cdot10^{-2}$
	14.57	15.62	$2.88\cdot10^{-3}$	$4.10\cdot10^{-2}$
	17.81	17.25	$1.07\cdot10^{-2}$	$3.13\cdot10^{-2}$
	21.04	18.33	$2.10\cdot10^{-2}$	$3.20\cdot10^{-2}$
	24.28	19.01	$3.20\cdot10^{-2}$	$3.81\cdot10^{-2}$
	27.52	19.42	$3.85\cdot10^{-2}$	$4.17\cdot10^{-2}$
	30.75	19.67	$4.30\cdot10^{-2}$	$4.47\cdot10^{-2}$
	33.99	19.81	$4.60\cdot10^{-2}$	$4.68\cdot10^{-2}$
	37.23	19.89	$4.78\cdot10^{-2}$	$4.82\cdot10^{-2}$
	40.47	19.94	$4.88\cdot10^{-2}$	$4.91\cdot10^{-2}$
	∞	20.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.90	17.28	24.48	$2.58\cdot10^{-13}$	$6.63\cdot10^{-2}$
	24.19	31.46	$1.81\cdot10^{-4}$	$3.40\cdot10^{-2}$
	31.10	36.47	$3.58\cdot10^{-3}$	$2.12\cdot10^{-2}$
	38.01	39.84	$1.26\cdot10^{-2}$	$2.17\cdot10^{-2}$
	44.92	41.98	$2.59\cdot10^{-2}$	$3.09\cdot10^{-2}$
	51.83	43.28	$3.45\cdot10^{-2}$	$3.70\cdot10^{-2}$
	58.74	44.04	$4.06\cdot10^{-2}$	$4.18\cdot10^{-2}$
	65.66	44.47	$4.46\cdot10^{-2}$	$4.52\cdot10^{-2}$
	72.57	44.71	$4.71\cdot10^{-2}$	$4.74\cdot10^{-2}$
	79.48	44.84	$4.85\cdot10^{-2}$	$4.86\cdot10^{-2}$
	86.39	44.92	$4.92\cdot10^{-2}$	$4.93\cdot10^{-2}$
	∞	45.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$

Table D.18: Expected waiting time and loss for FIFO-BW with $M = 5$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 5%

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	3.23	0.58	$1.67\cdot10^{-16}$	$4.30\cdot10^{-2}$
	3.55	0.62	$8.86\cdot10^{-5}$	$3.45\cdot10^{-2}$
	3.87	0.65	$3.08\cdot10^{-4}$	$2.79\cdot10^{-2}$
	4.20	0.68	$6.10\cdot10^{-4}$	$2.26\cdot10^{-2}$
	4.52	0.71	$9.64\cdot10^{-4}$	$1.86\cdot10^{-2}$
	4.84	0.73	$1.35\cdot10^{-3}$	$1.54\cdot10^{-2}$
	5.16	0.75	$1.76\cdot10^{-3}$	$1.30\cdot10^{-2}$
	5.49	0.77	$2.18\cdot10^{-3}$	$1.11\cdot10^{-2}$
	5.81	0.78	$2.60\cdot10^{-3}$	$9.75\cdot10^{-3}$
	6.13	0.80	$3.05\cdot10^{-3}$	$8.78\cdot10^{-3}$
	6.45	0.81	$3.49\cdot10^{-3}$	$8.05\cdot10^{-3}$
	∞	0.86	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.50	5.34	1.53	$1.39\cdot10^{-16}$	$3.41\cdot10^{-2}$
	5.87	1.61	$1.46\cdot10^{-4}$	$2.63\cdot10^{-2}$
	6.41	1.68	$4.99\cdot10^{-4}$	$2.06\cdot10^{-2}$
	6.94	1.74	$9.71\cdot10^{-4}$	$1.63\cdot10^{-2}$
	7.48	1.79	$1.51\cdot10^{-3}$	$1.33\cdot10^{-2}$
	8.01	1.83	$2.09\cdot10^{-3}$	$1.11\cdot10^{-2}$
	8.54	1.86	$2.70\cdot10^{-3}$	$9.62\cdot10^{-3}$
	9.08	1.89	$3.32\cdot10^{-3}$	$8.62\cdot10^{-3}$
	9.61	1.91	$3.95\cdot10^{-3}$	$8.01\cdot10^{-3}$
	10.15	1.93	$4.60\cdot10^{-3}$	$7.72\cdot10^{-3}$
	10.68	1.84	$5.23\cdot10^{-3}$	$7.62\cdot10^{-3}$
	∞	2.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.80	15.50	6.68	$0.00\cdot10^{+0}$	$1.43\cdot10^{-2}$
	17.05	6.95	$2.05\cdot10^{-4}$	$1.07\cdot10^{-2}$
	18.60	7.17	$6.86\cdot10^{-4}$	$8.36\cdot10^{-3}$
	20.15	7.35	$1.32\cdot10^{-3}$	$6.95\cdot10^{-3}$
	21.70	7.49	$2.03\cdot10^{-3}$	$6.16\cdot10^{-3}$
	23.25	7.60	$2.78\cdot10^{-3}$	$5.81\cdot10^{-3}$
	24.80	7.69	$3.60\cdot10^{-3}$	$5.85\cdot10^{-3}$
	26.35	7.76	$4.39\cdot10^{-3}$	$6.03\cdot10^{-3}$
	27.90	7.81	$5.18\cdot10^{-3}$	$6.39\cdot10^{-3}$
	29.45	7.86	$5.98\cdot10^{-3}$	$6.86\cdot10^{-3}$
	31.00	7.89	$6.78\cdot10^{-3}$	$7.42\cdot10^{-3}$
	∞	8.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$
0.90	32.15	15.32	$2.78\cdot10^{-16}$	$7.16\cdot10^{-3}$
	35.36	15.89	$2.19\cdot10^{-4}$	$5.41\cdot10^{-3}$
	38.58	16.35	$7.30\cdot10^{-4}$	$4.49\cdot10^{-3}$
	41.79	16.71	$1.40\cdot10^{-3}$	$4.12\cdot10^{-3}$
	45.01	17.00	$2.20\cdot10^{-3}$	$4.21\cdot10^{-3}$
	48.22	17.23	$2.99\cdot10^{-3}$	$4.44\cdot10^{-3}$
	51.43	17.41	$3.80\cdot10^{-3}$	$4.85\cdot10^{-3}$
	54.65	17.54	$4.62\cdot10^{-3}$	$5.39\cdot10^{-3}$
	57.86	17.65	$5.45\cdot10^{-3}$	$6.00\cdot10^{-3}$
	61.08	17.73	$6.28\cdot10^{-3}$	$6.68\cdot10^{-3}$
	64.29	17.80	$7.12\cdot10^{-3}$	$7.41\cdot10^{-3}$
	∞	18.00	$5.00\cdot10^{-2}$	$5.00\cdot10^{-2}$

