RED: Robust Earliest Deadline Scheduling

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Abstract

In this paper we introduce a robust earliest deadline scheduling algorithm for dealing with sporadic tasks under overloads in a hard real-time environment. The algorithm synergistically combines many features including a very important minimum level of guarantee, dynamic guarantees, graceful degradation in overloads, deadline tolerance, resource reclaiming, and dynamic re-guarantees. A necessary and sufficient schedulability test is presented, and an efficient $O(n)$ guarantee algorithm is proposed. The new algorithm is evaluated via simulation and compared to several baseline algorithms. The experimental results show excellent performance of the new algorithm in normal and overload conditions.

1 Introduction

Many real-time systems rely on the earliest deadline (EDF) first scheduling algorithm. This algorithm has been shown to be optimal under many different conditions. For example, for independent, preemptable tasks, on a uni-processor EDF is optimal in the sense that if any algorithm can find a schedule where all tasks meet their deadline then EDF can meet the deadlines [Der 74]. Also, Jackson's rule [Jac 55] says that ordering a set of tasks by deadline will minimize the maximum lateness. Further, it has also been shown that EDF is optimal under certain stochastic conditions [Tow 91].

In spite of these advantageous properties, EDF has one major negative aspect. That is, when using EDF in a dynamic system, if overload occurs, tasks may miss deadlines in an unpredictable manner, and in the worst case, the performance of the system can approach zero effective throughput [Loc 86]. This aspect of EDF is a well known fact and applies when using EDF in a dynamic system. However, for a static collection of tasks we see by Moore's rule [Moo 68], that to minimize the number of jobs which are late, simply order by earliest deadline and drop the late jobs. By using EDF in a planning mode that performs dynamic guarantees,

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we can make use of a result similar to Moore's theorem, thereby avoiding the catastrophic failure of EDF and making it more robust. Moreover, our approach synergistically combines many nice scheduling features, further enhancing its robustness. The main contribution of this paper is the development and the performance evaluation of a robust version of the EDF algorithm, which has the following characteristics:

- it operates in normal and overload conditions with excellent dynamic performance and avoids the major negative aspect of EDF scheduling;
- it is *a priori* analyzable together with offering certain minimum guarantees of system performance;
- it is used in planning mode with *additional* on-line evaluative properties such as depicting the size of the overload, when and where the overload will occur and dynamically assessing the overall impact of this overload on the system;
- it includes extended timing semantics based on a deadline tolerance per task that is suitable to certain applications such as robotics;
- it applies to simultaneously handling n classes of preemptable tasks where tasks can have different values;
- it is embedded in a general framework that:
  - is easily and cost effectively implementable ($O(n)$ complexity),
  - separates guarantee, dispatching and rejection policies,
  - maintains a state view of the system that accurately depicts overload (to accomplish this required us a novel on-line calculation of overload with respect to varying deadlines),
  - reclaims resources to improve performance, and
  - dynamically attempts re-guarantees when resources are reclaimed.

A performance study is accomplished via simulation. Our new algorithm is compared against the standard earliest deadline algorithm and a guarantee based earliest deadline algorithm [Ram 84]. We compare the algorithms under widely varying conditions with respect to load, arrival rates, value distributions, allowed tolerances, and actual versus worst case execution times. The new algorithm significantly outperforms these baselines in all tested situations.

The paper is organized as follows. Section 2 introduces terminology and basic concepts. In Section 3 we state the main properties of the proposed approach, give the basic guarantee algorithm, and present the efficient calculation of system load. In Section 4 we deal with overload conditions and present a new model for robust scheduling in overload. Section 5 discusses the problem of resource reclaiming, which adds flexibility to our approach, and presents a new framework in which all the parts of the algorithm synergistically integrate into a general scheme. Section 6 presents the performance study. Section 7 discusses related research, showing how this work is a significant extension to the state of the art of EDF scheduling. Section 8 summarizes the main results and presents the implications of the proposed approach.

2 Terminology and Basic Concepts

In order to motivate and more easily understand the following definitions, proofs, and subsequent additions to the basic algorithm, we begin by presenting an overview of the dynamic operation of the system. First, we assume that there are n known tasks given by $T_1, T_2, ..., T_n$ where each has a worst case computation time, a deadline relative to when the task is activated, and a specified deadline tolerance level which indicates the amount of lateness allowed for a given task (may be zero). Tasks are preemptable. Tasks are invoked dynamically, and at any point in time the system could have zero, all, or an arbitrary subset of these n tasks active.

When a task is activated at time t, it is inserted into an ordered list of tasks based on an earliest deadline first policy and an analysis is performed to determine feasibility of the entire set of currently active tasks, whether overload exists and, if so, where, when, for how long, and which tasks are involved in the overload, and how the active tasks would operate subject to allowed tolerances.

2.1 Minimum Levels of Guarantee

Static real-time systems are designed for worst case situations. Then, assuming that all the assumptions made in the design and analysis are correct, all tasks always make their deadlines. Theoretically, we can then say that the level of guarantee for these systems is absolute. Unfortunately, static systems are not always possible because for many applications the environment and system itself, being imperfect, violate their assumptions fairly often, or because to develop a static design with absolute guarantees is too costly. When either or both of these conditions exist, we find dynamic real-time systems. In these systems, absolute guarantees are not attained. However, we believe that a necessary property of dynamic systems must be a tradeoff between what is guaranteed and what is probabilistic. A property of a dynamic real-time system should be a *minimum level of guarantee* together with best effort beyond this minimum. For example, assume that we have an application with 2000 different tasks and this application exists in a hostile environment. To statically design a solution might be impossible, or if it were possible might require a very large number of processors (say 100s of them). Now it may be known that only 20 of these 2000 tasks are truly critical and that it is unlikely that more than 10 percent of the tasks are active at the same time. A good dynamic system design would give an absolute guarantee to the 20 critical tasks and provide enough resources to also meet the deadlines of all other tasks likely to be active at the same time, i.e., under normal loads. To achieve this may only require 5-6 processors. In other words, to build a system for a reasonable cost, we may miss deadlines (rarely) on non-critical tasks.

Unfortunately, many algorithms used in dynamic real-time systems do not have good minimum level of guarantee properties. For example, earliest deadline and rate monotonic have very limited minimum guarantees. Our RED algorithm considers minimum levels of guarantees at two different times: *a priori* and at run time. For the *a priori* guarantee of critical tasks, the designer applies all worst case assumptions, but only for the critical tasks and knowing that the run time algorithm is RED. In this way, an absolute minimum performance is guaranteed for the critical tasks. In the *a priori* analysis we show that using

RED all critical task deadlines are met, and that even in the presence of *other* real-time tasks from other classes of jobs, that there is no way that these non-critical tasks can negatively influence the scheduling of the critical tasks. This is a simple yet important property of our approach. Beyond this *a priori* analysis at run time, RED ensures that all critical and other real-time tasks will make their deadline under normal loads, simply due to the optimality feature of EDF scheduling. When overloads occur for non-critical tasks, RED still ensures that critical tasks can execute and that there is a best effort at maximizing value of non-critical tasks. Further, even if the load assumptions for the critical tasks are wrong and more critical tasks arrive than planned for *a priori*, then RED still guarantees a high value by ensuring that any critical task, once guaranteed, never misses its deadline. There is no domino effect. In summary, by maintaining a profile, analyzing it, and using resource reclaiming, the value output of the system is maintained at a high value. Catastrophic drops in value do not occur and effective cpu utilization is also maintained at a high value.

2.2 Notations and Assumptions

Before we describe the guarantee algorithm, we first state our definitions, notations, and assumptions.

Definitions:

- A *task* is a sequence of instructions that will continuously use the processor until its completion, if it is executing alone on the processor [Sha 90].
- A *sporadic task* is a task with irregular arrival times and minimum interarrival time.
- A task set is said to be *feasibly scheduled* by a certain algorithm if and only if all tasks can complete their execution within their time constraints.
- A newly arrived task is said to be *guaranteed* to complete its execution before its deadline if and only if a feasible schedule can be created for the newly arrived task and those still active tasks that have been previously guaranteed, such that all tasks will meet their timing constraints.
- We refer to the time interval between the finishing time of a task and its absolute deadline as a residual time.

For the proofs which provide a necessary and sufficient condition for the schedulability of task sets at each task activation, we require a notation that identifies the i^{th} task in the current ordered list at time t. For this we use the notation $J_1, J_2, ..., J_m$ where J_i is the task in the i^{th} position. At the next task activation, the task in each position may change completely due to the deadline of the new task and possible task completions since the last task activation. On task completions the order does not change, but all tasks conceptually move up one position. Further, at each task activation, once a task is inserted into the ordered list, then each task in the list has a current scheduled start and finish time. If a task completes before its worst case execution time, then the cpu time will be reclaimed, essentially moving all other still active tasks starting time forward. The time a task is dispatched is called the actual starting time. Since tasks are preemptable, a given task may have multiple actual start times, but only a single actual finish time. At each new actual start time, a task only requires its remaining execution time, not the full worst case time (in other words the task is not restarted, just continued).

Notation:

J denotes a set of active sporadic tasks J_i ordered by increasing deadline, J_1 being the task with the shortest absolute deadline.

a_i denotes the arrival time of task J_i, i.e., the time at which the task is activated and becomes ready to execute.

C_i denotes the maximum computation time of task J_i, i.e., the worst case execution time (*wcet*) needed for the processor to execute task J_i without interruption.

c_i denotes the dynamic computation time of task J_i, i.e., the remaining worst case execution time needed for the processor, at the current time, to complete task J_i without interruption.

d_i denotes the absolute deadline of task J_i, i.e., the time before which the task should complete its execution, without causing any damage to the system.

D_i denotes the relative deadline of task J_i, i.e., the time interval between the arrival time and the absolute deadline.

S_i denotes the first start time of task J_i, i.e., the time at which task J_i gains the processor for the first time.

s_i denotes the last start time of task J_i, i.e., the last time, before the current time, at which task J_i gained the processor.

f_i denotes the estimated finishing time of task J_i, i.e., the time according to the current schedule at which task J_i should complete its execution and leave the system.

L_i denotes the laxity of task J_i, i.e., the maximum time task J_i can be delayed before its execution begins.

