Fusing Structure by Kalman Filtering *

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Abstract

Deriving structure of the environment from motion using two successive images is error prone. However, if there are many measurements, they can be fused such that the reliability of the fused estimate is higher than for each of the noisy estimates. In a probabilistic framework, if the noise is assumed to be Gaussian, then reliability of each measurement is inversely proportional to the variance of its noise. Kalman filtering can be employed to combine noisy measurements such that the variance of the fused estimate is minimum. This paper describes a step by step understanding of the working of the Kalman filter as applied to a set of tasks, beginning from a very simple fusion, and ending in a more complex task of fusing structure obtained from known motion parameters.

1 Introduction

Deriving the structure of the environment from motion using two successive images is error prone [1]. Even small errors in the motion parameters or in the image measurements can introduce large errors in the the reconstructed 3 D coordinates of objects, represented as points in 3 space. However, if there are many such measurements of 3 D coordinates of the same point, then these measurements can be fused such that the reliability of this fused estimate will be higher than that of each individual measurement. It is assumed that the inverse of the variance is an estimate of the reliability of a measurement. Kalman filtering is a tool that can be used to fuse these measurements such that the variance of this combined estimate is optimal.

This project is the application of Kalman filtering to tasks that steadily increase in complexity. In the simplest task measurements of the distances to a static object from a fixed position are fused. The most complex task used structure obtained from motion and their associated covariance matrices. The main aim of each task is to determine the quantity being optimized in the application of the filter and the expression for the combined estimate in terms of all previous measurements.

2 Iterative, Linear, Optimal Estimation

The aim of this section is to introduce the Kalman filter by considering two simple tasks in which it is being applied.

2.1 Task 1 : Fixed process with constant corrupting noise

The task is to optimally combine a set of measurements, z_i, of a fixed distance from an observer to a point represented by a flag pole as shown in Fig 1. To measure this distance the person uses a digital laser range finder, which outputs the distance to the object it is pointed at. However, this measuring device is imperfect; its measurements being corrupted by an additive Gaussian noise whose variance is constant for all measurements.

The aim of this task is to linearly combine all the measurements to produce an optimal estimate. By optimal is meant that this combination has the minimum variance. Under this optimality condition it is shown in section 2.3 that the fused estimate has to be the mean of all the given measurements.

2.2 Task 2: Fixed process with changing corrupting noise

This task is similar to Task 1, but the difference is that each measurement is corrupted by a different additive, Gaussian noise process. However, the variance, r_i, of the noise associated with each measurement, z_i, is also output by the laser range finder.

The aim of this task, like Task 1, is to estimate the optimal, linear combination of all the measurements. The next step at this point will be to derive an expression for the value of this estimate.

2.3 Optimal, Linear Estimation

Given n measurements, z_i, $i = 1, ..., n$, of the same fixed underlying process x, each corrupted by zero-mean noise v_i whose variance is r_i, the optimal, linear estimate, $\hat{x}$, is one that is a linear combination of the measurements, which has the minimum variance. A detailed account of optimal, linear estimation can be found in [3].

Let us for simplicity consider the case when $n = 2$. The relation between the underlying process, the measurement and the noise in the measurement can be written as,

$$\begin{aligned} z_1 &= x + v_1 \\ z_2 &= x + v_2 \end{aligned} \tag{1}$$

The linear combination of these two measurements z_1 and z_2 will be

$$\hat{x} = k_1 z_1 + k_2 z_2 \tag{2}$$

If $\tilde{x}$ is the error in this estimate, $\hat{x}$, of the underlying process x, then

$$\tilde{x} = \hat{x} - x \tag{3}$$

If the estimate is unbiased, then k_1 and k_2 will be independent of x, and the mean of the estimation error will be zero, i.e.,

$$E[\tilde{x}] \;=\; E[\hat{x} - x] \;=\; 0 \tag{4}$$

Using Eqn. 1 and 2, this becomes,

$$E[(k_1(x + v_1) + k_2(x + v_2)) - x] = 0 \tag{5}$$

However, since the expectation of the noise in each measurement is zero, and $E[x] = x$, the above equation simplifies to give the relation,

$$k_2 = 1 - k_1 \tag{6}$$

Using this condition, the mean square estimation error can be written as

$$E[\tilde{x}^2] = k_1^2 r_1 + (1 - k_1)^2 r_2 \tag{7}$$

For maximum or minimum value of $E[\tilde{x}^2]$

$$\frac{dE[\tilde{x}^2]}{dk_1} = 2k_1 r_1 - 2(1 - k_1) r_2 = 0 \tag{8}$$

or

$$k_1 = \frac{r_2}{r_1 + r_2} \tag{9}$$

It can be shown that this value of k_1 gives the minimum mean square estimation error,

$$E[\tilde{x}^2] = \left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \tag{10}$$

Finally, the algorithm for determining the optimal estimate given this value of k_1 and k_2 is,

$$\hat{x} = \frac{\frac{1}{r_1}}{\frac{1}{r_1} + \frac{1}{r_2}} z_1 + \frac{\frac{1}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} z_2 \tag{11}$$

This algorithm shows that the optimal estimate is the normalized weighted sum of each measurement; the weights being the inverse of the variance of the noise associated with each measurement, and the normalizing term being the optimal mean square estimation error. If one of the variances approaches zero, then the other measurement will be almost discarded in determining the optimal estimate, which agrees with intuition. Also, if one of the variances is very large compared to the other, then it does not affect the value of the estimate much. Finally, if both the variances are the same, as in Task 1, then the optimal estimate becomes the mean of the two measurements.

An extension of this analysis can be done for the case when there are more than 2 measurements. The optimal, linear combination, $\hat{x}$, for the case when there are n measurements, z_i, $i = 1, ..., n$, each corrupted by zero-mean noise v_i having variances r_i, is given by

$$\hat{x} = \frac{\Sigma_{i=1}^{n} \frac{z_i}{r_i}}{\Sigma_{i=1}^{n} \frac{1}{r_i}} \tag{12}$$

and the variance, s, of the error in this optimal estimate is given by

$$s = \left[\Sigma_{i=1}^{n} \frac{1}{r_i} \right]^{-1} \tag{13}$$

Thus it can be seen that in Task 2, the optimal linear estimate is what can be defined as the *normalized* weighted mean.

2.4 Kalman Filtering: Iterative, optimal, linear estimation

So far we have described what the optimal, linear estimate is, given the input from Task 2. The Kalman filter is a recursive filter that determines this optimal, linear estimate iteratively. If $\hat{x}_k$ is the optimal,linear estimate of the first k measurements, and s_k is the variance of the error in this estimate, and if a new measurement, x_{k+1}, with a variance of corrupting noise, r_{k+1}, becomes available, then the optimal, linear estimate of all these $k+1$ measurements can be computed just in terms of the prior estimate, its associated variance and the new measurement as,

$$\hat{x}_{k+1} = \frac{\frac{1}{s_k}}{\frac{1}{s_k} + \frac{1}{r_{k+1}}} \hat{x}_k + \frac{\frac{1}{r_{k+1}}}{\frac{1}{s_k} + \frac{1}{r_{k+1}}} z_{k+1} \tag{14}$$

Kalman filtering obviates the need to store all past measurements. The required information is embodied in the prior estimate (plus the variance of this estimate). Hence, the Kalman filter can be termed a recursive, optimal, linear estimator.

3 Task 3: Changing Process

In this task, Task 3, as conveyed in Fig. 3, measurements are taken of the distance to the base of the flag pole as the person moves towards the flag pole, along a ray radiating from it. The exact distance moved each time is known. Thus, in this task, the underlying process being measured, i.e., the distance to the flag pole from the current location of the person, is changing in a known way ie., the observer moves exactly v meter each time.

The aim of this task is to understand the current, optimal, linear estimate, of all measurements obtained from the range finder. Just as in Task 2, the range finder also returns the variance of the noise associated with the measurement.

The variance of the optimal combined estimate of k measurements, s_k, is similar to the one obtained in Task 2, i.e.,

$$s_k = \left[\Sigma_{i=1}^{k} \frac{1}{r_i}\right]^{-1} \tag{15}$$

However, the optimal, linear estimate, x_k, can be shown to be

$$\hat{x_k} = \frac{\Sigma_{i=1}^{k} \frac{z_i-(k-i)v}{r_i}}{\Sigma_{i=1}^{k} \frac{1}{r_i}} \tag{16}$$

the normalized, *moved*, weighted mean of all measurements. By *moved* is meant that each past measurement is reckoned with respect to the current location of the person, or in other words, each past measurement is *moved* to the current reference frame. Since the exact nature of change in the distance to the object is known, the *ith* measurement,z_i, reckoned at the *kth* location is $z_i - (k-i)v$. The Kalman filter recursively determines this linear, optimal estimate.

It can be noticed in Table 3 that the estimation of the *standard deviation* of the error in the Kalman distance estimate decreases monotonically. However, all that can be expected of the actual error in the Kalman distance estimate, if it is assumed that the measurement errors are Gaussian, is that it be within one standard deviation of the actual underlying distance about 67% of the cases, and within three standard deviations close to 100% of the cases.

4 Task 4: Three Dimensional Process

So far, the process that has been measured has been the distance to an object, a 1 D quantity. If however, the goal is to determine the structure of the environment, represented as a set of 3 D coordinates of interesting points or landmarks in the world, then the underlying process measured for each landmark is a 3 D vector, $\mathbf{x}$.

In Task 4, it is assumed that the there is a special kind of laser range device that can be aligned to any coordinate frame. After alignment to a particular coordinate frame, when pointed at an object, it will return its 3 D coordinates, $\mathbf{p}$, and also a 3×3 covariance matrix, R, of the error in the vector $\mathbf{p}$. In this task where the covariance matrix is chosen to be diagonal, only the variance in each coordinate is displayed in Fig. 4.

The aim of this task, is to determine the optimal combination of the measurements, $\mathbf{p}_i$, $i = 1, ..., k$, of the same landmark, without moving the device. Each measurement has an associated covariance matrix R_i. If the covariance of the combined estimate is to be minimum, then this minimum covariance is given by

$$S_k = \left[\Sigma_{i=1}^{k} R_i^{-1}\right]^{-1} \tag{17}$$

and the optimal linear estimate is given by

$$\hat{\mathbf{x}}_k = S_k \Sigma_{i=1}^{k} R_i^{-1} \mathbf{p}_i \tag{18}$$

which is the normalized weighted mean of all the measurements.