Table D.19: Expected waiting time and loss for FIFO-BW with $M = 2$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 1%

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	2.57	0.70	$4.02 \cdot 10^{-16}$	$9.74 \cdot 10^{-2}$
	3.09	0.82	$1.99 \cdot 10^{-5}$	$6.91 \cdot 10^{-2}$
	3.60	0.93	$1.29 \cdot 10^{-4}$	$4.89 \cdot 10^{-2}$
	4.12	1.01	$4.09 \cdot 10^{-4}$	$3.48 \cdot 10^{-2}$
	4.63	1.07	$1.03 \cdot 10^{-3}$	$2.52 \cdot 10^{-2}$
	5.14	1.13	$1.83 \cdot 10^{-3}$	$1.87 \cdot 10^{-2}$
	5.66	1.16	$2.68 \cdot 10^{-3}$	$1.45 \cdot 10^{-2}$
	6.17	1.20	$3.58 \cdot 10^{-3}$	$1.19 \cdot 10^{-2}$
	6.69	1.22	$4.42 \cdot 10^{-3}$	$1.02 \cdot 10^{-2}$
	7.20	1.24	$5.21 \cdot 10^{-3}$	$9.26 \cdot 10^{-3}$
	7.72	1.25	$5.97 \cdot 10^{-3}$	$8.79 \cdot 10^{-3}$
	∞	1.29	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.50	4.35	1.96	$0.00 \cdot 10^{+0}$	$8.01 \cdot 10^{-2}$
	5.22	2.23	$4.95 \cdot 10^{-5}$	$5.27 \cdot 10^{-2}$
	6.09	2.44	$3.12 \cdot 10^{-4}$	$3.48 \cdot 10^{-2}$
	6.96	2.59	$8.99 \cdot 10^{-4}$	$2.34 \cdot 10^{-2}$
	7.83	2.71	$1.88 \cdot 10^{-3}$	$1.66 \cdot 10^{-2}$
	8.69	2.79	$3.00 \cdot 10^{-3}$	$1.25 \cdot 10^{-2}$
	9.56	2.86	$4.12 \cdot 10^{-3}$	$1.03 \cdot 10^{-2}$
	10.43	2.90	$5.23 \cdot 10^{-3}$	$9.27 \cdot 10^{-3}$
	11.30	2.93	$6.19 \cdot 10^{-3}$	$8.80 \cdot 10^{-3}$
	12.17	2.95	$7.05 \cdot 10^{-3}$	$8.74 \cdot 10^{-3}$
	13.04	2.97	$7.81 \cdot 10^{-3}$	$8.91 \cdot 10^{-3}$
	∞	3.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.80	12.93	8.99	$0.00 \cdot 10^{+0}$	$3.42 \cdot 10^{-2}$
	15.51	9.91	$9.00 \cdot 10^{-5}$	$2.07 \cdot 10^{-2}$
	18.10	10.57	$5.60 \cdot 10^{-4}$	$1.30 \cdot 10^{-2}$
	20.68	11.04	$1.54 \cdot 10^{-3}$	$8.99 \cdot 10^{-3}$
	23.27	11.37	$2.87 \cdot 10^{-3}$	$7.35 \cdot 10^{-3}$
	25.85	11.59	$4.34 \cdot 10^{-3}$	$7.06 \cdot 10^{-3}$
	28.44	11.73	$5.59 \cdot 10^{-3}$	$7.21 \cdot 10^{-3}$
	31.02	11.83	$6.69 \cdot 10^{-3}$	$7.65 \cdot 10^{-3}$
	33.61	11.89	$7.62 \cdot 10^{-3}$	$8.20 \cdot 10^{-3}$
	36.19	11.93	$8.39 \cdot 10^{-3}$	$8.73 \cdot 10^{-3}$
	38.78	11.96	$8.98 \cdot 10^{-3}$	$9.19 \cdot 10^{-3}$
	∞	12.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.90	26.98	20.84	$6.18 \cdot 10^{-15}$	$1.72 \cdot 10^{-2}$
	32.38	22.79	$1.00 \cdot 10^{-4}$	$1.02 \cdot 10^{-2}$
	37.77	24.18	$6.26 \cdot 10^{-4}$	$6.58 \cdot 10^{-3}$
	43.17	25.15	$1.71 \cdot 10^{-3}$	$5.20 \cdot 10^{-3}$
	48.56	25.80	$3.25 \cdot 10^{-3}$	$5.35 \cdot 10^{-3}$
	53.96	26.23	$4.65 \cdot 10^{-3}$	$5.87 \cdot 10^{-3}$
	59.36	26.51	$5.92 \cdot 10^{-3}$	$6.63 \cdot 10^{-3}$
	64.75	26.69	$7.01 \cdot 10^{-3}$	$7.43 \cdot 10^{-3}$
	70.15	26.81	$7.93 \cdot 10^{-3}$	$8.17 \cdot 10^{-3}$
	75.54	26.88	$8.65 \cdot 10^{-3}$	$8.79 \cdot 10^{-3}$
	80.94	26.93	$9.18 \cdot 10^{-3}$	$9.26 \cdot 10^{-3}$
	∞	27.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$