R_i denotes the residual time of task J_i, i.e., the length of time between the finishing time of J_i and its absolute deadline.

The meaning of the main task parameters defined above is illustrated in figure 1. It is easy to verify the following relationships among the parameters defined above:

$$d_i = a_i + D_i \tag{1}$$
$$L_i = d_i - a_i - C_i \tag{2}$$
$$R_i = d_i - f_i \tag{3}$$
$$f_1 = t + c_1; \qquad f_i = f_{i-1} + c_i \quad \forall i > 1 \tag{4}$$

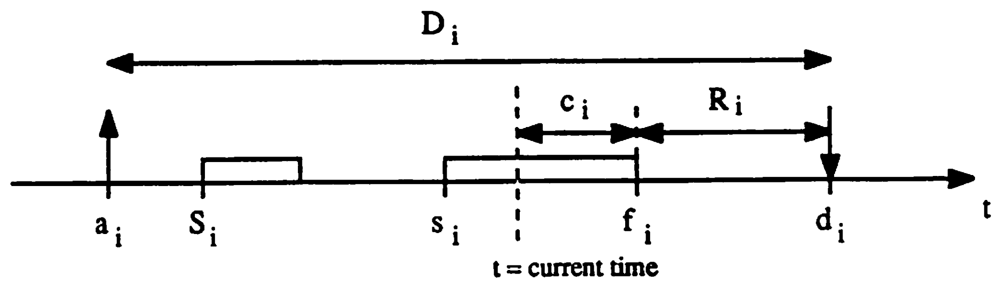


Figure 1: Task parameters

In our model, we assume that the minimum interarrival time of each sporadic task is equal to its relative deadline D_i, thus a sporadic task J_i can be completely characterized by specifying its relative deadline D_i and its worst case execution time C_i. Hence, a sporadic task set will be denoted as follows: $J = \{J_i(C_i, D_i),\ i = 1 \text{ to } n\}$.

Throughout our discussion, we assume that tasks are scheduled on a uniprocessor by the Earliest Deadline First (EDF) scheduling algorithm, according to a preemptive scheduling discipline, so that the processor is always assigned to the task whose deadline is the earliest. We assume that tasks arrive dynamically and arrival times are not known a priori. EDF algorithm is optimal in underload conditions, it is dynamic, it can be used for periodic and aperiodic tasks, and it can be easily extended to deal with precedence constraints. Group of tasks with precedence constraints can be scheduled with EDF by modifying their deadlines and release times so that both deadlines and precedence relations are met [Bla 76] [Che 90]. Overheads are assumed to be zero, but how to account for overheads is discussed in the conclusions.

3 Guarantee Algorithm

We formulate the dynamic, on-line, guarantee test in terms of residual time, which is a convenient parameter to deal with both normal and overload conditions. We first present the main results without the notion of deadline tolerance, and then we will extend the algorithm by including tolerance levels and task rejection policy. The basic properties stated by the following lemmas and theorems are used to derive an efficient $O(n)$ algorithm for analyzing the schedulability of the sporadic task set whenever a new task arrives in the systems.

Lemma 1 *Given a set $J = \{J_1, J_2, ..., J_n\}$ of active sporadic tasks ordered by increasing deadline, the residual time R_i of each task J_i at time t can be computed by the following recursive formula:*

$$R_1 = d_1 - t - c_1 \quad (5)$$

$$R_i = R_{i-1} + (d_i - d_{i-1}) - c_i. \quad (6)$$

Proof.
By the residual time definition (equation 3) we have:

$$R_i = d_i - f_i.$$

By the assumption on set J, at time t, task J_1 is executing and cannot be preempted by other tasks in the set J, hence its estimated finishing time is given by the current time plus its remaining execution time:

$$f_1 = t + c_1$$

and, by equation (3), we have:

$$R_1 = d_1 - f_1 = d_1 - t - c_1.$$

For any other task J_i, with $i > 1$, each task J_i will start executing as soon as J_{i-1} completes, hence we can write:

$$f_i = f_{i-1} + c_i \tag{7}$$

and, by equation (3), we have:

$$\begin{aligned} R_i = d_i - f_i &= d_i - f_{i-1} - c_i = \\ = d_i - (d_{i-1} - R_{i-1}) - c_i &= R_{i-1} + (d_i - d_{i-1}) - c_i \end{aligned}$$

and the lemma follows. □

Lemma 2 *A task J_i is guaranteed to complete within its deadline if and only if $R_i \geq 0$.*

Proof.
(If part): Suppose $R_i \geq 0$. By the result obtained in Lemma 1 we can write:

$$R_{i-1} + (d_i - d_{i-1}) - c_i \geq 0$$

that is, by equation (3):

$$d_i - f_{i-1} - c_i \geq 0$$

which can be written as:

$$f_{i-1} + c_i \leq d_i$$

and by equation (7):

$$f_i \leq d_i$$

which means that J_i completes its execution within its deadline, i.e., it is guaranteed.
(Only if part): Suppose task J_i completes its execution within its deadline. This means that

$$f_i \leq d_i$$

and since, by equation (7), $f_i = f_{i-1} + c_i$, we have

$$f_{i-1} + c_i \leq d_i.$$

By equation (3) we can write:

$$(d_{i-1} - R_{i-1}) + c_i - d_i \leq 0$$

and since, by equation (6) of Lemma 1,

$$(d_{i-1} - R_{i-1}) + c_i - d_i = -R_i$$

we have $R_i \geq 0$ and the lemma follows. □

Theorem 3 *A set $J = \{J_i,\ i = 1\ to\ n\}$ of n active sporadic tasks ordered by increasing deadline is feasibly schedulable if and only if $R_i \geq 0$ for all $J_i \in J$.*

Proof.
It follows directly by applying Lemma 2 to each task of the set. □

Notice that if we have a feasibly schedulable set J of n active sporadic tasks, and a new task J_a arrives at time t, to guarantee the new ordered task set $J' = J \cup \{J_a\}$ we only need to compute the residual time of task J_a and the residual times of tasks J_i such that $d_i > d_a$. This is because the execution of J_a does not influence those tasks having deadline less than or equal to d_a, which are scheduled before J_a.

We now summarize the basic guarantee algorithm in the form of pseudo code. This basic version is enhanced in later sections of the paper to produce the robust ED (RED) algorithm.

```
Algorithm GUARANTEE(J, J_a)

begin
        t = get_current_time();
        R_0 = 0;
        d_0 = t;
        Insert J_a in the ordered task list;
        J' = J ∪ J_a;
        k = position of J_a in the task set J';
        for each task J'_i such that i ≥ k do {
                R_i = R_{i-1} + (d_i - d_{i-1}) - c_i;
                if (R_i < 0) then return ("Not Guaranteed");
        }
        return ("Guaranteed");
end
```

Clearly, the above guarantee algorithm runs in $O(n)$ time in the worst case. This makes the proposed approach an efficient schedulability test to run whenever a sporadic task arrives.

Now we introduce a new framework for handling real-time sporadic tasks under overload conditions, and we propose a robust version of the Earliest Deadline algorithm. Before we describe such a robust algorithm, we define few more basic concepts.

3.1 Load Calculation

In some real-time environments, the workload of the system can be considered the same as the processor utilization factor. For example, for a set of n periodic tasks with computation time C_i and period T_i, with no resource conflicts, the utilization factor can be easily computed, as proposed in [Liu 73], by

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}.$$

For sporadic tasks, the utilization factor could be computed by considering the minimum interarrival time as a sort of period. However, this would lead to an overestimation of the workload, since it would refer to the (very pessimistic) case in which all sporadic tasks have the maximum arrival rate.

In a real-time environment with aperiodic tasks, a commonly accepted definition of workload refers to the standard queueing theory, according to which a load ρ, also called *traffic intensity*, represents the expected number of task arrivals per mean service time [Zlo 91]. This definition, however, does not say anything about task deadlines, hence it is not as useful in a hard real-time environment.

A more formal definition has been proposed in [Bar 92], in which is said that a sporadic real-time environment has a loading factor b if and only if it is guaranteed that there will be no interval of time $[t_x, t_y)$ such that the sum of the execution times of all tasks making requests and having deadlines within this interval is greater than $b(t_y - t_x)$. They use this definition in order to prove worst case bounds on the performance of on-line algorithms in the presence of overloads. Theoretically, this is fine for bounds analysis. For practical use, however, these intervals can be very numerous, requiring too much computation if used on-line. Moreover, having only a single load value can easily be misleading. Although such a definition is more precise than the first one, it is still of little practical use, since no on-line methods for calculating the load are provided, nor proposed.

One main purpose of RED is to operate well even in overload conditions. However, it is difficult to develop a good measure of load in a real-time system because each task has a unique start time and deadline. This gives rise to overloads occurring in specific intervals, even when the total processor utilization is very low. For actual use on-line, we need an efficient mechanism to detect and react to overload. Our approach uses a technique that is very similar to the off-line iterative analysis done in rate monotonic scheduling; however, we use it on-line and apply it to aperiodic tasks scheduled by earliest deadline rather than periodic tasks scheduled by frequency of period. The idea is to iteratively identify the cpu utilization required by all the tasks up to the i^{th} task. For example, first we compute the utilization required by the first task in the schedule. Then we compute the utilization required by the first two tasks in the schedule, etc. As we show below, this iterative procedure is efficient, avoids needing to know the theoretically worst case load in any interval, and has many nice properties including:

- a complete load profile is efficiently created,
- the time (intervals) at which overloads might occur is predicted,
- the magnitude of the overload is identified,
- the impact of the overload on the system as a whole is calculated, e.g., will only a few tasks miss their deadlines or is it possible that many (all) tasks will miss their deadlines, and
- many types of other analysis can be applied to the profile including
  - applying semantics of deadline tolerance levels (of specific tasks being late by specific amounts)

- shifting load by altering deadlines (possible, e.g., by slowing down operations in a robotics application).

Before we explain our method of computing system load, we introduce the following notation:

$\rho_i(t_a)$ indicates the processor load in the interval $[t_a, d_i)$, where t_a is the arrival time of the latest arrived task in the sporadic set,

ρ_{max} indicates the maximum processor load among all intervals $[t_a, d_i)$, $i = 1$ to n, where t_a is the arrival time of the latest arrived task in the sporadic set.