Task 4 is similar to Task 2; the only difference being that the underlying process is 3 D and not 1 D. Table 4.1 depicts the 3 D coordinates obtained from the device, and the fused Kalman estimate at each iteration. Table 4.2 shows the standard deviation of the error in the measured 3 D coordinates and in the combined optimal estimate. Again, only the latter standard deviation decreases monotonically over time, which defines the shape of the probability distribution function of the error in the Kalman estimate. The actual error in the measurement as well as in the Kalman estimate, from the knowledge of the known ground truth, are presented in Table 4.3.

5 Task 5: Process Changing under Pure Rotation

The natural extension to Task 4 will be to understand the behavior of the Kalman filter when the laser device undergoes a motion after each measurement. In this task the motion considered is pure rotation, implying that the distance to the corner of the building from the measurement device remains the same. To make matters simple, the coordinate system of the device was rotated by π radian about a fixed axis of rotation, as shown in Fig 5. At each rotated position of the device, the landmark was sighted by the device and its 3 D coordinates from this position noted. The device also provided the variance in each coordinate.

It can be shown that the optimal estimate given k measurements is the ***normalized moved weighted mean.*** By ***moved*** is meant each past measurement reckoned in the current coordinate system of the device; this being possible from the knowledge of the exact rotation between two successive positions of the device. If the rotation can be expressed as a transition matrix ϕ_i then the covariance of the optimal estimate, the moved covariance, is given by

$$S_k = \left[\Sigma_{i=1}^{k}(R_i')^{-1}\right]^{-1} \tag{19}$$

where

$$R_i' = (\phi_{k-1}\ \phi_{k-2}\ \ldots\ \phi_i)\ R_i(\phi_{k-1}\ \phi_{k-2}\ \ldots\ \phi_i)^T \tag{20}$$

and the optimal linear estimate, the normalized weighted mean of the measurements, is

$$\hat{\mathbf{x}}_k = S_k \Sigma_{i=1}^{k}(R_i')^{-1}\mathbf{p}_i' \tag{21}$$

where $\mathbf{p}'_i$, the *ith* measurement reckoned in frame k, termed the moved measurement, is

$$\mathbf{p}'_i = (\phi_{k-1}\ \phi_{k-2}\ ...\ \phi_i)\mathbf{p}_i \tag{22}$$

Table 5.1 depicts the Kalman estimate over time. From this table it can be seen that the error in this Kalman estimate diminishes as more measurements are available. Note that though the covariance matrix for any measurement is diagonal, it ceases to be so when it is moved, i.e., when multiplied by the transition matrix. Table 5.2 shows only the diagonal entries of the covariance matrix. From Table 5.3 it can be clearly observed that the Kalman estimate approaches the ground truth.

6 Task 6: Process Changing under General Motion

Observing the performance of the Kalman filter, and understanding the physical meaning of the optimal estimate given a set of measurements, when the device underwent both rotations and translations, is the aim of this Task 6, as depicted in Fig. 6. One representation of the transition involved in the underlying process is the use of homogeneous transforms, which would imply that the state vector be of the form $(x, y, z, 1)^T$. Figuring out what the Kalman filter computes at each step in this case turned out to be difficult. So, instead, the state vector remained in the form $(x, y, z)^T$ and the the motion was represented by a rotation matrix C_i and a translation vector $\mathbf{b}_i$. Since the motion of the device was a rotation followed by the translation, the transition in the underlying process is captured by

$$\mathbf{x}_k = C_{k-1}\ \mathbf{x}_{k-1}\ -\ \mathbf{b}_{k-1} \tag{23}$$

Under this representation the covariance of the optimal estimate given k measurements is

$$S_k = \left[\Sigma_{i=1}^{k}(R'_i)^{-1}\right]^{-1} \tag{24}$$

where

$$R'_i = (C_{k-1}\ C_{k-2}\ ...\ C_i)\ R_i(C_{k-1}\ C_{k-2}\ ...\ C_i)^T \tag{25}$$

and the optimal linear estimate, the normalized weighted mean of the measurements, is

$$\hat{\mathbf{x}}_k = S_k \Sigma_{i=1}^{k} (R_i')^{-1} [\mathbf{p}_i' + \Sigma_{j=1}^{k-1} \mathbf{b}_i'] \tag{26}$$

where the *ith* measurement reckoned in frame k, $\mathbf{p}_i'$, termed as the moved measurement, is

$$\mathbf{p}_i' = (\phi_{k-1}\ \phi_{k-2}\ \ldots\ \phi_i)\ \mathbf{p}_i \tag{27}$$

and the *ith* translation reckoned in frame k, $\mathbf{b}_i'$, called the moved translation, is

$$\mathbf{b}_i' = (\phi_{k-1}\ \phi_{k-2}\ \ldots\ \phi_{i+1})\ \mathbf{b}_i \tag{28}$$

From Table 6.1 and 6.3 one can observe that the Kalman estimate approaches the ground truth as more and more measurements are made available. This is in spite of the same order of magnitude of noise continuing to corrupt the measurements. As in Task 5, the covariance matrix is no longer diagonal, but the diagonal entries give an idea of the error that can be expected in the optimal estimate. Table 6.2 depicts these diagonal entries.

7 Task 7: Fusing Structure obtained from images and known motion parameters

7.1 Structure from motion paradigm

The laser device used to obtain the 3 D coordinates of the landmark in Task 6 is replaced by a structure from motion algorithm. A camera mounted on a robot moving in the environment is used to obtain images. It is assumed that tracking a particular landmark over the images is possible, which means that establishing image correspondences is not an issue. These tracked image coordinates from two successive camera positions, along with the motion parameters relating these two camera coordinate systems are the input to the structure from motion algorithm. The output of the motion algorithm is the reconstructed 3 D coordinates of the landmark in the second coordinate frame. These 3 D coordinates, that are obtained as the robot moves in the environment, are input as measurements to the Kalman filter, along with an associated covariance matrix of the error in them. The covariance matrix is obtained using a first order error analysis, which requires the variance of the noise in the image coordinates.

As shown in Fig. 7., in the structure from motion paradigm, given the exact motion parameters and image coordinates, the reconstruction of the 3 D coordinates in either of the two camera coordinate systems should be straightforward. In the figure it is clear that the rays emanating from the focal points and passing through the image coordinates, intersect in 3 D space at the landmark. However, this is true if the motion parameters as well as the image coordinates are noise free. If there is noise in the image coordinates, then the rays will no longer intersect, as shown in Fig 8. The reconstruction algorithm that is used in this task locates the landmark along the right ray at the point where they are the closest to each other, i.e., where the perpendicular distance between them is minimum. Conforming with the notation used in [2], the relationship between and the translation component of the camera motion, $\mathbf{b}$, and these two rays of unit magnitude, $\mathbf{r}_r$, and $\mathbf{r}_l'$ (the ray from the first camera position rotated into the second camera's coordinate system) is given by

$$\beta \mathbf{r}_l' = \mathbf{b} + \alpha \mathbf{r}_r + \gamma \mathbf{r}_l' \times \mathbf{r}_r \tag{29}$$

Deriving α from the above equation yields

$$\alpha = \frac{(\mathbf{b} \times \mathbf{r}_l') \cdot (\mathbf{r}_l' \times \mathbf{r}_r)}{\mid \mathbf{r}_l' \times \mathbf{r}_r \mid^2} \tag{30}$$

Since the errors that are allowed in this task are only in the image coordinates, the 3 D coordinate of the landmark in the coordinate frame of the right camera can be written as,

$$\mathbf{p} = \alpha \mathbf{r}_r = \frac{(\mathbf{b} \times \mathbf{I}_l') \cdot (\mathbf{I}_l' \times \mathbf{I}_r)}{\mid \mathbf{I}_l' \times \mathbf{I}_r \mid^2} \mathbf{I}_r \tag{31}$$

in which,

$$\mathbf{I}_r = (I_r(x), I_r(y), f)^T \tag{32}$$

where $(I(x), I(y))$ denote image coordinates, and the suffix r refers to the second camera position and f, the focal length,

while $\mathbf{I}_l'$ is the vector

$$\mathbf{I}_l = (I_l(x), I_l(y), f)^T \tag{33}$$

after reckoned in the coordinate system of the second camera.

7.2 Estimating Covariance of the Error in Structure

The assumption is that the underlying errors are in the image coordinates. Errors due to the flow algorithms used to track the point from the first to second image, digitization errors and other errors in the camera can be clumped together as a single error in the image coordinates. Further it seems reasonable to assume that this noise is zero mean, and that the noise along the two image coordinate axes are independent. Also assuming that this noise process is Gaussian, and not a more complex distribution will make the error analysis easier.

Since from equation 31 it is true that $\mathbf{p}$ is a function of $I_l(x)$, $I_l(y)$, $I_r(x)$ and $I_r(y)$, then, $COV(error\ in\ \mathbf{p})$, the covariance matrix of the error in $\mathbf{p}$ can be written as,

$$\begin{aligned} COV(error\ in\ \mathbf{p}) &= \left[\frac{\delta \mathbf{p}}{\delta I_l(x)}\right]\left[\frac{\delta \mathbf{p}}{\delta I_l(x)}\right]^T \sigma^2_{I_l(x)} + \left[\frac{\delta \mathbf{p}}{\delta I_l(y)}\right]\left[\frac{\delta \mathbf{p}}{\delta I_l(y)}\right]^T \sigma^2_{I_l(y)} + \\ & \left[\frac{\delta \mathbf{p}}{\delta I_r(x)}\right]\left[\frac{\delta \mathbf{p}}{\delta I_r(x)}\right]^T \sigma^2_{I_r(x)} + \left[\frac{\delta \mathbf{p}}{\delta I_r(y)}\right]\left[\frac{\delta \mathbf{p}}{\delta I_r(y)}\right]^T \sigma^2_{I_r(y)} \end{aligned} \tag{34}$$

where σ_a denotes the variance in the scalar a.

7.3 Performance of the Kalman Filter

Tables 7.1 and 7.2 show the input structure obtained from motion and the Kalman estimate at each iteration. The motion considered in this task is a typical motion that a robot on a flat road encounters; translations on the ground plane, and rotations about the vertical axis (other kinds of rotations however are not considered in this problem). In this fairly general motion case, the error in the Kalman estimate can be seen to decrease over time.