Table D.20: Expected waiting time and loss for FIFO-BW with $M = 3$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 1%

λ	τ	$E[w]$	$P[T\|A]$	$P[L]$
0.30	2.21	0.78	$3.06\cdot10^{-14}$	$1.58\cdot10^{-1}$
	2.87	1.02	$5.59\cdot10^{-6}$	$1.03\cdot10^{-1}$
	3.53	1.21	$1.02\cdot10^{-4}$	$6.68\cdot10^{-2}$
	4.19	1.35	$4.80\cdot10^{-4}$	$4.32\cdot10^{-2}$
	4.86	1.46	$1.25\cdot10^{-3}$	$2.85\cdot10^{-2}$
	5.52	1.54	$2.38\cdot10^{-3}$	$1.97\cdot10^{-2}$
	6.18	1.59	$3.65\cdot10^{-3}$	$1.47\cdot10^{-2}$
	6.84	1.63	$4.78\cdot10^{-3}$	$1.17\cdot10^{-2}$
	7.50	1.66	$5.81\cdot10^{-3}$	$1.02\cdot10^{-2}$
	8.17	1.68	$6.73\cdot10^{-3}$	$9.48\cdot10^{-3}$
	8.83	1.69	$7.54\cdot10^{-3}$	$9.26\cdot10^{-3}$
	∞	1.71	$1.00\cdot10^{-2}$	$1.00\cdot10^{-2}$
0.50	3.79	2.28	$2.59\cdot10^{-14}$	$1.32\cdot10^{-1}$
	4.93	2.82	$2.17\cdot10^{-5}$	$7.79\cdot10^{-2}$
	6.07	3.21	$2.99\cdot10^{-4}$	$4.56\cdot10^{-2}$
	7.21	3.49	$1.13\cdot10^{-3}$	$2.73\cdot10^{-2}$
	8.34	3.67	$2.47\cdot10^{-3}$	$1.75\cdot10^{-2}$
	9.48	3.79	$3.99\cdot10^{-3}$	$1.25\cdot10^{-2}$
	10.62	3.87	$5.49\cdot10^{-3}$	$1.04\cdot10^{-2}$
	11.76	3.92	$6.67\cdot10^{-3}$	$9.45\cdot10^{-3}$
	12.89	3.95	$7.64\cdot10^{-3}$	$9.21\cdot10^{-3}$
	14.03	3.97	$8.41\cdot10^{-3}$	$9.30\cdot10^{-3}$
	15.17	3.98	$8.99\cdot10^{-3}$	$9.50\cdot10^{-3}$
	∞	4.00	$1.00\cdot10^{-2}$	$1.00\cdot10^{-2}$
0.80	11.48	10.91	$0.00\cdot10^{+0}$	$5.74\cdot10^{-2}$
	14.92	12.81	$5.32\cdot10^{-5}$	$2.97\cdot10^{-2}$
	18.37	13.44	$6.37\cdot10^{-4}$	$1.59\cdot10^{-2}$
	21.81	14.39	$2.05\cdot10^{-3}$	$9.89\cdot10^{-3}$
	25.25	15.01	$4.07\cdot10^{-3}$	$8.16\cdot10^{-3}$
	28.70	15.40	$5.75\cdot10^{-3}$	$7.80\cdot10^{-3}$
	32.14	15.64	$7.12\cdot10^{-3}$	$8.14\cdot10^{-3}$
	35.59	15.79	$8.14\cdot10^{-3}$	$8.66\cdot10^{-3}$
	39.03	15.88	$8.88\cdot10^{-3}$	$9.14\cdot10^{-3}$
	42.47	15.97	$9.37\cdot10^{-3}$	$9.50\cdot10^{-3}$
	45.92	15.98	$9.67\cdot10^{-3}$	$9.74\cdot10^{-3}$
	∞	16.00	$1.00\cdot10^{-2}$	$1.00\cdot10^{-2}$
0.90	24.08	25.53	$0.00\cdot10^{+0}$	$2.89\cdot10^{-2}$
	31.30	29.59	$6.27\cdot10^{-5}$	$1.45\cdot10^{-2}$
	38.52	32.24	$7.39\cdot10^{-4}$	$7.93\cdot10^{-3}$
	45.74	33.86	$2.52\cdot10^{-3}$	$6.25\cdot10^{-3}$
	52.97	34.81	$4.44\cdot10^{-3}$	$6.24\cdot10^{-3}$
	60.19	35.36	$6.14\cdot10^{-3}$	$7.01\cdot10^{-3}$
	67.41	35.65	$7.47\cdot10^{-3}$	$7.89\cdot10^{-3}$
	74.64	35.82	$8.44\cdot10^{-3}$	$8.64\cdot10^{-3}$
	81.86	35.90	$9.11\cdot10^{-3}$	$9.20\cdot10^{-3}$
	89.08	35.95	$9.53\cdot10^{-3}$	$9.58\cdot10^{-3}$
	96.30	35.97	$9.77\cdot10^{-3}$	$9.79\cdot10^{-3}$
	∞	36.00	$1.00\cdot10^{-2}$	$1.00\cdot10^{-2}$