In practice, the load is computed only when a new task arrives, and it is of significant importance only within those time intervals $[t_a, d_i)$ from the latest arrival time t_a, which is the current time, and a deadline d_i. Thus the load computation can be simplified as:

$$\rho_i(t_a) = \frac{\sum_{d_k \leq d_i} c_k}{d_i - t_a}.$$

Theorem 4 *The load $\rho_i(t_a)$ in the interval $[t_a, d_i)$ can be directly related to the residual time R_i of task J_i, according to the following relation:*

$$\rho_i = 1 - \frac{R_i}{d_i - t_a} \tag{8}$$

Proof.
Since tasks are ordered by increasing deadline, after the arrival time t_a, they will be executed in such a way that $f_1 = t_a + c_1$ and $f_i = f_{i-1} + c_i$. Therefore, we can write:

$$\sum_{d_k \leq d_i} c_k + t_a = f_i$$

and since, by equation (3),

$$f_i = d_i - R_i$$

we have:

$$\sum_{d_k \leq d_i} c_k = (d_i - t_a) - R_i.$$

Hence the load $\rho_i(t_a)$ can be directly related to the residual time R_i of task J_i, as follows:

$$\rho_i(t_a) = 1 - \frac{R_i}{d_i - t_a}.$$

□

3.2 Load function

It is important to point out that, within the interval $[t_a, d_n]$ between the latest arrival time t_a and the latest deadline of task J_n, the processor work load is not constant, but it varies in each interval $[t_a, d_i)$. To express this fact, we define the following *load function*:

$$\rho(t_a, t) = \begin{cases} \rho_1 & \text{for } t_a \le t < d_1 \\ \rho_i & \text{for } t \in [d_{i-1}, d_i) \\ 0 & \text{for } t \ge d_n \end{cases}$$

Definition 1 *Let ρ_{max} be the maximum of the load function $\rho(t_a, t)$ in the interval $[t_a, d_n]$. We say that the system is* **underloaded** *if $\rho_{max} \le 1$, and* **overloaded** *if $\rho_{max} > 1$.*

Definition 2 *We define Exceeding Time E_i of a task J_i as the time that task J_i will execute after its deadline, that is: $E_i = max_i(0, -R_i)$. We then define Maximum Exceeding Time E_{max} the maximum among all E_i in the tasks set, that is: $E_{max} = max_i(E_i)$.*

Notice that, in underloaded conditions ($\rho_{max} \le 1$), $E_{max} = 0$, whereas in overload conditions ($\rho_{max} > 1$), $E_{max} > 0$.

Observation 1 *Once we have computed the load factor ρ_i for task J_i, the next load factor ρ_{i+1} can be computed as follows:*

$$\rho_{i+1} = \frac{\rho_i(d_i - t_a) + c_{i+1}}{d_{i+1} - t_a}.$$

Proof.
By Theorem 4 we can write:

$$\rho_{i+1} = 1 - \frac{R_{i+1}}{d_{i+1} - t_a}$$

and by Lemma 1:

$$\rho_{i+1} = 1 - \frac{R_i + d_{i+1} - d_i - c_{i+1}}{d_{i+1} - t_a} = \frac{d_i - t_a - R_i + c_{i+1}}{d_{i+1} - t_a}.$$

Applying equation (8) we have:

$$(d_i - t_a - R_i) = \rho_i(d_i - t_a)$$

therefore we obtain:

$$\rho_{i+1} = \frac{\rho_i(d_i - t_a) + c_{i+1}}{d_{i+1} - t_a}.$$

□

task	a_i	C_i	d_i
J_0	7	4	12
J_1	0	14	16
J_2	3	4	21
J_3	5	5	28

Table 1: Task parameters for the Example 1

3.3 Example 1

To clarify the concepts defined above, we present an example to illustrate how the residual times and the load function help to identify the time at which the overload occurs, the tasks involved, and the effects that this produces on the system.

Let us consider a set of four tasks as indicated in Table 1. At time $t_a = 7$, when task J_0 arrives, the residual times R_i and the corresponding load factors ρ_i have the following values:

$$\left\{ \begin{array}{l} R_0 = d_0 - t_a - c_0 = 1 \\ R_1 = R_0 + d_1 - d_0 - c_1 = -2 \\ R_2 = R_1 + d_2 - d_1 - c_2 = -1 \\ R_3 = R_2 + d_3 - d_2 - c_3 = 1 \end{array} \right. \qquad \left\{ \begin{array}{l} \rho_0 = 1 - R_0/(d_0 - t_a) = 0.8 \\ \rho_1 = 1 - R_1/(d_1 - t_a) = 1.22 \\ \rho_2 = 1 - R_2/(d_2 - t_a) = 1.07 \\ \rho_3 = 1 - R_3/(d_3 - t_a) = 0.95 \end{array} \right.$$

Figure 2a illustrates the scheduling sequence of the four tasks in the time domain. Figure 2b and 2c shows the residual values and the resulting load function calculated at time $t_a = 7$. As we see, since $\rho_{max} > 1$, the system is overloaded. However, the load function shows that after time $t = 21$ the overload condition disappears. Moreover, from the residual values we know that the maximum exceeding time is $E_{max} = 2$, and it is caused by task J_1. This a very useful information since, if tasks have deadline tolerances, the peak load may be tolerated by the system. For example, if task J_1 can tolerate an exceeding time of 2 time units and task J_2 can tolerate an exceeding time of 1 time unit, then the task set can still be scheduled so that all tasks will meet their timing constraints.

3.4 Localization of exceeding time

As we have shown in the above example, by computing the load function, we can have a global picture of the system load, and we can see in advance the effects of an overload in the system. For instance, we can see whether the overload will cause a "*domino effect*", in which all task will miss their deadlines, or whether it is transient and it will extinguish after a while. In other words we are able to locate the time or times at which the system will experience the overload, identify the exact tasks that will miss their deadlines, and we can easily compute the amount of computation time required above the capacity of the system – the exceeding time.

This global view of the system allows us to plan an action to recover from the overload condition. Our approach is general enough that many recovering strategies can be used to solve this problem. The recovery strategy we propose in this paper is described in Section 6. The important advantages of our approach are:

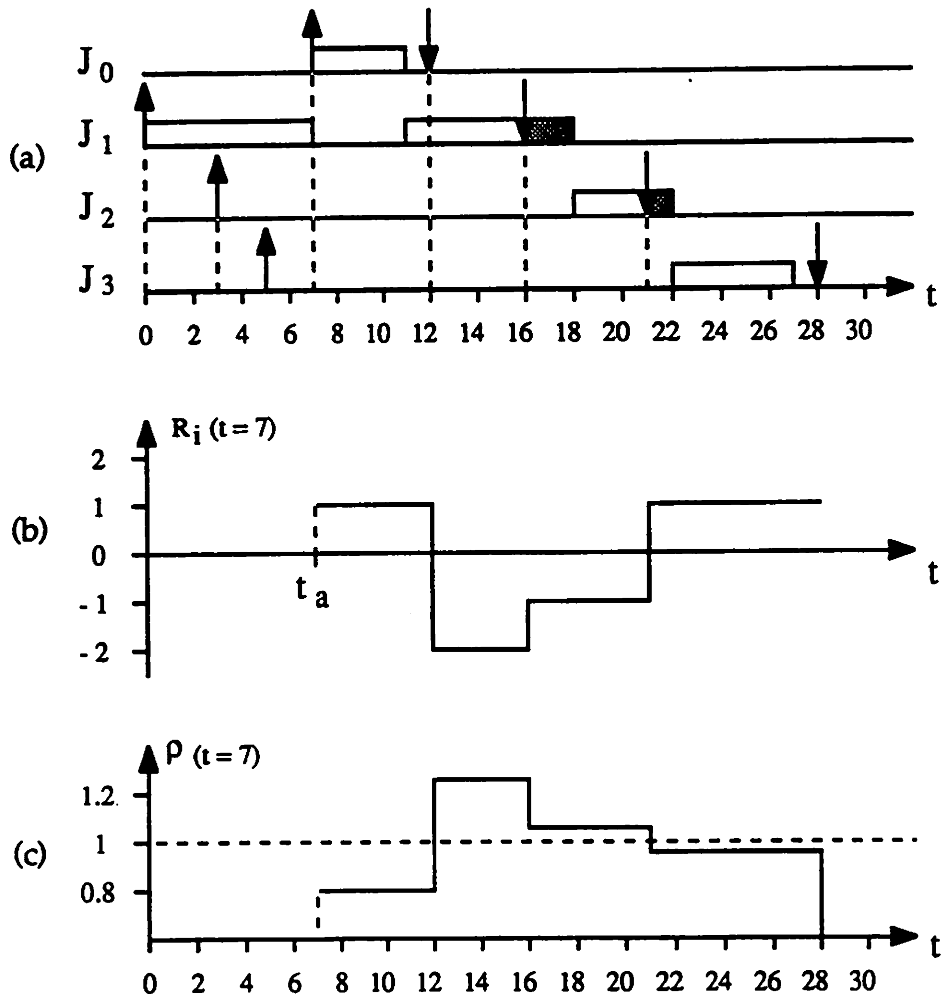


Figure 2: Scheduling sequence of the tasks defined in Table 1 (a); residual values calculated at time $t_a = 7$ (b); resulting load function at $t_a = 7$ (c);

- it increases flexibility in expressing time constraints via the deadline tolerance mechanism,
- it makes the scheduling algorithm more robust in the sense of giving better and more predictable performance under all loads; this is achieved via a combination of using value, deadline tolerance, and resource reclaiming,
- it allows the integration of different rejection policies for different classes of tasks, and
- it provides a minimum guarantee even in overload conditions.