An interesting feature that came up in these experiments, being discussed in detail in our next paper, is that fusing two successive measurements weighed by these covariance matrices, is in effect a triangulation using these narrow cylindrical probability distributions. Given two of these cylindrical distributions along with the means of these distributions, the measured 3 D coordinates, then the optimal fused estimate lies in the intersection region of these two distributions (if any). Hence the optimal estimate, or the weighted mean, can be seen at the tip of the triangle whose base is the translation vector, and the sides are these two cylindrical distributions. More experiments need to be done to understand more about this phenomenon.

8 Conclusion

The above tasks give a clear idea of what Kalman filtering optimizes, given the input measurements and the covariance of the noise in these measurements. Task 7, the most complex task considered here, can be extended to the case when the motion parameters are noisy. By removing the assumption that motion parameters need to be known accurately, the motion parameter oracle used in Task 7 can be replaced by a motion determination algorithm. In the extension to this work, Horn's relative orientation algorithm is being considered. An error analysis of this algorithm has been performed to determine the covariance of the error in the motion parameters. These covariances will model the noise in the transition of the underlying process that is being estimated by the filter. The understanding of the application of Kalman filtering to this realistic task will also be a good stopping point for this line of research.

References

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Experiment 1

Repeated measurements are taken to the base of the flag pole from the same position using a laser range finder. The output of the range finder also includes the std. dev. of the noise corrupting the measurement. In this experiment this std. dev. remains constant over measurements and it is shown that the combined optimal estimate is the mean of the measurements.

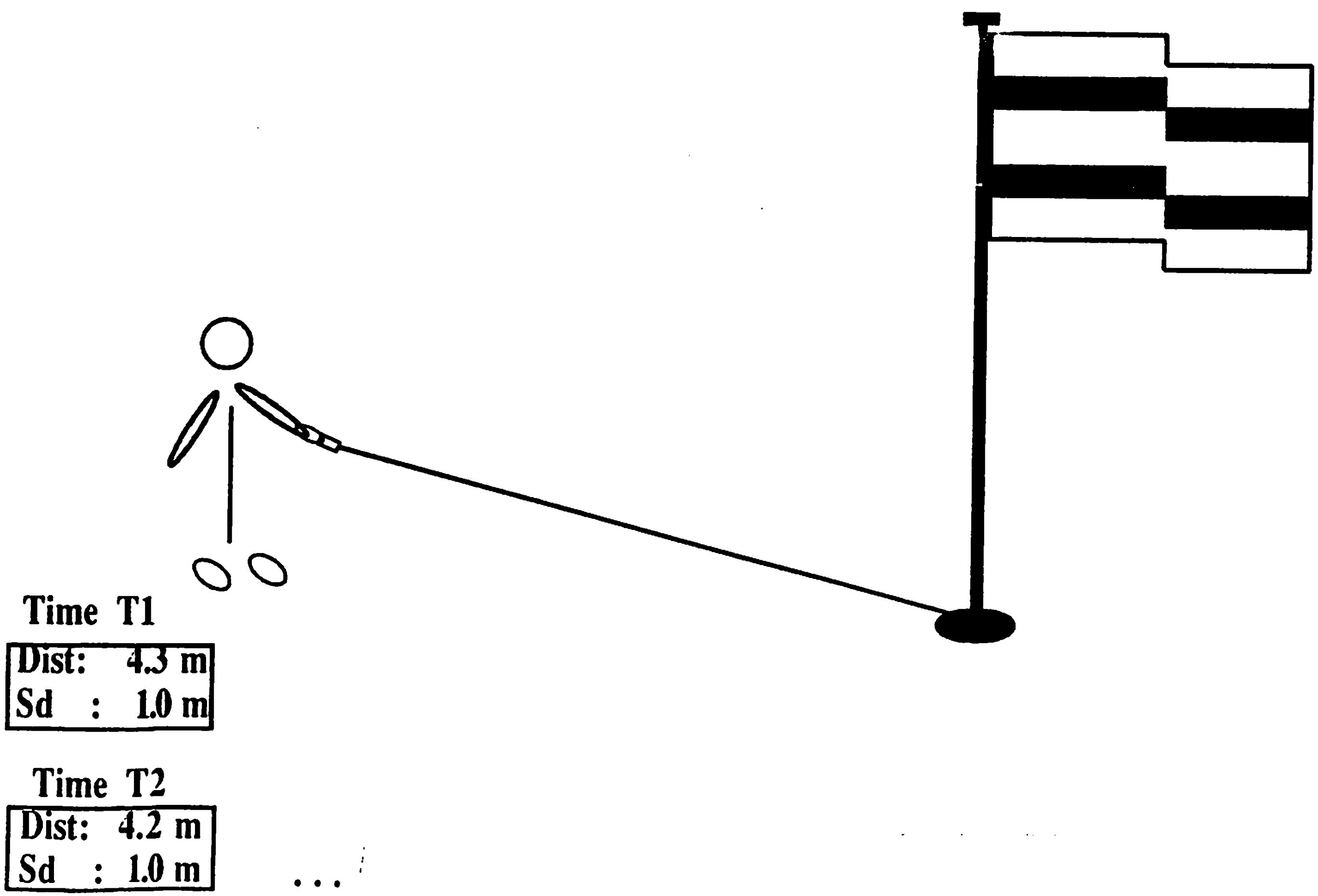
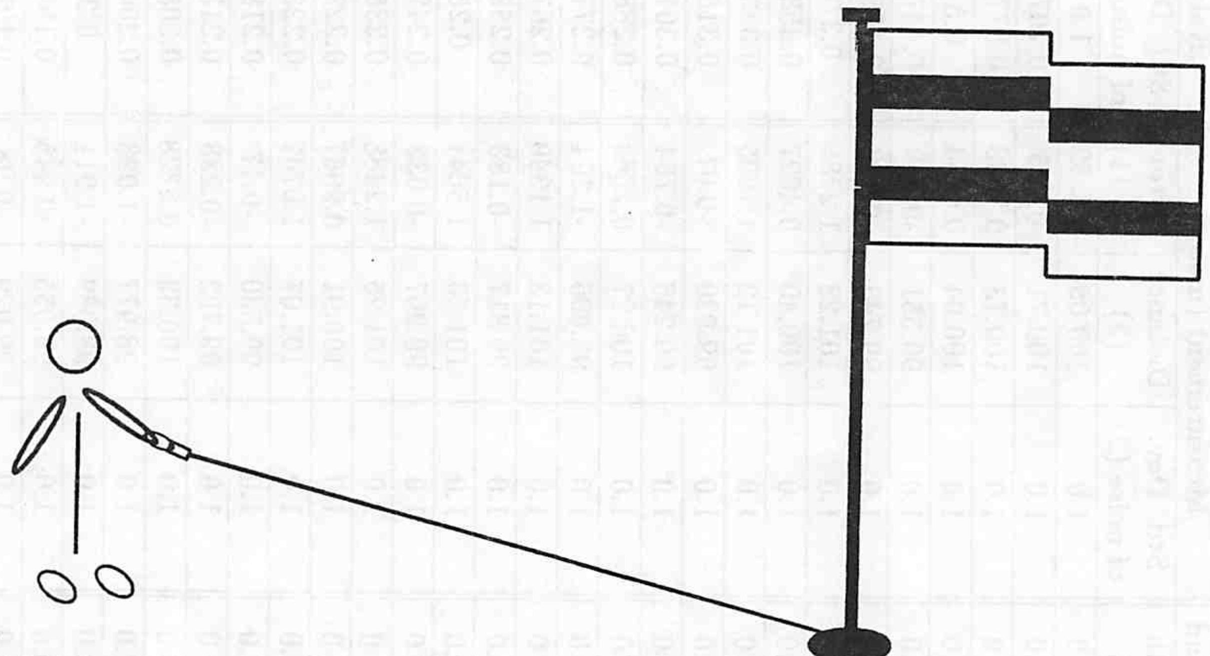


Table 1 (Task 1): Input measurements from device and Kalman filter output.

Iter	Ground	Measurement (meter)			Kalman output (meter)		
num ber	Truth (1)	Std. Dev. of noise (2)	Distance (3)	Error (3)-(1)	Std. Dev. of noise (5)	Distance (6)	Error (6)-(1)
1	100.0	1.0	100.06	0.0562	1.0	100.06	0.0563
2	100.0	1.0	100.71	0.706	0.7071	100.38	0.3811
3	100.0	1.0	100.73	0.7346	0.5774	100.5	0.4989
4	100.0	1.0	100.04	0.0354	0.5	100.38	0.3831
5	100.0	1.0	99.551	-0.449	0.4472	100.22	0.2166
6	100.0	1.0	99.743	-0.257	0.4082	100.14	0.1378
7	100.0	1.0	101.28	1.2786	0.378	100.30	0.3007
8	100.0	1.0	100.40	0.4027	0.3536	100.31	0.3135
9	100.0	1.0	101.12	1.1205	0.3333	100.40	0.4031
10	100.0	1.0	99.930	-0.07	0.3162	100.36	0.3559
11	100.0	1.0	99.246	-0.754	0.3015	100.25	0.255
12	100.0	1.0	100.27	0.2664	0.2887	100.26	0.2559
13	100.0	1.0	98.696	-1.304	0.2774	100.14	0.1359
14	100.0	1.0	101.13	1.1340	0.2673	100.21	0.2072
15	100.0	1.0	99.817	-0.183	0.2582	100.18	0.1812
16	100.0	1.0	101.56	1.5641	0.25	100.27	0.2676
17	100.0	1.0	99.967	-0.033	0.2425	100.25	0.25
18	100.0	1.0	101.25	1.2488	0.2357	100.31	0.3055
19	100.0	1.0	100.91	0.9067	0.2294	100.34	0.3371
20	100.0	1.0	101.07	1.0747	0.2236	100.37	0.374
21	100.0	1.0	99.530	-0.47	0.2182	100.33	0.3338
22	100.0	1.0	99.762	-0.238	0.2132	100.31	0.3078
23	100.0	1.0	100.33	0.3328	0.2085	100.31	0.3089
24	100.0	1.0	98.977	-1.023	0.2041	100.25	0.2534
25	100.0	1.0	98.089	-1.911	0.2	100.17	0.1669
26	100.0	1.0	99.055	-0.945	0.1961	100.12	0.1241
27	100.0	1.0	99.020	-0.98	0.1925	100.08	0.0832
28	100.0	1.0	101.13	1.1343	0.189	100.12	0.1207
29	100.0	1.0	98.506	-1.494	0.1857	100.07	0.0650
30	100.0	1.0	100.71	0.7087	0.1826	100.09	0.0865

Experiment 2

Measurements are taken to the base of the flag pole just as in expt. 1, using a laser range finder. But in this experiment, each measurement is corrputed by a different noise process, and hence the std. dev. of the noise in each measurement is not the same.
It is shown that the optimal combined estimate is the normalized weighted mean of the measurements, where the weight is the inverse of the corresponding variance.