Table D.21: Expected waiting time and loss for FIFO-BW with $M = 4$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 1%

λ	τ	$E[w]$	$P[TA]$	$P[L]$
0.30	1.97	0.84	$0.00 \cdot 10^{+0}$	$2.18 \cdot 10^{-1}$
	2.76	1.21	$2.29 \cdot 10^{-6}$	$1.34 \cdot 10^{-1}$
	3.55	1.50	$9.31 \cdot 10^{-5}$	$8.09 \cdot 10^{-2}$
	4.33	1.71	$5.85 \cdot 10^{-4}$	$4.85 \cdot 10^{-2}$
	5.12	1.86	$1.59 \cdot 10^{-3}$	$2.97 \cdot 10^{-2}$
	5.91	1.96	$3.11 \cdot 10^{-3}$	$1.97 \cdot 10^{-2}$
	6.70	2.03	$4.54 \cdot 10^{-3}$	$1.41 \cdot 10^{-2}$
	7.49	2.07	$5.82 \cdot 10^{-3}$	$1.13 \cdot 10^{-2}$
	8.27	2.10	$6.90 \cdot 10^{-3}$	$1.01 \cdot 10^{-2}$
	9.06	2.11	$7.78 \cdot 10^{-3}$	$9.61 \cdot 10^{-3}$
	9.85	2.13	$8.51 \cdot 10^{-3}$	$9.56 \cdot 10^{-3}$
	∞	2.14	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.50	3.43	2.53	$0.00 \cdot 10^{+0}$	$1.85 \cdot 10^{-1}$
	4.80	3.41	$1.27 \cdot 10^{-5}$	$9.99 \cdot 10^{-2}$
	6.18	4.02	$3.26 \cdot 10^{-4}$	$5.30 \cdot 10^{-2}$
	7.55	4.42	$1.44 \cdot 10^{-3}$	$2.88 \cdot 10^{-2}$
	8.92	4.66	$3.17 \cdot 10^{-3}$	$1.72 \cdot 10^{-2}$
	10.29	4.81	$5.11 \cdot 10^{-3}$	$1.23 \cdot 10^{-2}$
	11.67	4.89	$6.59 \cdot 10^{-3}$	$1.02 \cdot 10^{-2}$
	13.04	4.94	$7.73 \cdot 10^{-3}$	$9.56 \cdot 10^{-3}$
	14.41	4.97	$8.57 \cdot 10^{-3}$	$9.49 \cdot 10^{-3}$
	15.78	4.98	$9.15 \cdot 10^{-3}$	$9.61 \cdot 10^{-3}$
	17.15	4.99	$9.53 \cdot 10^{-3}$	$9.77 \cdot 10^{-3}$
	∞	5.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.80	10.53	12.55	$2.59 \cdot 10^{-13}$	$8.14 \cdot 10^{-2}$
	14.75	15.72	$4.08 \cdot 10^{-5}$	$3.70 \cdot 10^{-2}$
	18.96	17.69	$7.80 \cdot 10^{-4}$	$1.75 \cdot 10^{-2}$
	23.17	18.81	$2.70 \cdot 10^{-3}$	$1.01 \cdot 10^{-2}$
	27.39	19.41	$5.16 \cdot 10^{-3}$	$8.49 \cdot 10^{-3}$
	31.60	19.71	$6.93 \cdot 10^{-3}$	$8.37 \cdot 10^{-3}$
	35.81	19.86	$8.18 \cdot 10^{-3}$	$8.80 \cdot 10^{-3}$
	40.03	19.93	$9.00 \cdot 10^{-3}$	$9.26 \cdot 10^{-3}$
	44.24	19.97	$9.49 \cdot 10^{-3}$	$9.60 \cdot 10^{-3}$
	48.45	19.99	$9.76 \cdot 10^{-3}$	$9.81 \cdot 10^{-3}$
	52.67	19.99	$9.89 \cdot 10^{-3}$	$9.92 \cdot 10^{-3}$
	∞	20.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$
0.90	22.18	29.64	$3.20 \cdot 10^{-14}$	$4.10 \cdot 10^{-2}$
	31.05	36.44	$5.08 \cdot 10^{-5}$	$1.78 \cdot 10^{-2}$
	39.92	40.53	$9.27 \cdot 10^{-4}$	$8.61 \cdot 10^{-3}$
	48.79	42.79	$3.35 \cdot 10^{-3}$	$6.78 \cdot 10^{-3}$
	57.66	43.95	$5.60 \cdot 10^{-3}$	$7.01 \cdot 10^{-3}$
	66.53	44.51	$7.33 \cdot 10^{-3}$	$7.91 \cdot 10^{-3}$
	75.40	44.78	$8.49 \cdot 10^{-3}$	$8.73 \cdot 10^{-3}$
	84.27	44.90	$9.22 \cdot 10^{-3}$	$9.31 \cdot 10^{-3}$
	93.14	44.95	$9.63 \cdot 10^{-3}$	$9.67 \cdot 10^{-3}$
	102.01	44.98	$9.84 \cdot 10^{-3}$	$9.85 \cdot 10^{-3}$
	110.88	44.99	$9.93 \cdot 10^{-3}$	$9.94 \cdot 10^{-3}$
	∞	45.00	$1.00 \cdot 10^{-2}$	$1.00 \cdot 10^{-2}$