4 Deadline Tolerance

In many real applications, such as robotics, the deadline timing semantics is more flexible than scheduling theory generally permits. For example, most scheduling algorithms and accompanying theory treat the deadline as an absolute quantity. However, it is often acceptable for a task to continue to execute and produce an output even if it is late – but not too late. In order to more closely model this real world situation, we permit each task to be characterized by a computation time, deadline, and deadline tolerance. The deadline tolerance is then the amount of time by which a specific task is permitted to be late. Further, when using a dynamic guarantee paradigm, a deadline tolerance provides a sort of compensation for the pessimistic evaluation of using the worst case execution time. For example, without tolerance, we could find that a task set is not feasibly schedulable, and hence decide to reject a task. But, in reality, the system could have been scheduled because with the tolerance and full assessment of the load we might determine that overload is simply for this task and it is within its tolerance level. Another effect is that various tasks could actually finish before their worst case times so the resource reclaiming part of our algorithm could then compensate and the guaranteed task with tolerance could actually finish on time. Basically, our approach minimizes the pessimism found in a basic guarantee algorithm.

Another real application issue is that once some task has to miss a deadline, it should be the least valuable task. Again, many algorithms do not address this fact. In summary, we modify our previous task model by introducing two additional parameters: a deadline tolerance and a task value.

Notation:

m_i denotes the deadline tolerance of task J_i, i.e., the maximum time that task J_i may execute after its deadline, and still produce a valid result.

v_i denotes the task value, i.e, the relative importance of task J_i with respect to the other tasks in the set.

Moreover, we split real-time tasks into two classes:

- *HARD* tasks are those tasks that are guaranteed to complete within their deadline in underload conditions;

- *CRITICAL* tasks are those tasks that are dynamically guaranteed to complete within their deadline in underload conditions and in overload conditions. *CRITICAL* tasks may also be guaranteed *a priori* for specified loads (in both under and overloads) by ensuring enough cpu power for them.

In our more robust model, a sporadic task J_i is completely characterized by specifying its class, its relative deadline D_i, the deadline tolerance m_i, the worst case execution time C_i, and its value v_i. In the following, we assume that the task class can be derived from the task value. For instance, tasks with maximum value $v_i = V_{max}$ can be considered as *CRITICAL*.

In summary, a sporadic task set will be denoted as follows:

$$J = \{J_i(C_i, D_i, m_i, v_i),\ i = 1\ to\ n\}$$

Within this framework, different policies are used for handling sporadic tasks in a robust fashion. In particular, tasks are scheduled based on their deadline, guaranteed based on C_i, D_i, m_i, v_i, and rejected based on v_i.

4.1 The general RED scheduling strategy

When dealing with the deadline tolerance factor m_i, each Exceeding Time has to be computed with respect to the tolerance factor m_i, so we have: $E_i = max(0, -(R_i + m_i))$.

To guarantee the execution time of *CRITICAL* tasks in overload conditions, the algorithm uses a rejection strategy to reject tasks based on their values, to remove the overload condition. Several rejection strategies can be used for this purpose. As discussed in the next section on performance evaluation, two rejection strategies have been implemented and compared. The first policy rejects a single task (the least value one), while the second strategy tries to reject more tasks, but only if the newly arrived task is a *CRITICAL* task. In the single task policy, since the algorithm knows the *Maximum Exceeding Time* E_{max} and the task J_w which would cause such a time overflow, it tries to reject the least value task in the sporadic set, whose execution time is greater than E_{max}. The multiple task policy may choose to reject more tasks such that $\sum_i c_i \geq E_{max}$ and whose deadlines are less than the deadline of the newly arrived *CRITICAL* task. Both of these options are evaluated in the performance study. Another strategy for rejecting tasks is based on the concept of *value density*, defined as the ratio v_i/C_i. According to this strategy, tasks with longer computation times have lower value density and hence have more chances to be rejected. To be general, we will describe the RED algorithm by assuming that, in overload conditions, some rejection policy will search for a subset J^* of least value (non critical) tasks to reject in order to make the current set schedulable. If J^* is returned empty, then the overload cannot be recovered, and the newly arrived task cannot be accepted. Notice that *CRITICAL* tasks previously guaranteed cannot be rejected.

If J_w is the task causing the maximum exceeding time overflow, the rejectable tasks that can remove the overload condition are only those tasks whose deadline is earlier than or equal to d_w. This means that the algorithm has to search only for tasks J_i, with $i \leq w$.

```
Algorithm RED_guarantee(J, J_a)

begin
      t = get_current_time();
      E = 0;            /* Maximum Exceeding Time */
      w = 1;            /* Exceeding task index */
      R_0 = 0;
      d_0 = t;
      J' = J ∪ {J_a};   /* Insert J_a in the ordered task list */
      k = position of J_a in the task set J';
      for each task J'_i such that i ≥ k do {
            R_i = R_{i-1} + (d_i - d_{i-1}) - c_i;
            if (R_i + m_i < -E) then {
                        E = -(R_i + m_i);
                        w = i;
            }
      }
      if (E = 0) then return (''Guaranteed");
      else {   Let J* be the set of least value tasks
               selected by the rejection policy;
               if (J* is not empty) then {
                        reject all task in J*;
                        return (''Guaranteed");
               }
               else return (''Not Guaranteed");
      }
end
```

4.2 Example 2

To show how the RED algorithm works, we present an example on a set of five tasks, whose parameters are given in Table 2. The rejection policy adopted in the example is to remove the least value task, such that its execution time is greater than the maximum exceeding time E_{max}. The estimated scheduling sequence of the set at time $t = 3$, before J_0 arrives, is shown in figure 3a, while the situation at time $t = 4$ is illustrated in figure 3b. When task J_0 arrives, the set is found not schedulable, because $R_3 + m_3 = -2 < 0$, meaning that task J_3 would exceed its (tolerant) time constraint by 2 units.

To make the set schedulable, some task has to be rejected, so that all residual times become non negative. According to this strategy, the least value task in the set, task J_4 (with value 2) cannot be removed, since it would not recover the overload situation. The next least value task, J_2 (with value 3) cannot be removed, because its computation time ($c_2 = 1$) is not sufficient to remove the exceeding load. The first least value task that can be removed is task J_1, since its computation time ($c_1 = 5$) is greater than E_{max}.

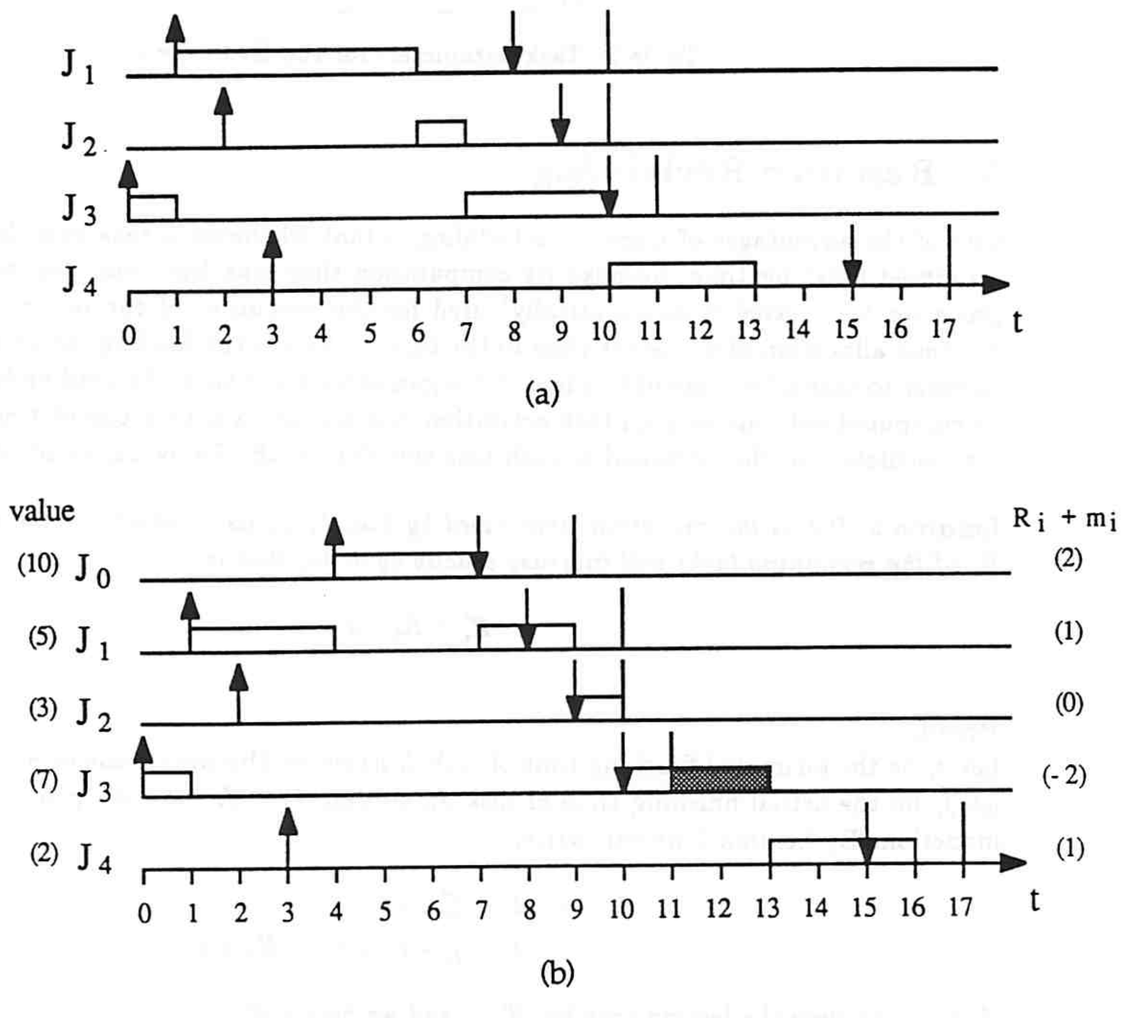


Figure 3: Estimated scheduling sequence at time $t = 3$ (a); Estimated scheduling sequence at time $t = 4$, when task J_0 arrives (b). Deadline tolerances are indicated with a vertical bar with no arrow.

task	a_i	C_i	d_i	m_i	v_i
J_0	4	3	7	2	10
J_1	1	5	8	2	5
J_2	2	1	9	1	3
J_3	0	4	10	1	7
J_4	3	3	15	2	2

Table 2: Task parameters for the Example 2

5 Resource Reclaiming

One of the advantages of dynamic scheduling is that whenever a task completes before its estimated finishing time, because its computation time was less than the worst case, the processor time saved is automatically[1] used for the execution of the other tasks. Such a dynamic allocation of processor time to the task set lowers the loading factor of the system. In order to take advantage of this fact in the guarantee algorithm, the loading function has to be computed not only at each task activation, but also at each task completion. To simplify the calculation of the workload at each task completion, the following results can be used.