Time T1

Dist: 4.3 m
Std dev: 0.9m

Time T2

Dist: 4.1 m
Std Dev: 0.6m

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Table 2 (Task 2): The table shows the input measurements from the device and the output of the Kalman filter. Note that the error in the output of the Kalman filter is always within 3 standard deviations in the noise, and within one standard deviation 70% of the time.

Iter	Ground	Measurement (meter)			Kalman output (meter)		
num ber	Truth (1)	Std. Dev. of noise (2)	Distance (3)	Error (3)-(1)	Std. Dev. of noise (5)	Distance (6)	Error (6)-(1)
1	40.0	1.6247	39.602	-0.398	1.6247	39.602	-0.398
2	40.0	1.7141	39.53	-0.470	1.1792	39.568	-0.432
3	40.0	1.1132	35.984	-4.016	0.8095	37.673	-2.327
4	40.0	1.4612	42.948	2.9480	0.7081	38.911	-1.089
5	40.0	1.9499	40.975	0.9747	0.6655	39.152	-0.848
6	40.0	1.4211	41.075	1.0751	0.6027	39.498	-0.502
7	40.0	1.6206	39.092	-0.908	0.5649	39.448	-0.552
8	40.0	1.4744	46.134	6.1343	0.5275	40.304	0.3043
9	40.0	1.9170	36.255	-3.745	0.5086	40.019	0.0192
10	40.0	1.7965	39.120	-0.88	0.4894	39.953	-0.047
11	40.0	1.8742	42.791	2.7906	0.4735	40.134	0.1337
12	40.0	1.9181	41.865	1.8653	0.4597	40.233	0.2332
13	40.0	1.8545	40.477	0.4768	0.4462	40.247	0.2473
14	40.0	1.1878	38.154	-1.846	0.4177	39.988	-0.012
15	40.0	1.9327	39.546	-0.454	0.4083	39.969	-0.031
16	40.0	1.2718	33.262	-6.738	0.3887	39.342	-0.658
17	40.0	1.3776	37.014	-2.986	0.3741	39.170	-0.83
18	40.0	1.7893	39.385	-0.615	0.3662	39.179	-0.821
19	40.0	1.4457	42.756	2.7565	0.355	39.395	-0.605
20	40.0	1.3158	43.336	3.336	0.3427	39.662	-0.338

Experiment 3

Distance to the foot of the flag pole is obtained from the range finder from different positions along a line radiating from the base of the pole. The distance between each adjacent measurement positions, depicted schematically by a person or by blobs, is exactly 1m.

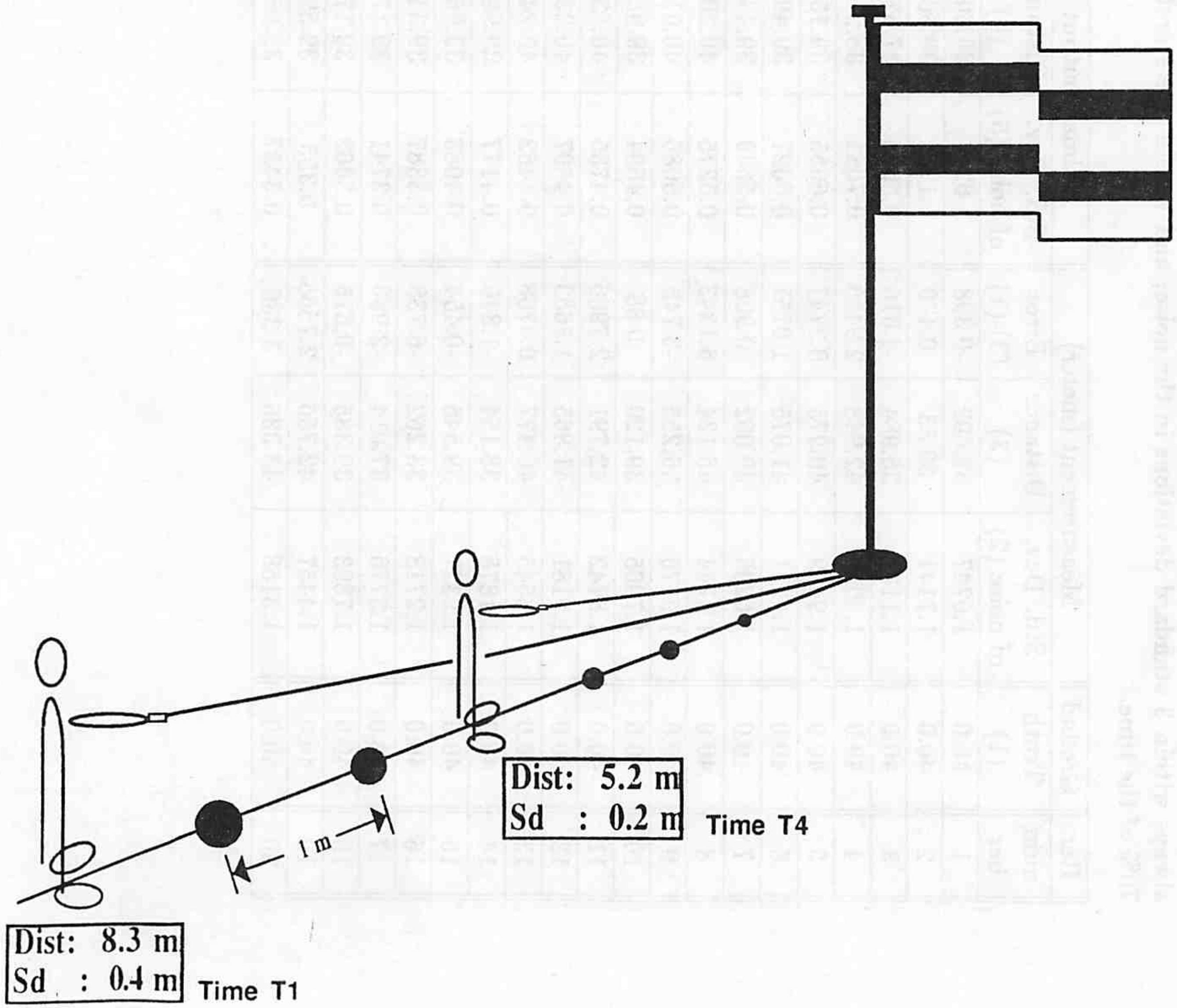
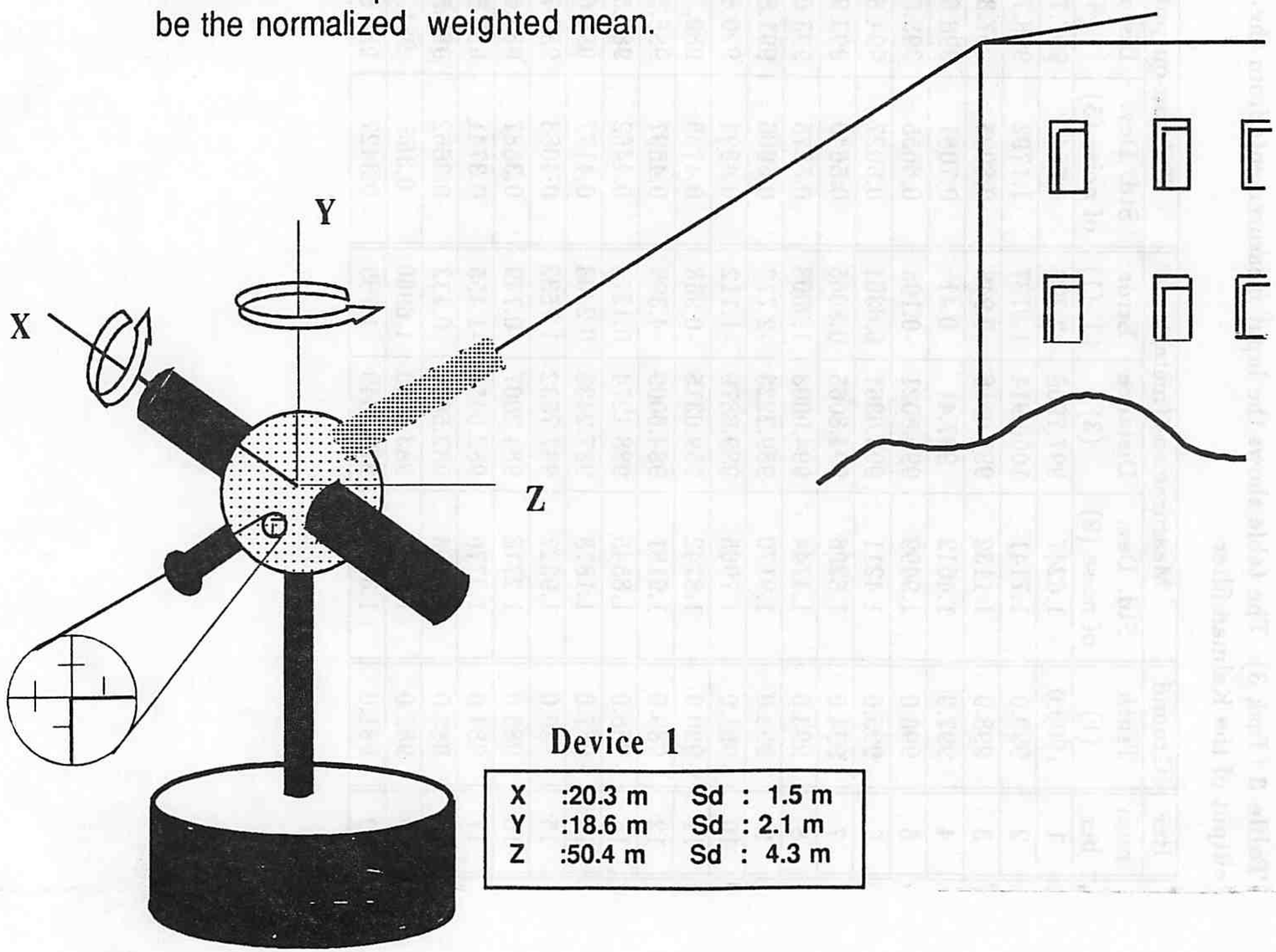