Table D.22: Expected waiting time and loss for FIFO-BW with $M = 5$, homogeneous deadlines, decreasing traffic, uncontrolled loss of 1%

τ	ρ	$P[L]$ $M/M/1$	uni λ	uni τ	red. net.
15.0	0.30	$5.469\cdot10^{-14}$	$2.891\cdot10^{-5}$	$2.891\cdot10^{-5}$	$2.891\cdot10^{-5}$
	0.35	$1.091\cdot10^{-12}$	$6.631\cdot10^{-5}$	$6.630\cdot10^{-5}$	$6.629\cdot10^{-5}$
	0.40	$1.974\cdot10^{-11}$	$1.481\cdot10^{-4}$	$1.480\cdot10^{-4}$	$1.480\cdot10^{-4}$
	0.45	$3.276\cdot10^{-10}$	$3.233\cdot10^{-4}$	$3.230\cdot10^{-4}$	$3.227\cdot10^{-4}$
	0.50	$5.024\cdot10^{-9}$	$6.913\cdot10^{-4}$	$6.898\cdot10^{-4}$	$6.884\cdot10^{-4}$
	0.55	$7.134\cdot10^{-8}$	$1.449\cdot10^{-3}$	$1.442\cdot10^{-3}$	$1.435\cdot10^{-3}$
	0.60	$9.364\cdot10^{-7}$	$2.974\cdot10^{-3}$	$2.944\cdot10^{-3}$	$2.915\cdot10^{-3}$
	0.65	$1.128\cdot10^{-5}$	$5.968\cdot10^{-3}$	$5.844\cdot10^{-3}$	$5.723\cdot10^{-3}$
	0.70	$1.233\cdot10^{-4}$	$1.167\cdot10^{-2}$	$1.120\cdot10^{-2}$	$1.074\cdot10^{-2}$
	0.75	$1.197\cdot10^{-3}$	$2.216\cdot10^{-2}$	$2.048\cdot10^{-2}$	$1.893\cdot10^{-2}$
	0.80	$9.952\cdot10^{-3}$	$4.055\cdot10^{-2}$	$3.529\cdot10^{-2}$	$3.072\cdot10^{-2}$
	0.85	$6.650\cdot10^{-2}$	$7.118\cdot10^{-2}$	$5.673\cdot10^{-2}$	$4.524\cdot10^{-2}$
	0.90	$3.159\cdot10^{-1}$	$1.194\cdot10^{-1}$	$8.470\cdot10^{-2}$	$6.027\cdot10^{-2}$
	0.95	$8.346\cdot10^{-1}$	$1.916\cdot10^{-1}$	$1.179\cdot10^{-1}$	$7.345\cdot10^{-2}$
18.3	0.30	$5.469\cdot10^{-14}$	$2.870\cdot10^{-6}$	$2.870\cdot10^{-6}$	$2.870\cdot10^{-6}$
	0.35	$1.091\cdot10^{-12}$	$7.763\cdot10^{-6}$	$7.763\cdot10^{-6}$	$7.762\cdot10^{-6}$
	0.40	$1.974\cdot10^{-11}$	$2.045\cdot10^{-5}$	$2.045\cdot10^{-5}$	$2.044\cdot10^{-5}$
	0.45	$3.276\cdot10^{-10}$	$5.265\cdot10^{-5}$	$5.264\cdot10^{-5}$	$5.263\cdot10^{-5}$
	0.50	$5.024\cdot10^{-9}$	$1.328\cdot10^{-4}$	$1.327\cdot10^{-4}$	$1.326\cdot10^{-4}$
	0.55	$7.134\cdot10^{-8}$	$3.282\cdot10^{-4}$	$3.278\cdot10^{-4}$	$3.273\cdot10^{-4}$
	0.60	$9.364\cdot10^{-7}$	$7.946\cdot10^{-4}$	$7.920\cdot10^{-4}$	$7.893\cdot10^{-4}$
	0.65	$1.128\cdot10^{-5}$	$1.881\cdot10^{-3}$	$1.866\cdot10^{-3}$	$1.851\cdot10^{-3}$
	0.70	$1.233\cdot10^{-4}$	$4.346\cdot10^{-3}$	$4.261\cdot10^{-3}$	$4.177\cdot10^{-3}$
	0.75	$1.197\cdot10^{-3}$	$9.774\cdot10^{-3}$	$9.344\cdot10^{-3}$	$8.934\cdot10^{-3}$
	0.80	$9.952\cdot10^{-3}$	$2.148\cdot10^{-2}$	$1.949\cdot10^{-2}$	$1.770\cdot10^{-2}$
	0.85	$6.650\cdot10^{-2}$	$4.671\cdot10^{-2}$	$3.829\cdot10^{-2}$	$3.160\cdot10^{-2}$
	0.90	$3.159\cdot10^{-1}$	$1.016\cdot10^{-1}$	$7.000\cdot10^{-2}$	$4.961\cdot10^{-2}$
	0.95	$8.346\cdot10^{-1}$	$2.142\cdot10^{-1}$	$1.166\cdot10^{-1}$	$6.796\cdot10^{-2}$
20.0	0.30	$5.469\cdot10^{-14}$	$8.731\cdot10^{-7}$	$8.775\cdot10^{-7}$	$8.731\cdot10^{-7}$
	0.35	$1.091\cdot10^{-12}$	$2.571\cdot10^{-6}$	$2.571\cdot10^{-6}$	$2.571\cdot10^{-6}$
	0.40	$1.974\cdot10^{-11}$	$7.373\cdot10^{-6}$	$7.373\cdot10^{-6}$	$7.373\cdot10^{-6}$
	0.45	$3.276\cdot10^{-10}$	$2.067\cdot10^{-5}$	$2.067\cdot10^{-5}$	$2.067\cdot10^{-5}$
	0.50	$5.024\cdot10^{-9}$	$5.675\cdot10^{-5}$	$5.674\cdot10^{-5}$	$5.672\cdot10^{-5}$