Lemma 5 *If δ is the execution time saved by task J_1 at its completion, the residual times R_i of the remaining tasks will increase exactly by delta, that is:*

$$R'_i = R_i + \delta. \tag{9}$$

Proof.
Let f_i be the estimated finishing time of task J_i based on the worst case execution time, and let f'_i be the actual finishing time of task J_i, so that $f'_i = f_i - \delta$. We prove the lemma by induction. By Lemma 1 we can write:

$$\begin{aligned} R'_1 &= d_1 - f'_i - c_1 = \\ &= d_1 - f_i - c_1 + \delta = R_1 + \delta. \end{aligned}$$

Now, we assume the lemma true for R'_{i-1}, and we derive R'_i:

$$\begin{aligned} R'_i &= R'_{i-1} + (d_i - d_{i-1}) - c_i = \\ &= R_{i-1} + (d_i - d_{i-1}) - c_i + \delta = \\ &= R_i + \delta \end{aligned}$$

which proves the lemma. □

[1] If resources can be locked or multiprocessing is being used then resource reclaiming is not automatic. See [She 90] for a full discussion and solutions.

Observation 2 *If δ is the execution time saved by task J_1 at its completion, the loading factors ρ_i of the remaining tasks will decrease by the following amount:*

$$\Delta\rho_i = \frac{\delta\rho_i(1-\rho_i)}{R_i + \delta(1-\rho_i)} \tag{10}$$

Proof.
Let ρ_i' be the workload due to the residual time R_i'. By equation (8) of Theorem 4 we can write:

$$\Delta\rho_i = \rho_i - \rho_i' = \rho_i - (1 - \frac{R_i'}{d_i - f_i'})$$

and by equation (9) of Lemma 5 we can write:

$$\rho_i' = 1 - \frac{R_i + \delta}{d_i - f_i + \delta} = \frac{d_i - f_i - R_i}{d_i - f_i + \delta}$$

and since, by Lemma 4, $(d_i - f_i) = \frac{R_i}{(1-\rho_i)}$, we have:

$$\rho_i' = \frac{\rho_i R_i}{R_i + \delta(1-\rho_i)}$$

and hence:

$$\Delta\rho_i = \frac{\delta\rho_i(1-\rho_i)}{R_i + \delta(1-\rho_i)}$$

□

5.1 Increasing flexibility

The result obtained in Lemma 5 suggests that if a sporadic task cannot be guaranteed by the RED algorithm at its arrival time, there are chances that it could be guaranteed at later time, by using the execution time saved by other tasks. Scheduling tasks at an "opportune" time, rather than at arrival time has been proposed in [Zlo 91] as a technique called *Well-Timed Scheduling*. However, this technique has been mainly used to reduce the scheduling overhead in highly loaded systems, rather than focusing on increasing the probability of a successful guarantee by utilizing reclaimed time. Also it did not treat holding a rejected task for possible re-guarantee at a later time.

In a more general framework, a task J_r rejected in an overload condition can still be guaranteed if the sum of the execution time saved by all tasks completing within the laxity of J_r is greater or equal to the Maximum Exceeding Time found when J_r was rejected. This result is formally expressed in the following theorem.

Theorem 6 *Let J_r be a sporadic task rejected at time t_r in an overload condition because $E_{max}(t_r) > 0$, and let a_r and L_r be its arrival time and its current laxity. If f_i' is the actual finishing time of the current running task J_1, δ_p is the execution time saved by previous tasks* *after t_r, and δ_1 is the execution time saved by J_1, then the task set $J' = (J - \{J_1\}) \cup \{J_r\}$ can be guaranteed at time f'_i if and only if*

$$(f'_i \leq t_r + L_r) \quad \text{and} \quad (\delta \geq E_{max})$$

where $\delta = \delta_p + \delta_1$ is the total execution time saved in the interval $[t_r, f'_i]$.

Proof.
Without loss of generality, we assume $m_i = 0$ for all tasks.
(If part). If task J_r was rejected, there was a task J_k such that $R_k = -E_{max}(t_r) < 0$ and $R_i \geq R_k$ for all task J_i. Let $\delta = \delta_p + \delta_1$ be the total execution time saved in the interval $[t_r, t'_f]$. If $\delta \geq E_{max}$, then at time t'_f, by Lemma 5, we have that

$$R'_k = \delta + R_k = \delta - E_{max} \geq 0 \text{ and}$$
$$R'_i \geq R'_k \text{ for all tasks } J_i$$

therefore:

$$R'_i \geq 0 \textit{ for all tasks } J_i$$

and by Theorem 1 the task set is feasibly schedulable.
(Only if part). Left to the reader. □

5.2 General scheduling scheme

Theorem 6 provides a necessary and sufficient condition for guaranteeing a previously rejected task as soon as a running task completes its execution. This result can be used to propose a more general framework for scheduling sporadic tasks, as illustrated in figure 4.

Within this framework, if a task cannot be guaranteed by the system at its arrival time, it is not removed forever, but it is temporarly rejected in a queue of non guaranteed tasks, called *Reject Queue*, ordered by decreasing values, to give priority to the most important tasks. As soon as the running task completes its execution before its worst case finishing time, the highest value task in the Reject Queue having positive laxity and causing a $Maximum\_Exceeded\_Time < \delta$ will be reinserted in the Ready Queue and scheduled by earliest deadline. All rejected tasks with negative laxity are removed from the system, and inserted in another queue, called *Miss Queue*, containing all late tasks; whereas all tasks that complete within their timing constraints are inserted in a queue of regularly terminated jobs, called *Term Queue*. The purpose of the Miss and Term Queues is to record the history of the system, which aids in debugging and understanding the operation of system.

6 Performance Evaluation

Simulations were conducted to evaluate the performance of the RED algorithm with respect to several other baseline algorithms including EDF which is commonly used in dynamic hard real-time environments.

In all the experiments, the following scheduling algorithms have been compared:

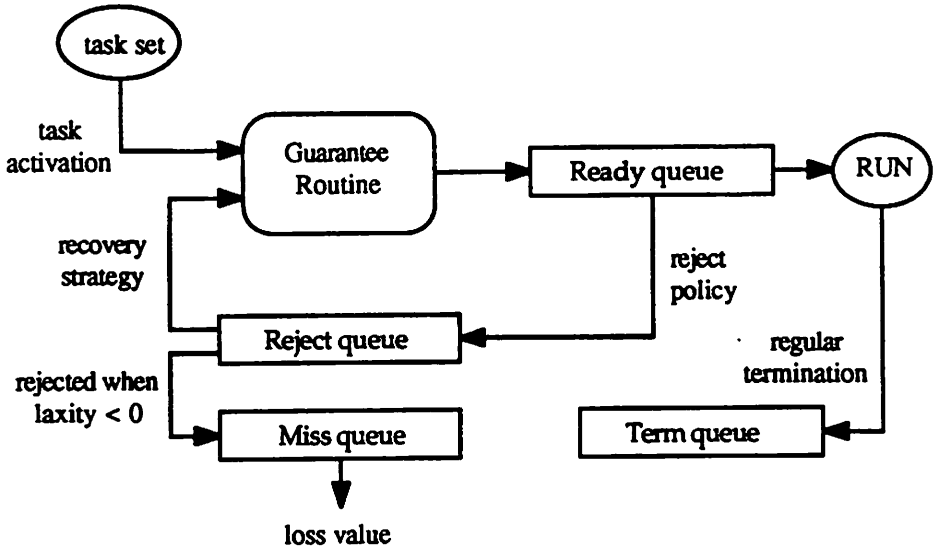


Figure 4: RED Scheduling Block Diagram

- **EDF** - Earliest Deadline First algorithm, without any form of guarantee. As soon as a new task arrives in the system, it is inserted in the ready queue by its deadline and scheduled according to the EDF policy.
- **GED** - Guaranteed Earliest Deadline algorithm. When a new task arrives, a guarantee routine verifies whether the new task set is schedulable: if yes, the newly arrived task is inserted in the ready queue and scheduled by its deadline; if no, the newly arrived task is rejected.
- **RED** - Robust Earliest Deadline algorithm with single task rejection. When a new task arrives, a guarantee routine verifies whether the new task set is feasibly schedulable: if yes, the newly arrived task is accepted; if no, the system will reject the least value task, if any, such that the remaining set is schedulable, else the newly arriving task is rejected. Everytime a task completes its execution, a recovery routine tries to reaccept the greatest value task, among those rejected tasks whose laxity is positive.
- **MED** - Robust Earliest Deadline algorithm with multiple task rejection. The same as the RED algorithm, with the following difference: if the new task set is found unschedulable and the newly arrived task is critical, then the system may reject more than one lower value tasks to make the remaining task set schedulable.

The main performance metrics we used are:

Loss Value Ratio (LVR) : ratio of the sum of the values of late HARD processes to the total set value.

atime	arrival time
dline	deadline
wcet	estimated worst case computation time
ctime	actual computation time
toler	deadline tolerance
value	relative importance

Table 3: Task parameters

N	number of tasks in the set
λ	arrival rate
load	initial system workload
α	load rate or growth factor: $\alpha \in [-1, 1]$ $\alpha > 0$ increasing load $\alpha = 0$ constant load $\alpha < 0$ decreasing load
crit	(critical factor) ratio of the average number of critical tasks to the total number of tasks
dw	average difference between the estimated wcet and the real computation time

Table 4: Set parameters

Loss Critical Ratio (LCR) : ratio of the number of critical tasks that missed their deadline to the total number of critical tasks. This is used to show how the system operates in a region beyond which the a priori guarantee had accounted for.

While all the graphs plot average values obtained over 50 runs, the standard deviations were also computed and are reported in the figures. A summary of the task, task set, and test parameters is given in Table 3, Table 4, and Table 5, respectively.

The *wcet* parameter is a random variable with uniform distribution within a range [*wcet_min*, *wcet_max*].