Table 3 (Task 3): The table shows the input measurements from the device and the output of the Kalman filter

Iter number	Ground Truth (1)	Measurement (meter) Std. Dev. of noise (2)	Measurement (meter) Distance (3)	Measurement (meter) Error (3)-(1)	Kalman output (meter) Std. Dev. of noise (5)	Kalman output (meter) Distance (6)	Kalman output (meter) Error (6)-(1)
1	1000.0	1.6247	997.7908	-2.209	1.6247	997.7908	-2.209
2	999.0	1.7141	1000.914	1.9137	1.1792	998.7419	-0.258
3	998.0	1.1132	997.0516	-0.948	0.8095	997.3769	-0.623
4	997.0	1.4612	997.41	0.41	0.7081	996.6195	-0.381
5	996.0	1.9499	995.8021	-0.198	0.6655	995.6408	-0.359
6	995.0	1.4211	995.6361	0.6361	0.6027	994.8198	-0.180
7	994.0	1.6206	994.8065	0.8065	0.5649	993.9397	-0.060
8	993.0	1.4744	994.0008	1.0008	0.5275	993.0755	0.0755
9	992.0	1.9170	989.2223	-2.778	0.5086	991.8747	-0.125
10	991.0	1.7965	989.8879	-1.112	0.4894	990.8014	-0.199
11	990.0	1.8742	989.0315	-0.968	0.4735	989.7523	-0.248
12	989.0	1.9181	984.6009	-4.399	0.4597	988.5138	-0.486
13	988.0	1.8545	988.1274	0.1274	0.4462	987.5494	-0.451
14	987.0	1.1878	987.2433	0.2433	0.4177	986.6352	-0.365
15	986.0	1.9327	987.7532	1.7532	0.4083	985.7297	-0.270
16	985.0	1.2718	984.2007	-0.799	0.3887	984.6803	-0.32
17	984.0	1.3776	982.5454	-1.455	0.3741	983.5966	-0.403
18	983.0	1.7893	982.5667	-0.433	0.3662	982.5953	-0.405
19	982.0	1.4457	983.6960	1.6960	0.355	981.722	-0.278
20	981.0	1.3158	979.2349	-1.765	0.3427	980.6211	-0.379

Experiment 4

A device schematically shown below, is used to measure the (x,y,z) coordinates of a point in space w.r.t the fixed coordinate axes. The device also ouputs the std. dev. of the noise corrupting these measurements. In this expt. a set of measurements of the 3 D coordinates is obtained from a set of these measuring devices. The optimal fusion of this set of measurements will be the normalized weighted mean.



Device 1

X	:20.3 m	Sd :	1.5 m
Y	:18.6 m	Sd :	2.1 m
Z	:50.4 m	Sd :	4.3 m

Table 4.1(Task 4): Coordinates of the landmark from the measuring device are inputs to the Kalman filter. The device also gives the variance of the noise corrupting the measurements. At the *ith* iteration the previous Kalman estimate is combined with the *ith* coordinate obtained from the device.

Iter	X-Coordinate (meter)			Y-Coordinate (meter)			Z-Coordinate (meter)		
num ber	Ground truth	From device	Kalman output	Ground truth	From device	Kalman output	Ground truth	From device	Kalman output
1	40.0	37.647	37.647	40.0	37.603	37.603	60.0	57.324	57.324
2	40.0	32.96	37.302	40.0	35.857	36.429	60.0	60.794	59.959
3	40.0	41.320	37.902	40.0	42.901	38.167	60.0	61.375	60.257
4	40.0	38.517	38.065	40.0	34.232	37.413	60.0	55.156	59.913
5	40.0	41.786	38.493	40.0	38.922	38.151	60.0	60.784	60.063
6	40.0	38.458	38.491	40.0	40.763	38.335	60.0	61.462	60.138
7	40.0	36.186	38.414	40.0	38.749	38.359	60.0	59.749	60.089
8	40.0	43.061	38.836	40.0	38.693	38.385	60.0	64.153	60.361
9	40.0	40.939	39.604	40.0	38.905	38.464	60.0	57.313	60.238
10	40.0	38.731	39.464	40.0	37.429	38.414	60.0	62.517	60.447
11	40.0	36.749	39.365	40.0	41.495	38.587	60.0	60.350	60.403
12	40.0	43.876	39.465	40.0	37.502	38.51	60.0	62.606	60.716
13	40.0	44.304	39.548	40.0	39.939	38.828	60.0	58.076	60.603
14	40.0	39.823	39.556	40.0	39.002	38.843	60.0	63.151	60.676
15	40.0	42.034	39.64	40.0	41.85	38.988	60.0	57.677	60.612
16	40.0	35.079	39.56	40.0	38.046	38.946	60.0	58.682	60.563
17	40.0	40.22	39.626	40.0	37.124	38.899	60.0	64.643	60.637
18	40.0	43.116	39.696	40.0	41.666	39.376	60.0	64.443	60.706
19	40.0	39.556	39.685	40.0	41.926	39.464	60.0	63.750	60.833
20	40.0	43.568	39.740	40.0	42.814	39.538	60.0	59.729	60.8

Table 4.2: Standard deviation in the measurement from the device and Kalman estimate.

Iter	X: Std Dev. (m)		Y: Std Dev. (m)		Z: Std Dev. (m)	
num ber	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	1.2958	1.2958	4.1181	4.1181	3.0265	3.0265
2	4.6007	1.2473	2.8749	2.3573	1.7039	1.4848
3	2.9772	1.1504	3.8901	2.0160	2.8753	1.3193
4	1.9151	0.9861	4.1418	1.8127	4.908	1.2740
5	2.7346	0.9277	1.8520	1.2955	2.8	1.1596
6	3.5011	0.8967	4.7124	1.2491	4.8431	1.1278
7	4.8028	0.8815	4.9868	1.2117	2.9589	1.0538
8	2.7872	0.8405	4.1826	1.1638	3.9325	1.0179
9	1.1081	0.6696	2.7455	1.0715	4.9608	0.9971
10	1.5332	0.6137	4.727	1.0450	3.143	0.9504
11	3.1545	0.6024	4.2842	1.0152	1.0509	0.7049
12	4.0182	0.5957	3.6773	0.9786	1.7316	0.6529
13	4.4978	0.5905	1.8295	0.8629	3.083	0.6387
14	3.3965	0.5818	2.7881	0.8244	3.7098	0.6295
15	3.1098	0.5719	3.6672	0.8043	4.2435	0.6226
16	4.2739	0.5668	3.7182	0.7861	3.8558	0.6147
17	1.6970	0.5376	4.8283	0.7759	4.5108	0.6090
18	3.7426	0.5322	1.6998	0.7058	4.4937	0.6035
19	1.7963	0.5103	3.7195	0.6935	2.8904	0.5908
20	4.2448	0.5066	4.6242	0.6858	3.3542	0.5818

Table 4.3: The errors, computed as a difference of the values from Table 4.1 and the ground truth. Note how the Kalman estimate error decreases steadily.

Iter	X error (m)		Y error (m)		Z error (m)		Distance error (m)	
num ber	From device	Kalman output	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	-2.353	-2.353	-2.397	-2.397	-2.676	-2.676	-4.249	-4.249
2	-7.040	-2.698	-4.143	-3.571	0.7937	-0.041	-4.565	-3.004
3	1.3204	-2.098	2.9014	-1.833	1.3749	0.2568	3.0644	-1.69
4	-1.483	-1.935	-5.768	-2.587	-4.844	-0.087	-6.98	-2.223
5	1.7863	-1.507	-1.078	-1.849	0.784	0.0625	0.9388	-1.562
6	-1.542	-1.509	0.7634	-1.665	1.4624	0.1384	0.7141	-1.420
7	-3.814	-1.586	-1.251	-1.641	-0.251	0.0890	-2.582	-1.482
8	3.0612	-1.164	-1.307	-1.615	4.1533	0.3613	3.9499	-1.067
9	0.9392	-0.396	-1.095	-1.536	-2.687	0.2381	-1.999	-0.751
10	-1.269	-0.536	-2.571	-1.586	2.517	0.4465	0.0572	-0.689
11	-3.251	-0.635	1.4955	-1.413	0.3504	0.4033	-0.52	-0.687
12	3.8757	-0.535	-2.498	-1.490	2.606	0.7164	2.6904	-0.444
13	4.3039	-0.452	-0.061	-1.172	-1.924	0.6031	0.7891	-0.338
14	-0.177	-0.444	-0.998	-1.157	3.1515	0.6764	1.7705	-0.273
15	2.0342	-0.360	1.8497	-1.012	-2.323	0.6119	0.2720	-0.212
16	-4.921	-0.440	-1.954	-1.054	-1.318	0.5628	-4.221	-0.306
17	0.22	-0.374	-2.876	-1.101	4.6434	0.6372	2.2410	-0.242
18	3.1156	-0.304	1.666	-0.624	4.4429	0.7059	5.5600	0.0695
19	-0.444	-0.315	1.926	-0.536	3.7503	0.8331	3.4829	0.1999
20	3.5676	-0.26	2.8140	-0.462	-0.271	0.7998	2.9704	0.2372