	0.55	$7.134\cdot10^{-8}$	$1.527\cdot10^{-4}$	$1.526\cdot10^{-4}$	$1.525\cdot10^{-4}$
	0.60	$9.364\cdot10^{-7}$	$4.027\cdot10^{-4}$	$4.019\cdot10^{-4}$	$4.012\cdot10^{-4}$
	0.65	$1.128\cdot10^{-5}$	$1.039\cdot10^{-3}$	$1.034\cdot10^{-3}$	$1.029\cdot10^{-3}$
	0.70	$1.233\cdot10^{-4}$	$2.622\cdot10^{-3}$	$2.588\cdot10^{-3}$	$2.554\cdot10^{-3}$
	0.75	$1.197\cdot10^{-3}$	$6.500\cdot10^{-3}$	$6.286\cdot10^{-3}$	$6.080\cdot10^{-3}$
	0.80	$9.952\cdot10^{-3}$	$1.610\cdot10^{-2}$	$1.483\cdot10^{-2}$	$1.368\cdot10^{-2}$
	0.85	$6.650\cdot10^{-2}$	$4.092\cdot10^{-2}$	$3.384\cdot10^{-2}$	$2.821\cdot10^{-2}$
	0.90	$3.159\cdot10^{-1}$	$1.057\cdot10^{-1}$	$7.205\cdot10^{-2}$	$5.060\cdot10^{-2}$
	0.95	$8.346\cdot10^{-1}$	$2.519\cdot10^{-1}$	$1.346\cdot10^{-1}$	$7.596\cdot10^{-2}$
23.3	0.30	$5.469\cdot10^{-14}$	$8.667\cdot10^{-8}$	$8.667\cdot10^{-8}$	$8.667\cdot10^{-8}$
	0.35	$1.091\cdot10^{-12}$	$3.010\cdot10^{-7}$	$3.010\cdot10^{-7}$	$3.010\cdot10^{-7}$
	0.40	$1.974\cdot10^{-11}$	$1.018\cdot10^{-6}$	$1.018\cdot10^{-6}$	$1.018\cdot10^{-6}$
	0.45	$3.276\cdot10^{-10}$	$3.366\cdot10^{-6}$	$3.366\cdot10^{-6}$	$3.366\cdot10^{-6}$
	0.50	$5.024\cdot10^{-9}$	$1.090\cdot10^{-5}$	$1.090\cdot10^{-5}$	$1.090\cdot10^{-5}$
	0.55	$7.134\cdot10^{-8}$	$3.462\cdot10^{-5}$	$3.461\cdot10^{-5}$	$3.460\cdot10^{-5}$
	0.60	$9.364\cdot10^{-7}$	$1.079\cdot10^{-4}$	$1.078\cdot10^{-4}$	$1.077\cdot10^{-4}$
	0.65	$1.128\cdot10^{-5}$	$3.307\cdot10^{-4}$	$3.300\cdot10^{-4}$	$3.294\cdot10^{-4}$
	0.70	$1.233\cdot10^{-4}$	$1.008\cdot10^{-3}$	$1.002\cdot10^{-3}$	$9.961\cdot10^{-4}$
	0.75	$1.197\cdot10^{-3}$	$3.152\cdot10^{-3}$	$3.092\cdot10^{-3}$	$3.034\cdot10^{-3}$
	0.80	$9.952\cdot10^{-3}$	$1.059\cdot10^{-2}$	$9.977\cdot10^{-3}$	$9.411\cdot10^{-3}$
	0.85	$6.650\cdot10^{-2}$	$3.808\cdot10^{-2}$	$3.244\cdot10^{-2}$	$2.777\cdot10^{-2}$
	0.90	$3.159\cdot10^{-1}$	$1.295\cdot10^{-1}$	$9.205\cdot10^{-2}$	$6.639\cdot10^{-2}$
	0.95	$8.346\cdot10^{-1}$	$3.472\cdot10^{-1}$	$2.004\cdot10^{-1}$	$1.167\cdot10^{-1}$
25.0	0.40	$1.974\cdot10^{-11}$	$3.671\cdot10^{-7}$	$4.109\cdot10^{-7}$	$3.671\cdot10^{-7}$
	0.45	$3.276\cdot10^{-10}$	$1.321\cdot10^{-6}$	$1.320\cdot10^{-6}$	$1.321\cdot10^{-6}$
	0.50	$5.024\cdot10^{-9}$	$4.661\cdot10^{-6}$	$4.660\cdot10^{-6}$	$4.660\cdot10^{-6}$
	0.55	$7.134\cdot10^{-8}$	$1.613\cdot10^{-5}$	$1.613\cdot10^{-5}$	$1.613\cdot10^{-5}$
	0.60	$9.364\cdot10^{-7}$	$5.490\cdot10^{-5}$	$5.488\cdot10^{-5}$	$5.486\cdot10^{-5}$
	0.65	$1.128\cdot10^{-5}$	$1.852\cdot10^{-4}$	$1.850\cdot10^{-4}$	$1.848\cdot10^{-4}$
	0.70	$1.233\cdot10^{-4}$	$6.338\cdot10^{-4}$	$6.312\cdot10^{-4}$	$6.286\cdot10^{-4}$
	0.75	$1.197\cdot10^{-3}$	$2.306\cdot10^{-3}$	$2.273\cdot10^{-3}$	$2.241\cdot10^{-3}$
	0.80	$9.952\cdot10^{-3}$	$9.297\cdot10^{-3}$	$8.856\cdot10^{-3}$	$8.440\cdot10^{-3}$
	0.85	$6.650\cdot10^{-2}$	$3.924\cdot10^{-2}$	$3.418\cdot10^{-2}$	$2.987\cdot10^{-2}$
	0.90	$3.159\cdot10^{-1}$	$1.460\cdot10^{-1}$	$1.079\cdot10^{-1}$	$8.049\cdot10^{-2}$
	0.95	$8.346\cdot10^{-1}$	$3.982\cdot10^{-1}$	$2.441\cdot10^{-1}$	$1.489\cdot10^{-1}$