The *ctime* parameter is computed from the *wcet*, by subtracting a factor *dw*, which is also

N_VAL	number of different values for a test
DELTA	parameter increment
N_IT	number of iterations for each value

Table 5: Test parameters

a random variable with uniform distribution within a range [*dw_min*, *dw_max*]. Deadline tolerances are also uniformly distributed in the interval [*tol_min*, *tol_max*].

Task arrival times are generated in sequence based on the arrival rate λ specified in the set parameters. The arrival time of task i is computed as:

$$atime(i) = atime(i-1) + r(1/\lambda, \sigma)$$

where $r(a, b)$ is a Gaussian random variable with mean a and variance b^2. Task deadline is calculated according to the workload specification given by the two parameters ρ and α:

$$d_{i+1} = d_i + \frac{c_{i+1}}{\rho} - r(\alpha\frac{c_{i+1}}{\rho}, \sigma).$$

Finally, the value of non critical tasks is defined as a random variable uniformly distributed in the interval $[1, N]$. The value of critical tasks is defined as CRIT_VALUE, which is a value greater than N. The number of critical tasks in the set is controlled by the *crit* parameter.

Default values were set at: $N = 50$, $\lambda = 0.2$ (a task every 5 ticks), $load = 0.9$, $\alpha = 0.5$, $crit = 0.2$ (20 percent of critical tasks), $wcet = 30$, $dw = 0$, $tol = 0$.

6.1 Experiment 1: Critical Factor

In the first experiment, we tested the capability of the algorithms of handling critical tasks in overload conditions. Figure 5a and 5b plot the Loss Critical Ratio (LCR) and the Loss Value Ratio (LVR) obtained for the four algorithms as a function of the critical factor. In this experiment, the initial workload was 0.9, with a growth factor $\alpha = 0.5$. For each task, the deadline tolerance was set to zero, and the computation time (*ctime*) was set equal to the estimated *wcet* ($dw = 0$).

As shown in Figure 5, both LVR and LCR for EDF go over 0.9 as soon as the critical factor become greater than 0.2. This is clearly due to the *domino effect* caused by the heavy load. Although the guarantee routine used in the GED algorithm avoids such a domino effect typical of the EDF policy, it does not work as well as RED nor MED, since critical tasks are rejected as normal hard tasks, if they cause an overload. For example, when the percentage of critical tasks is 50% (critical factor = 0.5) we see a gain of about 15% for RED and MED over GED.

Another important implication from Figure 5 is that RED and MED are able to provide almost no loss for critical tasks in overload conditions, until the number of critical tasks in the set is above 50% of the total number of tasks; the LCR is practically zero for both algorithms. Above this percentage, however, some loss is experienced and, by around 80% of the load being critical tasks, we start to see the multiple task rejection policy used in MED begin to be slightly more effective than RED.

To understand the behavior of RED and MED depicted in figure 5a, remember that the LVR is computed from the value of HARD tasks only, since CRITICAL tasks belong to a different class. Therefore, as the critical factor increases, RED and MED have to reject more HARD tasks to keep the LCR value low, whereas GED does not make any dinstiction between HARD and CRITICAL tasks.

As a matter of fact, an important result shown in this experiment is that for task sets in which the number of critical tasks is less than the number of hard tasks, it is not worthwhile

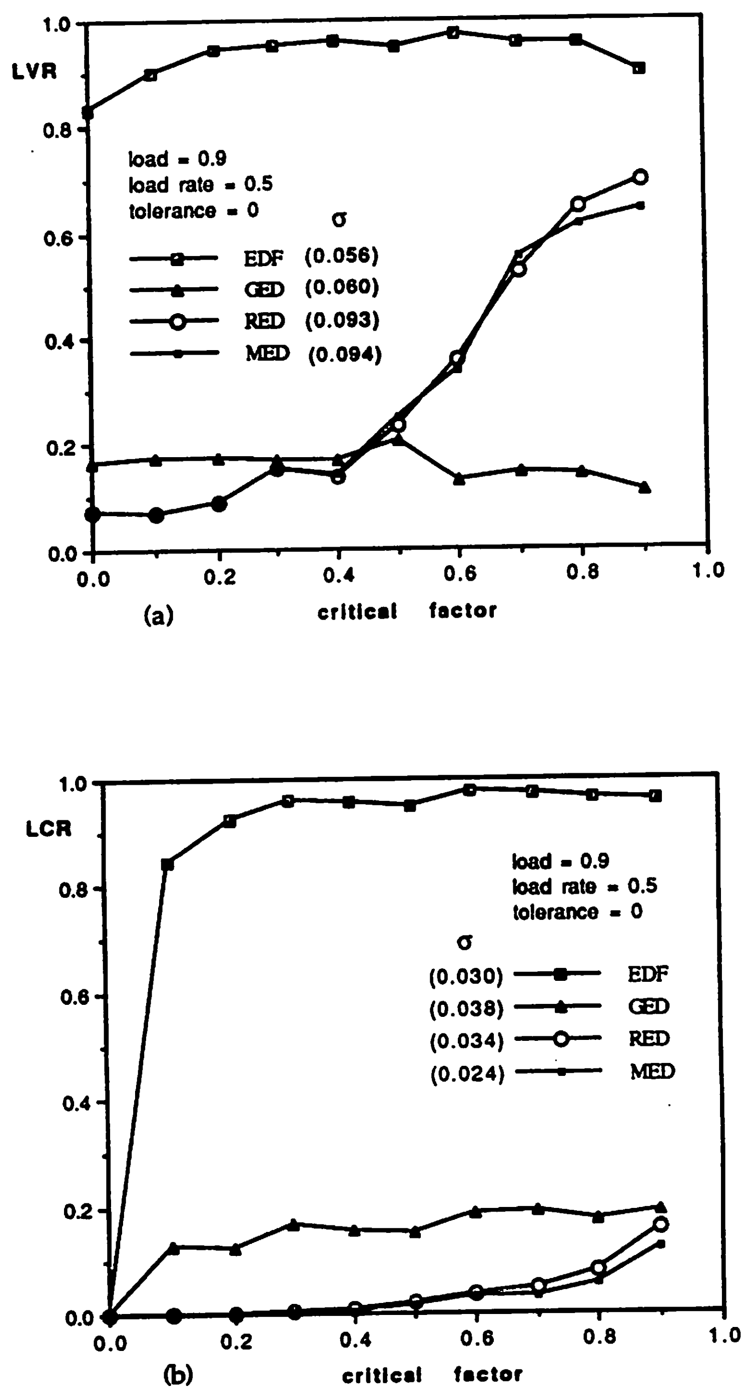


Figure 5: LVR vs critical factor (a); LCR vs critical factor (b).

to use complicated rejection strategies. In this cases, the simple ($O(n)$) strategy used in RED, in which the least value task is rejected, performs as well as more sophisticated and time consuming policies.

Notice that in all experiments presented in this paper, no assumption has been made on the minimum interarrival time of critical tasks. Therefore, even when the percentage of critical tasks is low, there is always a (low) probability that a critical task can be rejected with the MED algorithm, if it arrives just after another critical task and the deadlines of both are close. Note that this condition is an overload.

6.2 Experiment 2: Load Rate

In this experiment, the number of critical tasks was 20 percent of the total number of tasks (which means 10 in the average out of 50). The initial workload was 0.9, and the load growth factor α, was varied from 0 to 0.75, with a step of 0.05. For each task, the deadline tolerance was set to zero, and the computation time (ctime) was set equal to the estimated wcet (dw = 0).

Figure 6a plots the loss value ratio (LVR) obtained with the four algorithms as a function of the load growth factor α, and figure 6b plots the loss critical ratio (LCR). When $\alpha = 0$ the system workload is maintained on the average around its initial value $\rho = 0.9$, therefore the loss value is negligible for all algorithms. By increasing α, the load increases as new tasks arrive in the system.

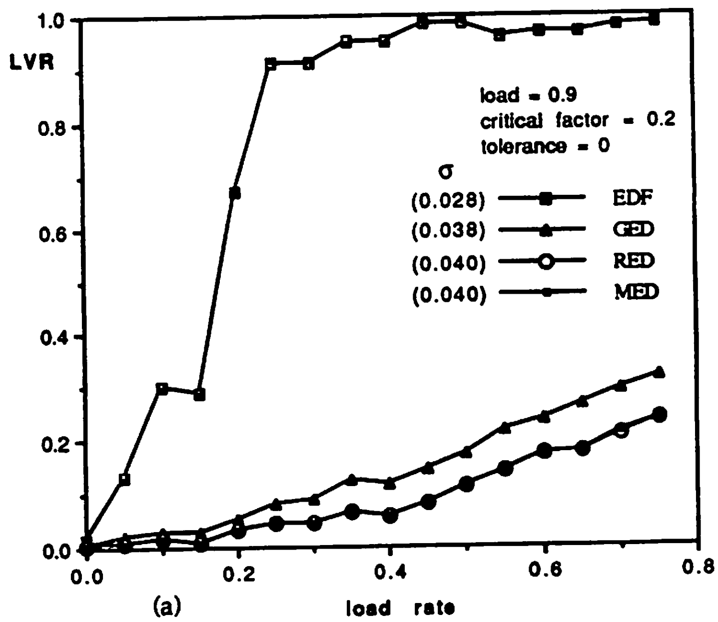
As it is clear from the figure, the EDF algorithm without guarantee was not capable of handling overloads, so that the loss in value increased rapidly towards its maximum (equal to the total set value). At this level, only the first tasks were able to finish in time, while all other tasks missed their deadlines. Again, RED and MED did not show any significant difference between themselves during this test, but a very significant improvement was achieved over EDF and GED. For example, using a growth factor $\alpha = 0.5$, which causes a heavy load, the LVR is 0.98 with EDF, 0.17 with GED, and only 0.11 for both RED and MED. Notice that the loss value obtained running RED and MED is entirely due to hard tasks, since from figure 5b we see that the number of critical tasks missing their deadlines is practically zero for RED and MED.