EXPERIMENT 5

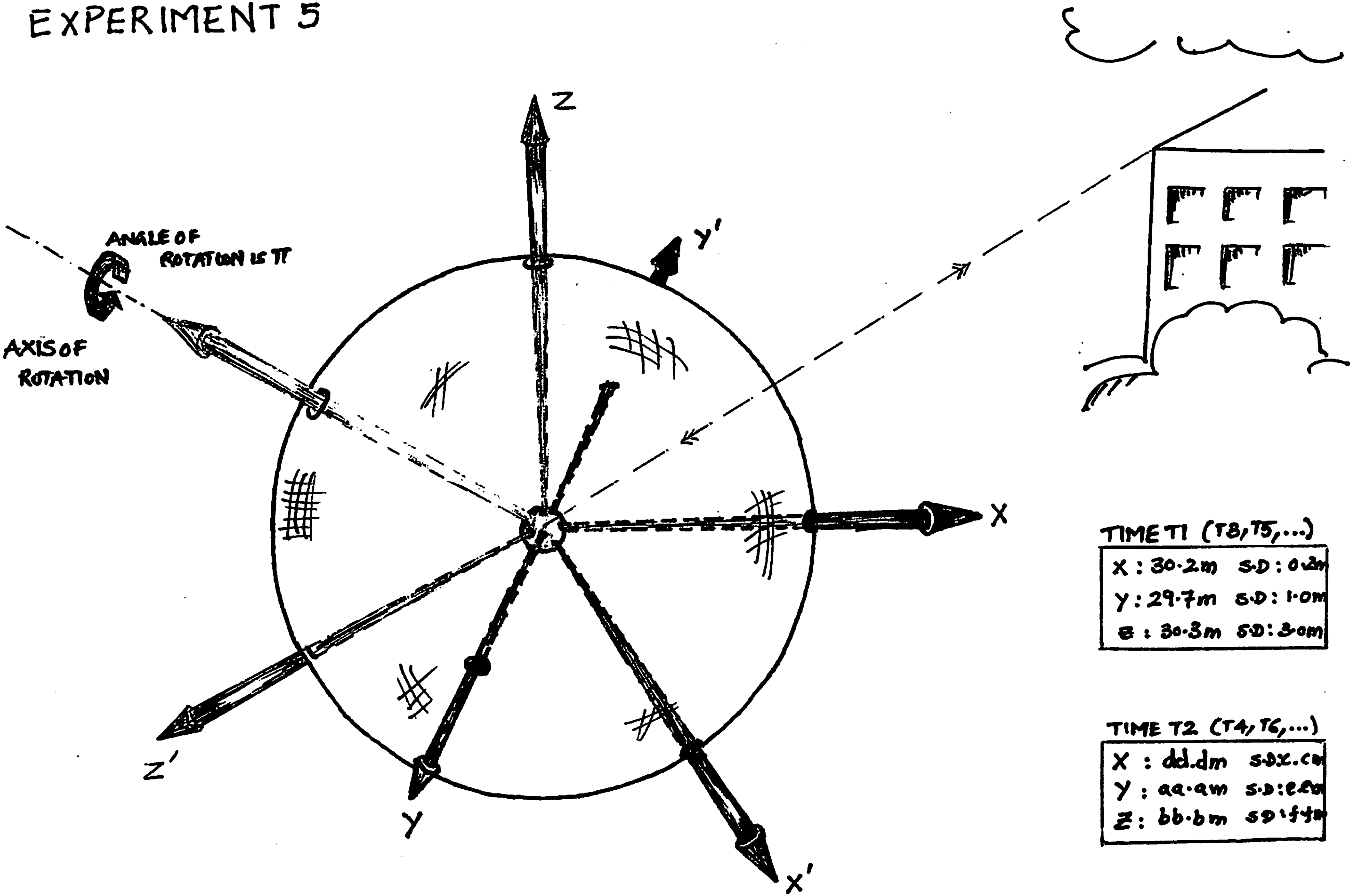


Table 5.1(Task 5): Structure is obtained from measuring device which also returns the variance of the corrupting noise. The measuring device rotates in the environment and an oracle is assumed to make available the values of these rotations on demand. Table shows the measured estimates and the Kalman filter output. Note Kalman estimate approach ground truth.

Iter	X-Coordinate (meter)			Y-Coordinate (meter)			Z-Coordinate (meter)		
num ber	Ground truth	From motion	Kalman output	Ground truth	From device	Kalman output	Ground truth	From device	Kalman output
1	40.0	40.811	40.811	40.0	35.409	35.409	60.0	56.529	56.529
2	34.286	31.981	30.844	71.429	60.777	65.408	-22.86	-24.5	-23.17
3	40.0	42.032	39.403	40.000	36.98	36.218	60.000	58.924	58.088
4	34.286	28.184	30.371	71.429	71.12	69.506	-22.86	-21.81	-22.86
5	40.0	38.748	39.842	40.000	30.163	35.320	60.000	61.142	59.06
6	34.286	26.845	29.802	71.429	69.077	69.566	-22.86	-23.04	-24.05
7	40.0	40.828	40.087	40.000	40.924	35.58	60.000	58.734	58.908
8	34.286	33.761	30.372	71.429	72.772	70.791	-22.86	-20.10	-23.36
9	40.0	38.796	40.834	40.000	41.357	36.57	60.000	53.958	58.700
10	34.286	32.582	30.652	71.429	74.174	71.168	-22.86	-27.00	-23.02
11	40.0	33.400	40.854	40.000	42.212	37.273	60.000	59.963	59.049
12	34.286	36.428	31.792	71.429	77.686	71.205	-22.86	-23.56	-22.94
13	40.0	37.302	40.583	40.000	41.893	38.166	60.000	63.083	59.439
14	34.286	34.837	32.563	71.429	64.955	70.952	-22.86	-21.07	-22.9
15	40.0	38.580	39.991	40.000	39.994	38.435	60.000	62.355	59.483
16	34.286	35.761	33.114	71.429	69.246	70.726	-22.86	-19.40	-22.79
17	40.0	40.088	39.928	40.000	41.217	39.194	60.000	55.097	59.033
18	34.286	29.304	33.200	71.429	75.482	70.99	-22.86	-22.46	-22.39
19	40.0	41.241	40.314	40.000	39.726	39.179	60.000	56.372	59.044
20	34.286	33.632	33.197	71.429	64.652	70.902	-22.86	-25.93	-22.4

Table 5.2: The standard deviation in the measurements for each coordinate and the Kalman output.

Iter	X: Std Dev. (m)		Y: Std Dev. (m)		Z: Std Dev. (m)	
num ber	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	4.0132	4.0132	3.2222	3.2222	2.6894	2.6894
2	3.1946	2.3002	4.3978	2.8133	3.4524	2.2148
3	2.6844	1.9503	2.2525	1.6129	1.8042	1.3816
4	4.1668	1.5133	3.7038	1.6649	4.9929	1.4075
5	4.3985	1.6169	3.9485	1.4080	1.9193	1.0889
6	4.5208	1.3415	4.4663	1.4409	4.2265	1.1619
7	4.2947	1.4327	4.5552	1.2877	2.4135	0.967
8	4.1849	1.2212	1.91	1.1035	4.3986	1.0386
9	4.1241	1.1506	4.8796	1.1817	3.4483	0.8988
10	4.7846	1.1446	2.3668	0.9698	4.8865	0.9776
11	4.6403	1.0348	3.6026	1.0889	4.0981	0.8592
12	3.2858	1.0383	4.5590	0.9258	3.0616	0.9047
13	3.7823	0.9662	3.1827	0.9824	3.7965	0.8095
14	3.1166	0.9435	4.7572	0.8812	3.447	0.8487
15	1.9873	0.8491	4.9641	0.9231	3.6902	0.771
16	2.7846	0.8647	4.2913	0.7991	2.8677	0.7905
17	4.7906	0.8143	2.0697	0.8085	2.8670	0.7222
18	4.3554	0.7947	3.2393	0.7596	1.8512	0.7005
19	2.4957	0.7512	2.9272	0.7572	4.7691	0.6743
20	3.5049	0.7481	4.7953	0.7215	4.3102	0.6793

Table 5.3: Depicts the error in the input measurements and the Kalman estimate. These values are computed as the difference of the ground truth and the values in table 5.1

Iter	X error (m)		Y error (m)		Z error (m)	
num ber	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	0.811	0.811	-4.591	-4.591	-3.471	-3.471
2	-2.304	-3.442	-10.65	-6.020	-1.639	-0.314
3	2.0323	-0.597	-3.020	-3.782	-1.076	-1.912
4	-6.102	-3.914	-0.309	-1.923	1.0492	-0.005
5	-1.252	-0.158	-9.837	-4.68	1.1419	-0.940
6	-7.440	-4.483	-2.352	-1.863	-0.181	-1.193
7	0.8275	0.0872	0.9236	-4.420	-1.266	-1.092
8	-0.524	-3.914	1.3437	-0.637	2.756	-0.507
9	-1.204	0.8337	1.3570	-3.430	-6.042	-1.3
10	-1.703	-3.633	2.7450	-0.260	-4.146	-0.159
11	-6.6	0.8536	2.2122	-2.727	-0.037	-0.951
12	2.142	-2.494	6.257	-0.224	-0.702	-0.078
13	-2.698	0.583	1.8929	-1.834	3.0829	-0.561
14	0.5515	-1.723	-6.474	-0.477	1.7901	-0.038
15	-1.42	-0.009	-0.006	-1.565	2.3546	-0.517
16	1.4756	-1.172	-2.182	-0.703	3.4527	0.0673
17	0.0883	-0.072	1.2170	-0.806	-4.903	-0.967
18	-4.982	-1.085	4.0539	-0.439	0.3932	0.4700
19	1.2406	0.3144	-0.274	-0.821	-3.628	-0.956
20	-0.653	-1.089	-6.776	-0.527	-3.072	0.4588