Table D.23: Loss performance for fixed end-to-end deadline, $M = 5$, $\mu = 1$, $d = 52.7$

λ	$\tau = 15$	$\tau = 18.3$	$\tau = 20.0$	$\tau = 23.3$	$\tau = 25.0$	$\tau = \infty$
0.30	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.35	0.3500	0.3500	0.3500	0.3500	0.3000	0.3500
0.40	0.3999	0.4000	0.4000	0.4000	0.4000	0.4000
0.45	0.4499	0.4500	0.4500	0.4500	0.4500	0.4500
0.50	0.4997	0.4999	0.5000	0.5000	0.5000	0.5000
0.55	0.5492	0.5498	0.5499	0.5500	0.5500	0.5500
0.60	0.5982	0.5995	0.5998	0.6000	0.6000	0.6000
0.65	0.6462	0.6488	0.6493	0.6498	0.6499	0.6500
0.70	0.6922	0.6970	0.6982	0.6993	0.6996	0.6999
0.75	0.7346	0.7430	0.7453	0.7477	0.7483	0.7491
0.80	0.7718	0.7844	0.7881	0.7920	0.7929	0.7920
0.85	0.8018	0.8175	0.8212	0.8224	0.8209	0.7933
0.90	0.8238	0.8370	0.8352	0.8172	0.8029	0.6153
0.95	0.8380	0.8392	0.8221	0.7596	0.7181	0.1569

Table D.24: Goodput of FIFO-BW tandem system with $M = 5$ for uncontrolled loss of 1%

M	ρ	d	τ	samples	theory	average	SCI	cycles	RCI
5	0.80	40.47	8.09	497000	13.000	11.640	11.490...11.780	1186	11.460...11.800
			11.33	465909	7.020	6.224	6.047... 6.400	724	6.054... 6.364
			14.57	5173630	4.100	3.483	3.406... 3.560	5899	3.439... 3.527
			17.81	3793875	3.130	2.447	2.368... 2.525	3771	2.382... 2.512
			21.04	2528250	3.200	2.503	2.369... 2.636	2413	2.379... 2.626
			24.28	2528250	3.810	3.035	2.821... 3.249	2431	2.871... 3.194
			27.52	2528250	4.170	3.725	3.550... 3.900	2319	3.504... 3.945
			30.75	1684500	4.470	4.338	3.993... 4.684	1618	4.036... 4.643
			33.99	747000	4.680	4.469	4.048... 4.890	657	4.018... 4.909
			37.23	2528250	4.820	5.025	4.658... 5.391	2334	4.743... 5.312
			40.47	1684500	4.910	5.062	4.700... 5.424	1629	4.689... 5.436
			∞	1684500	5.000	4.834	4.419... 5.248	1474	4.466... 5.180
5	0.80	52.67	10.53	497000	8.140	7.206	7.006... 7.406	772	7.033... 7.376
			14.75	497000	3.700	3.243	3.073... 3.413	567	3.108... 3.382
			18.96	5072692	1.750	1.508	1.473... 1.543	4959	1.477... 1.538
			23.17	5031148	1.010	0.823	0.783... 0.863	4552	0.789... 0.857
			27.39	5011915	0.849	0.642	0.614... 0.671	4513	0.597... 0.687
			31.60	5003670	0.837	0.763	0.674... 0.851	4463	0.697... 0.829
			35.81	5000067	0.880	0.866	0.791... 0.942	4383	0.785... 0.944
			40.03	4998322	0.926	0.899	0.848... 0.949	4552	0.817... 0.981
			44.24	4997680	0.960	1.004	0.872... 1.137	4388	0.908... 1.010
			48.45	4997232	0.981	1.000	0.870... 1.131	4543	0.906... 1.092
			52.67	4997086	0.992	1.012	0.920... 1.103	4348	0.918... 1.106
			∞	4997000	1.000	0.984	0.904... 1.064	4358	0.889... 1.074