6.3 Experiment 3: Arrival Rate

In the third experiment, we monitored the loss value by varying the task arrival rate. In this case, task deadlines were generated in a slightly different fashion. Task laxity was generated first as a random variable uniformly distributed in the interval $[lax\_min, lax\_max]$. Then, task deadline was computed as follows:

$$deadline = arrival\_time \; + \; wcet \; + \; laxity.$$

As shown in the graphs reported in figure 7, the results of this test are consistent with those discussed in the previous experiment. So the results hold over a wide range of system conditions.



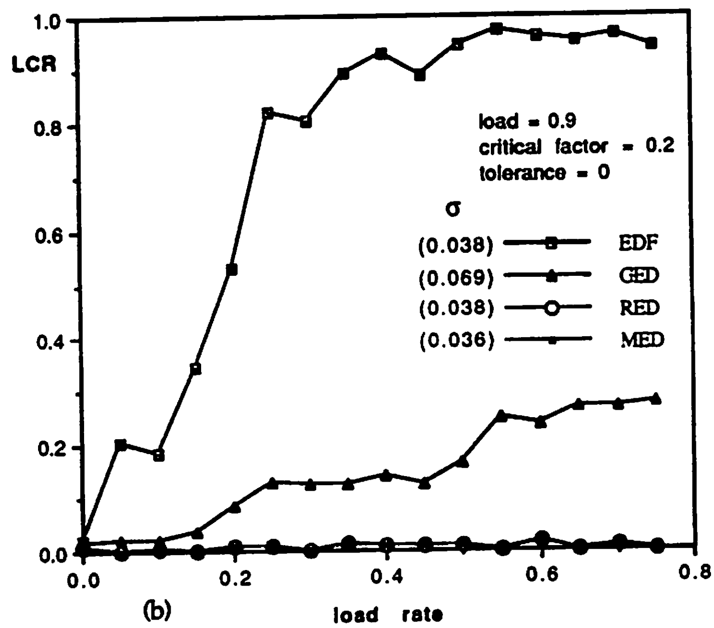
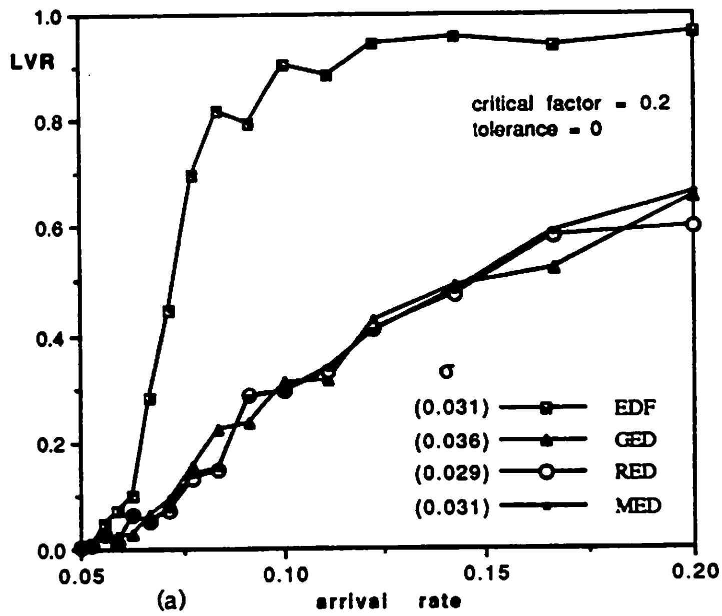


Figure 6: (a)LVR vs load rate; (b) LCR vs load rate.



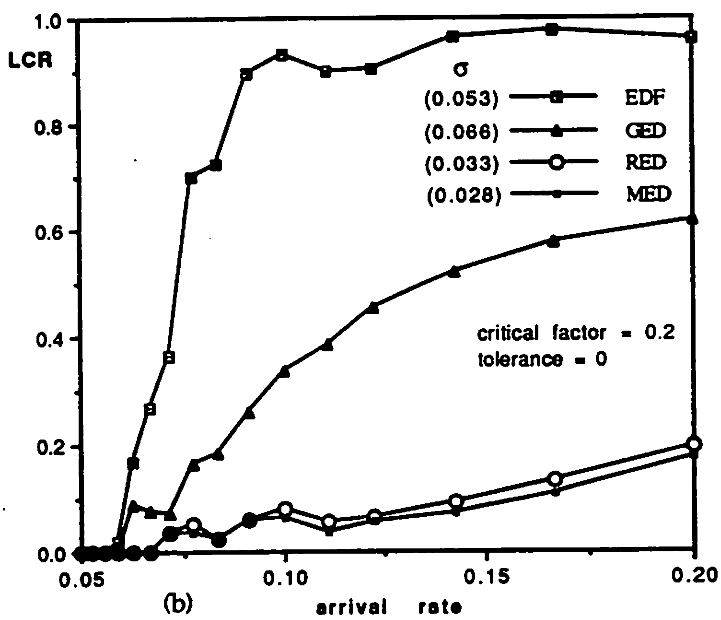


Figure 7: (a) LVR vs arrival rate λ; (b) LCR vs arrival rate λ.

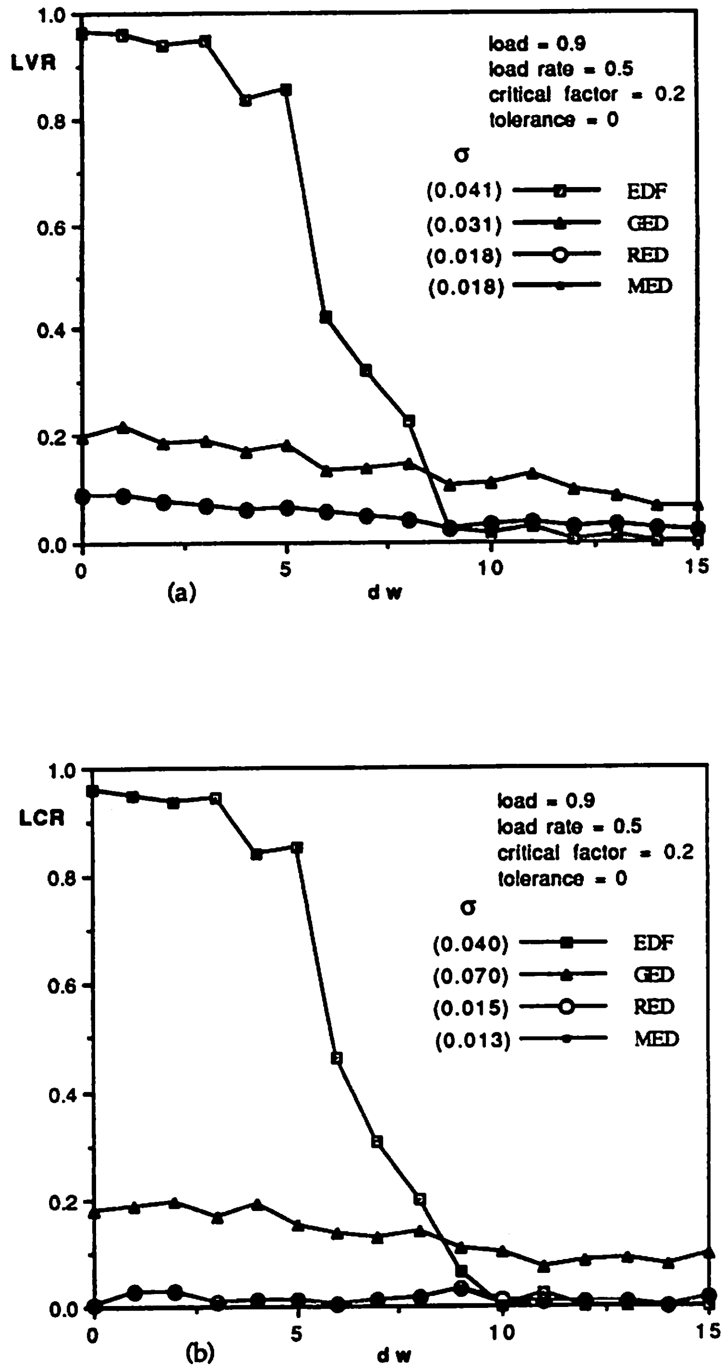


Figure 8: (a) LVR vs dw; (b) LCR vs dw.

6.4 Experiment 4: Early Completion

Experiment 4 is intended to show the effectiveness of the recovery strategy used in the RED (and MED) algorithm. When a task completes its execution before its estimated finishing time, the recovery strategy tries to reaccept rejected tasks (based on their values) until they have positive laxity. This is a very important feature of RED since, due to the pessimistic evaluation of the *wcet*, the actual computation time of a task may be very different from its estimated value. We call *computation time error dw* the difference between the estimated wcet and the actual computation time of a task. Figure 8a and 8b show the loss value ratio LVR and the loss critical ratio LCR as a function of *dw*. In this test, the initial load was set to 0.9, with a growth factor $\alpha = 0.5$. The number of critical tasks was 20 percent of the total number, and computation time error *dw* varied from 0 to 15, with an estimated *wcet* ranging from 30 to 40 time units.

Notice that, although EDF does not use any recovery strategy, EDF performs better than GED for high values of *dw*. This is due to the fact that for high computation time errors, the actual workload of the system is much less than the one estimated at the arrival time, so EDF is able to execute more tasks. On the other hand, GED cannot take advantage of saved time, since it rejects tasks at arrival time based on the estimated load. RED and MED also reject tasks based on the current estimated workload, but the recovery strategy makes use of saved execution time for reaccepting rejected tasks. Since tasks are reaccepted based on their value, RED and MED perform as well as EDF for large computation time errors.

For example, with an average computation time error of $dw = 5$, the LVR obtained with RED is about 3 times less than the one obtained with GED, and about 13 times less than the one obtained with EDF. For large errors, say $dw > 10$, EDF shows about the same performance of RED and MED, in terms of both LVR and LCR, whereas GED performs almost as before.