EXPERIMENT 6

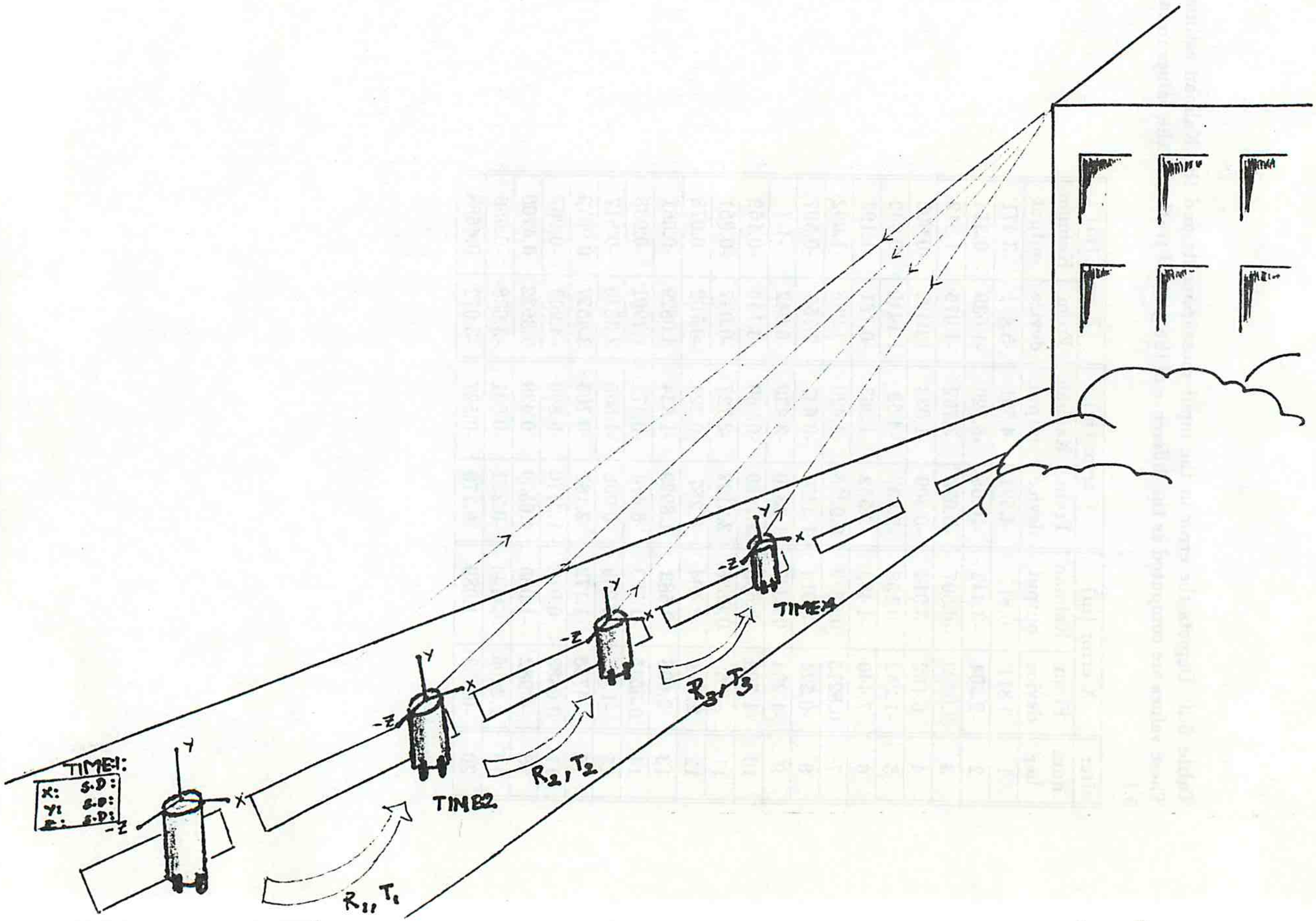


Table 6.1(Task 6): Structure measurements are obtained from the device. The table shows the Kalman estimate at each iteration.

Iter	X-Coordinate (meter)			Y-Coordinate (meter)			Z-Coordinate (meter)		
num ber	Ground truth	From device	Kalman output	Ground truth	From device	Kalman output	Ground truth	From device	Kalman output
1	40.0	40.117	40.117	40.0	35.636	35.636	60.0	60.729	60.729
2	41.041	41.386	41.298	40.0	45.793	39.288	58.293	61.345	60.180
3	40.017	39.690	40.015	40.0	37.993	39.091	58.000	61.808	60.995
4	41.024	38.123	40.531	40.0	41.748	39.707	56.293	58.591	59.125
5	40.035	40.986	40.269	40.0	41.492	40.441	56.000	59.073	58.856
6	41.006	39.261	41.132	40.0	38.151	40.207	54.293	53.126	55.163
7	40.052	37.655	39.876	40.0	36.059	39.394	54.000	52.890	54.651
8	40.989	39.819	40.758	40.0	42.943	39.607	52.293	54.640	53.061
9	40.07	44.563	40.543	40.0	42.238	39.843	52.001	49.539	52.374
10	40.971	43.495	41.602	40.0	38.722	39.403	50.293	47.634	50.483
11	40.087	38.694	40.584	40.0	41.072	39.495	50.001	49.472	50.110
12	40.954	39.057	41.135	40.0	41.306	39.801	48.294	45.254	48.173
13	40.105	37.626	39.525	40.0	34.859	39.685	48.001	50.432	48.008
14	40.936	43.843	40.457	40.0	39.237	39.663	46.294	43.781	46.123
15	40.122	40.884	40.095	40.0	41.985	39.718	46.001	50.392	45.997
16	40.919	43.538	40.996	40.0	39.668	39.717	44.294	37.341	44.051
17	40.14	41.267	40.261	40.0	44.463	39.872	44.001	45.680	43.963
18	40.901	44.303	41.189	40.0	38.693	39.821	42.294	43.861	42.387
19	40.157	41.556	40.470	40.0	43.963	39.974	42.001	41.53	41.965
20	40.884	39.995	41.178	40.0	39.258	39.932	40.294	42.858	40.915
21	40.175	42.098	40.484	40.0	40.082	40.011	40.002	44.929	40.699
22	40.867	43.175	41.242	40.0	40.452	40.02	38.294	38.113	38.954
23	40.192	41.517	40.572	40.0	40.281	40.022	38.002	43.002	38.798
24	40.849	40.385	41.221	40.0	40.537	40.028	36.294	35.157	37.019

Table 6.2: Standard deviation in the measurement from the device and Kalman estimate.

Iter num ber	X: Std Dev. (m)		Y: Std Dev. (m)		Z: Std Dev. (m)	
	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	3.5667	3.5667	2.1003	2.1003	3.8555	3.8555
2	2.9922	2.2924	2.8029	1.6808	3.8642	2.7293
3	2.7738	1.7671	3.9647	1.5475	2.3444	1.7784
4	3.7212	1.5962	2.8151	1.3561	3.2163	1.5564
5	1.5336	1.1059	1.6231	1.0407	3.9565	1.4483
6	3.9744	1.0655	3.0868	0.9861	1.4689	1.0313
7	2.9743	1.003	1.9966	0.8842	2.9251	0.9727
8	3.7642	0.9692	3.5001	0.8572	3.6086	0.9391
9	2.2902	0.8925	2.7283	0.8178	2.5171	0.8799
10	3.1546	0.8589	1.0175	0.6374	3.5376	0.8538
11	3.2725	0.8307	2.6340	0.6196	2.2634	0.7989
12	2.1274	0.7738	1.3741	0.5648	2.9075	0.7703
13	1.2212	0.6536	3.6477	0.5582	3.3745	0.7511
14	3.7781	0.6441	2.4145	0.5438	2.6500	0.7225
15	0.8539	0.5142	3.4756	0.5373	3.6422	0.7088
16	2.5148	0.5039	3.8129	0.5320	3.7419	0.6963
17	2.5181	0.494	2.8997	0.5233	2.0249	0.6585
18	2.1433	0.4814	2.4765	0.512	2.2115	0.6311
19	3.0416	0.4755	2.6145	0.5024	1.1323	0.5513
20	3.5167	0.4712	2.0096	0.4874	0.9434	0.476
21	3.7117	0.4674	0.4606	0.3348	3.651	0.4720
22	2.8154	0.4611	2.3736	0.3315	2.3873	0.4630
23	3.6116	0.4574	3.3630	0.3299	2.6337	0.4560
24	2.8619	0.4517	3.0312	0.328	2.4406	0.4483

Table 6.3: The errors, computed as a difference of the values from Table 6.1 and the ground truth. Note how the Kalman estimate error decreases steadily.

Iter	X error (m)		Y error (m)		Z error (m)		Distance error (m)	
num ber	From device	Kalman output	From device	Kalman output	From device	Kalman output	From device	Kalman output
1	0.1165	0.1165	-4.364	-4.364	0.7294	0.7294	-1.423	-1.423
2	0.3453	0.2573	5.7926	-0.712	3.0521	1.8873	5.2769	1.1438
3	-0.327	-0.002	-2.007	-0.909	3.8077	2.9946	1.6710	1.7354
4	-2.900	-0.493	1.7481	-0.293	2.2979	2.8321	1.0965	1.6231
5	0.9511	0.2345	1.4916	0.4408	3.0725	2.8559	3.3953	2.3658
6	-1.745	0.1258	-1.849	0.2071	-1.167	0.8699	-2.641	0.7702
7	-2.397	-0.176	-3.941	-0.606	-1.110	0.6509	-3.965	0.0545
8	-1.170	-0.231	2.9433	-0.393	2.347	0.7683	2.5407	0.1981
9	4.4935	0.4734	2.2376	-0.157	-2.461	0.3737	2.0183	0.4192
10	2.5241	0.6305	-1.278	-0.597	-2.659	0.1898	-0.976	0.1559
11	-1.393	0.4966	1.0720	-0.505	-0.529	0.1094	-0.501	0.072
12	-1.897	0.1809	1.3055	-0.199	-3.04	-0.121	-2.236	-0.085
13	-2.479	-0.579	-5.141	-0.315	2.4315	0.0069	-2.313	-0.477
14	2.9067	-0.479	-0.763	-0.337	-2.512	-0.170	-0.274	-0.556
15	0.762	-0.027	1.9855	-0.282	4.3906	-0.004	4.3105	-0.171
16	2.6192	0.0772	-0.332	-0.283	-6.953	-0.242	-2.624	-0.261
17	1.1277	0.1219	4.4626	-0.128	1.6791	-0.038	4.1935	-0.026
18	3.4012	0.2878	-1.307	-0.179	1.5671	0.0932	2.2275	0.1211
19	1.3991	0.3132	3.9633	-0.026	-0.471	-0.037	2.8328	0.1423
20	-0.889	0.2944	-0.742	-0.068	2.5634	0.6209	0.5869	0.4925
21	1.9232	0.3099	0.0817	0.0112	4.9275	0.6979	4.0831	0.5899
22	2.3081	0.3751	0.4522	0.0198	-0.181	0.6598	1.5553	0.6029
23	1.3253	0.3796	0.2811	0.0223	5.0007	0.7963	3.8188	0.6822
24	-0.464	0.3718	0.5367	0.0283	-1.137	0.7245	-0.562	0.6312