Table D.25: Total loss (simulation)

M	ρ	d	τ	samples	theory	average	SCI	cycles	RCI
5	0.80	40.47	8.09	438827	0.000	0.000	0.000...0.000	1186	0.000...0.000
			11.33	465509	0.014	0.000	0.000...0.000	724	0.000...0.000
			14.57	4997000	0.288	0.073	0.064...0.081	5899	0.065...0.081
			17.81	3722524	1.070	0.578	0.536...0.621	3771	0.538...0.620
			21.04	2502031	2.100	1.482	1.357...1.608	2413	1.378...1.586
			24.28	2514275	3.200	2.497	2.304...2.689	2431	2.347...2.642
			27.52	2528250	3.850	3.444	3.272...3.616	2319	3.233...3.654
			30.75	1120311	4.300	4.115	3.702...4.528	1117	3.765...4.470
			33.99	746394	4.600	4.392	3.976...4.808	657	3.950...4.823
			37.23	2527150	4.780	4.983	4.619...5.347	2334	4.706...5.267
			40.47	1121798	4.880	5.040	4.709...5.372	1629	4.670...5.412
			∞	1684499	5.000	4.834	4.419...5.248	1474	4.466...5.188
5	0.80	52.67	10.53	460948	0.000	0.000	0.000...0.000	772	0.000...0.000
			14.75	480771	0.000	0.000	0.000...0.000	567	0.000...0.000
			18.96	4997000	0.080	0.016	0.010...0.023	4959	0.012...0.021
			23.17	4997000	0.270	0.145	0.117...0.174	4552	0.125...0.167
			27.39	4997000	0.516	0.346	0.328...0.364	4513	0.308...0.384
			31.60	4997000	0.693	0.630	0.543...0.718	4463	0.569...0.692
			35.81	4997000	0.818	0.805	0.733...0.878	4383	0.727...0.880
			40.03	4997000	0.900	0.872	0.821...0.924	4552	0.793...0.953
			44.24	4997000	0.949	0.991	0.860...1.121	4388	0.895...1.080
			48.45	4997000	0.976	0.996	0.865...1.127	4543	0.902...1.087
			52.67	4997000	0.989	1.010	0.918...1.102	4348	0.916...1.104
			∞	4997000	1.000	0.984	0.904...1.064	4358	0.889...1.074

Table D.26: Late loss (simulation)

M	ρ	d	τ	samples	theory	average	SCI	cycles	RCI
5	0.80	40.47	8.09	438827	10.06	9.88	9.80... 9.95	1186	9.83... 9.92
			11.33	465309	13.26	12.99	12.86...13.11	724	12.93...13.03
			14.57	4997000	15.62	15.40	15.34...15.45	5899	15.35...15.44
			17.81	3722524	17.25	17.06	17.02...17.11	3771	17.00...17.13
			21.04	2502031	18.33	18.18	18.04...18.32	2413	18.08...18.28
			24.28	2514275	19.01	18.85	18.69...19.01	2431	18.73...18.97
			27.52	2520893	19.42	19.26	19.09...19.44	2319	19.13...19.40
			30.75	1681897	19.67	19.54	19.33...19.75	1618	19.37...19.71
			33.99	746394	19.81	19.63	19.33...19.94	657	19.37...19.88
			37.23	2527150	19.89	19.87	19.67...20.06	2334	19.71...20.03
			40.47	1121798	19.94	19.87	19.65...20.08	1629	19.67...20.06
			∞	1684499	20.00	19.81	19.59...20.03	1474	19.61...20.00
5	0.80	52.67	10.53	460948	12.55	12.31	12.20...12.42	772	12.25...12.36
			14.75	480771	15.72	15.49	15.31...15.67	567	15.40...15.59
			18.96	4997000	17.69	17.49	17.43...17.55	4959	17.43...17.56
			23.17	4997000	18.81	18.67	18.58...18.76	4552	18.59...18.75
			27.39	4997000	19.41	19.24	19.12...19.36	4513	19.15...19.33
			31.60	4997000	19.71	19.62	19.53...19.71	4463	19.52...19.73
			35.81	4997000	19.86	19.82	19.74...19.81	4383	19.71...19.94
			40.03	4997000	19.93	19.88	19.79...19.97	4552	19.77...20.00
			44.24	4997000	19.97	19.95	19.80...20.09	4388	19.83...20.06
			48.45	4997000	19.99	19.98	19.85...20.12	4543	19.86...20.10
			52.67	4997000	20.00	19.92	19.77...20.06	4348	19.80...20.04
			∞	4997000	20.00	19.93	19.82...20.04	4358	19.81...20.04

Table D.27: Expected end-to-end waiting time (simulation)

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