EXPERIMENT 7

STRUCTURE FROM MOTION PARADIGM

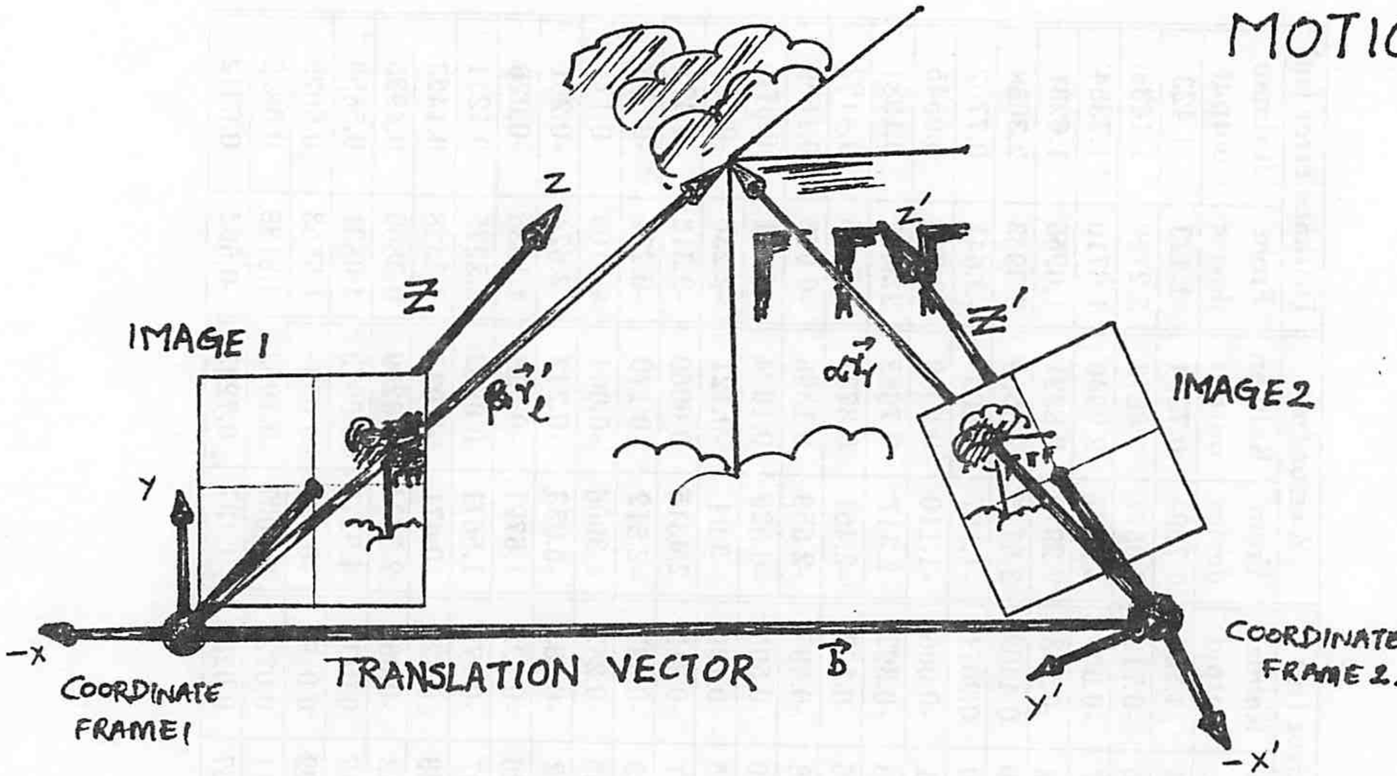


Table 7.1(Task 7): Structure measurements are obtained from known motion parameters. The table shows the Kalman estimate at each iteration.

Iter	X-Coordinate (meter)			Y-Coordinate (meter)			Z-Coordinate (meter)		
num ber	Ground truth	From motion	Kalman output	Ground truth	From motion	Kalman output	Ground truth	From motion	Kalman output
1	41.041	40.430	40.434	40.0	39.677	39.681	58.293	57.340	57.346
2	40.017	78.72	44.363	40.0	78.752	44.524	58.000	115.17	64.405
3	41.024	22.952	34.352	40.0	22.604	33.548	56.293	31.551	47.069
4	40.035	49.16	35.698	40.0	49.135	35.725	56.000	69.123	49.842
5	41.006	28.602	38.548	40.0	28.164	37.705	54.293	38.012	51.047
6	40.052	24.304	39.098	40.0	23.996	39.015	54.000	32.669	52.698
7	40.989	54.870	40.563	40.0	52.802	39.536	52.293	70.057	51.757
8	40.07	24.840	39.559	40.0	24.588	39.370	52.001	31.989	51.301
9	40.971	22.629	37.953	40.0	22.336	37.010	50.293	27.594	46.286
10	40.087	19.727	37.170	40.0	19.387	36.951	50.001	24.238	45.917
11	40.954	52.581	38.164	40.0	50.852	37.14	48.294	61.565	44.523
12	40.105	15.263	39.622	40.0	15.511	39.621	48.001	18.563	47.473
13	40.936	20.204	40.507	40.0	19.589	39.673	46.294	22.769	45.855
14	40.122	7.9047	39.391	40.0	8.2171	39.923	46.001	9.2988	45.638
15	40.919	12.613	39.98	40.0	12.311	39.682	44.294	13.648	43.671
16	40.14	41.084	39.255	40.0	41.313	39.721	44.001	45.481	43.416
17	40.901	30.607	39.986	40.0	30.102	39.696	42.294	31.672	41.696
18	40.157	31.626	39.074	40.0	31.354	39.501	42.001	32.809	41.156
19	40.884	39.968	39.838	40.0	39.471	39.547	40.294	39.678	39.527
20	40.175	27.395	39.141	40.0	27.035	39.520	40.002	27.161	39.200
21	40.867	36.566	39.888	40.0	35.716	39.577	38.294	34.169	37.588
22	40.192	37.782	39.16	40.0	37.240	39.498	38.002	35.488	37.190
23	40.849	34.359	39.928	40.0	33.867	39.612	36.294	30.723	35.644
24	40.209	34.549	39.361	40.0	34.205	39.661	36.002	30.939	35.402

Table 7.2: Standard deviation in the 3 D coordinates from known motion parameters and the standard deviation in the Kalman estimates. The error analysis from Section 7.2 is based on known variance of the noise in the image coordinates.

Iter	X: Std Dev. (m)		Y: Std Dev. (m)		Z: Std Dev. (m)	
num ber	From motion	Kalman output	From motion	Kalman output	From motion	Kalman output
1	46.474	46.472	46.038	46.036	66.534	66.531
2	540.61	39.595	542.06	40.129	791.67	58.729
3	13.237	9.4121	12.843	9.2418	18.26	13.160
4	112.34	8.3361	113.03	8.3768	158.6	12.055
5	20.760	5.3125	20.164	5.2136	27.619	7.3148
6	36.622	3.7759	36.523	3.7774	49.467	5.3191
7	62.282	3.5445	60.235	3.453	80.188	4.7880
8	12.917	2.3601	12.649	2.3521	16.763	3.2451
9	16.384	1.795	15.876	1.7525	19.881	2.3445
10	28.572	1.4116	28.358	1.4076	35.247	1.879
11	58.456	1.3905	56.056	1.3556	68.497	1.7795
12	26.495	1.0684	26.719	1.055	32.044	1.3764
13	15.895	0.9685	15.683	0.9354	18.096	1.1889
14	8.932	0.6640	9.183	0.6589	10.415	0.8062
15	10.520	0.6171	10.437	0.6047	11.461	0.7164
16	46.918	0.5989	46.661	0.5994	51.789	0.7195
17	14.156	0.5871	14.038	0.5757	14.909	0.6775
18	19.954	0.5511	20.112	0.5508	21.015	0.6554
19	57.753	0.5474	56.509	0.5366	57.116	0.6273
20	21.496	0.5000	21.560	0.4997	21.541	0.5896
21	20.986	0.4778	20.2	0.4692	19.643	0.5414
22	23.403	0.44	23.417	0.4401	22.313	0.513
23	36.160	0.4213	35.21	0.414	32.182	0.4721
24	23.128	0.384	23.282	0.3841	20.988	0.4419

Table 7.3: The errors, computed as a difference of the values from Table 7.1 and the ground truth. Note how the Kalman estimate error decreases steadily.

Iter	X error (m)		Y error (m)		Z error (m)		Distance error (m)	
num ber	From motion	Kalman output	From motion	Kalman output	From motion	Kalman output	From motion	Kalman output
1	-0.611	-0.607	-0.323	-0.319	-0.953	-0.947	-1.143	-1.136
2	38.702	4.3451	38.752	4.5236	57.172	6.4046	79.171	8.9637
3	-18.07	-6.671	-17.4	-6.452	-24.74	-9.224	-35.23	-13.08
4	9.1248	-4.337	9.1346	-4.275	13.123	-6.158	18.408	-8.66
5	-12.40	-2.458	-11.84	-2.295	-16.28	-3.246	-23.64	-4.673
6	-15.75	-0.954	-16.00	-0.985	-21.33	-1.302	-30.97	-1.891
7	13.881	-0.426	12.802	-0.464	17.764	-0.536	25.92	-0.826
8	-15.23	-0.511	-15.41	-0.63	-20.01	-0.700	-29.49	-1.067
9	-18.34	-3.018	-17.66	-2.99	-22.7	-4.007	-34.11	-5.836
10	-20.36	-2.917	-20.61	-3.049	-25.76	-4.084	-38.77	-5.865
11	11.627	-2.79	10.852	-2.860	13.271	-3.771	20.711	-5.484
12	-24.84	-0.482	-24.49	-0.379	-29.44	-0.528	-45.64	-0.806
13	-20.73	-0.43	-20.41	-0.327	-23.52	-0.439	-37.41	-0.693
14	-32.22	-0.732	-31.78	-0.077	-36.70	-0.363	-58.27	-0.672
15	-28.31	-0.939	-27.69	-0.318	-30.65	-0.623	-50.07	-1.087
16	0.9443	-0.884	1.3135	-0.279	1.4803	-0.586	2.1692	-1.008
17	-10.29	-0.915	-9.898	-0.304	-10.62	-0.598	-17.8	-1.051
18	-8.531	-1.083	-8.646	-0.499	-9.193	-0.846	-15.23	-1.402
19	-0.916	-1.046	-0.529	-0.453	-0.616	-0.767	-1.192	-1.311
20	-12.78	-1.034	-12.97	-0.48	-12.84	-0.801	-22.28	-1.336
21	-4.301	-0.979	-4.284	-0.423	-4.125	-0.707	-7.339	-1.219
22	-2.41	-1.032	-2.76	-0.502	-2.514	-0.811	-4.435	-1.353
23	-6.49	-0.921	-6.133	-0.388	-5.572	-0.651	-10.52	-1.133
24	-5.661	-0.849	-5.795	-0.339	-5.063	-0.599	-9.552	-1.03