6.5 Experiment 5: Deadline Tolerance

Experiment 5 shows the effect of having a deadline tolerance. Remember that a task with deadline d_i and tolerance m_i is not treated as a task with deadline $(d_i + m_i)$: deadline is used for scheduling, and tolerance is used for guarantee. This means that the algorithm always tries to schedule all tasks to meet their deadlines; only in overload conditions there is a chance that some task may exceed its deadline. In order to compare the four algorithms in a consistent fashion, the concept of tolerance has been used also for EDF and GED. In this experiment, the initial load was 0.9, with a growth factor $\alpha = 0.2$. The number of critical tasks was 70 percent of the total number (i.e., about 35 out of 50), and tolerance level was varied from 0 to 15.

Figures 9a and 9b show the LVR and the LCR values for the four algorithms, as the tolerance level varies from 0 to 15. LCR can be considered as the probability for a critical task of missing its deadline. LCR values obtained with EDF were more than an order of magnitude bigger than those obtained with the other algorithms. Notice that, as the number of critical tasks in the set was relatively high, for low tolerance values, the MED algorithm performs better than RED, whereas for higher values the two algorithms have the same performance.

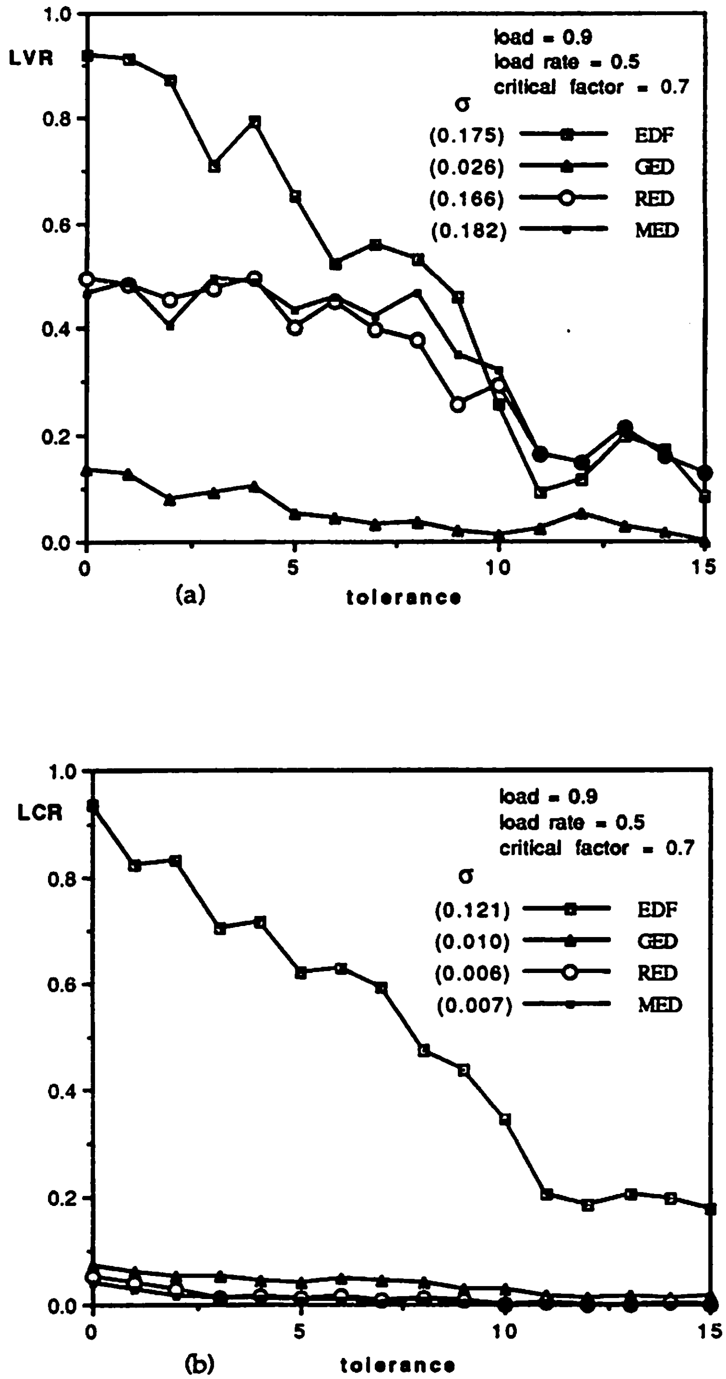


Figure 9: LVR vs tolerance (a); LCR vs tolerance (b).

For example, with a tolerance level of 5 units, the LCR factor is 0.62 with EDF, 0.042 with GED, 0.0014 with RED, and only 0.009 with MED. As we can see from the graph, for tolerance values greater than 10, RED and MED cause no late critical tasks.

Note from figure 9a that GED performs better than the other three algorithms in terms of LVR. This is due to the fact that RED and MED reject more hard tasks than GED, to keep the LCR as low as possible.

6.6 Performance Evaluation Summary

The performance results clearly show the poor performance of EDF scheduling in overload. This fact has been often stated, but rarely shown with performance data. The results also show the clear advantage of on-line planning (dynamic guarantee) algorithms under such conditions. More importantly, the results also show that the new RED algorithm is significantly better than even the basic guarantee approach because it uses the entire load profile, deadline tolerance, and resource reclaiming. The better performance occurs across different loads, deadline distributions, arrival rates, tolerance levels, and errors on the estimates of worst case execution time. RED also is significantly better than EDF and better than GED in handling unexpected overloads of critical tasks; an important property for safety critical systems. This implies that RED keeps the system safe, longer, when unanticipated overloads occur. We also see that the very simple rejection policy for RED suffices in almost all conditions, and that MED only improves RED in a small part of the parameter space.

7 Related Work

Earliest deadline scheduling has received much attention. It has been formally analyzed proving that it is an optimal algorithm for preemptive, independent tasks when there is no overload [Der 74], and proving that it is not optimal for multiprocessors [Mok 83]. It has been used in many real-time systems, often assuming that overload will not occur. However, it is now well known that earliest deadline scheduling has potentially catastrophically poor performance in overload. Various research has been performed attempting to rectify this problem. We briefly present EDF work related to overload as well as some general work on overload in real-time systems.

Ramamritham and Stankovic [Ram 84] used EDF to dynamically guarantee incoming work and if a newly arriving task could not be guaranteed the task was either dropped or distributed scheduling was attempted. All tasks had the same value. The dynamic guarantee performed in this paper had the effect of avoiding the catastrophic effects of overload on EDF. In [Biy 88] the previous work was extended to tasks with different values, and various policies were studied to decide which tasks should be dropped when a newly arriving task could not be guaranteed. However, the work did not fully analyze the overload characteristics, build a profile, allow deadline tolerance, nor integrate with resource reclaiming and re-guarantee, as we do in this paper.

Haritsa, Livny and Carey [Har 91] present the use of a feedback controlled EDF algorithm for use in real-time database systems. The purpose of their work is to get good average performance for transactions even in overload. Since they are working in a database environment their assumptions are quite different than ours, e.g., they assume no knowledge of transaction characteristics, they consider firm deadlines not hard deadlines, there is no guarantee, and no detailed analysis of overload. On the other hand, for their environment, they produce a very nice result, a robust EDF algorithm. The robust EDF algorithm we present is very different because of the different application areas studied, and because we also include additional features not found in [Har 91].

In real-time Mach [Tok 87] tasks were ordered by EDF and overload was predicted using a statistical guess. If overload was predicted, tasks with least value were dropped. Jeffay, Stanat and Martel [Jef 91] studied EDF scheduling for non-preemptive tasks, rather than the preemptive model used here, but did not address overload.

Other general work on overload in real-time systems has also been done. For example, Sha [Sha 86] shows that the rate monotonic algorithm has poor properties in overload. Thambidurai and Trivedi [Tha 89] study transient overloads in fault tolerant real-time systems, building and analyzing a stochastic model for such a system. However, they provide no details on the scheduling algorithm itself. Schwan and Zhou [Sch 92] do on-line guarantees based on keeping a slot list and searching for free time intervals between slots. Once schedulability is determined in this fashion, tasks are actually dispatched using EDF. If a new task cannot be guaranteed, it is discarded. Zlokapa, Stankovic and Ramamritham [Zlo 91] propose an approach called well-time scheduling which focuses on reducing the guarantee overhead in heavily loaded systems by delaying the guarantee. Various properties of the approach are developed via queueing theoretic arguments, and the results are a multi-level queue (based on an analytical derivation), similar to that found in [Har 91] (based on simulation).

In [Loc 86] an algorithm is presented which makes a best effort at scheduling tasks based on Earliest Deadline with a rejection policy based on removing tasks with the minimum value density. He also suggests that removed tasks remain in the system until their deadline has passed. The algorithm computes the variance of the total slack time in order to find the probability that the available slack time is less than zero. The calculated probability is used to detect a system overload. If it is less than the user prespecified threshold, the algorithm removes the tasks in increasing value density order. Consequently, detection of overload is performed in a statistical manner rather than based on an exact profile as in our work. This gives us the ability to perform specific analysis on the load rather than a probabilistic analysis. While many features of our algorithm are similar to Locke's algorithm, we extend his work, in a significant manner, by the careful and exact analysis of overload, support n classes, provide a minimum level of guarantee even in overload, allow deadline tolerance, more formally address resource reclaiming, and provide performance data for the impact of re-guarantees. We also formally prove all our main results.

Finally, Gehani and Ramamritham [Geh 91] propose programming language features to allow specification of deadline and a deadline slop factor (similar to our deadline tolerance), but propose no algorithms for supporting this feature.

8 Conclusions

We have developed a robust earliest deadline scheduling algorithm for hard real-time environments with preemptive tasks, multiple classes of tasks, and tasks with deadline tolerances.

We have formally proven several properties of the approach, developed an efficient on-line mechanism to detect overloads, and provided a complete load profile which can be usefully exploited in various ways. A performance study shows the excellent performance of the algorithm in both normal and overload conditions. Precedence constraints can be handled by *a priori* converting precedence constraints to deadlines. A future extension we are working on is to include resource sharing among tasks.

In this paper we assumed zero overheads for dynamic scheduling. This, of course, is not true. Basically, there are two distinct and important aspects to scheduling overhead, one is the cpu time it steals from application tasks, and two, is the latency it creates while making the decision. We advocate the use of a scheduling chip that does the dynamic guarantees and which runs in parallel with the application processor. This eliminates the first aspect of the overhead, i.e., scheduling no longer steals cpu cycles from application tasks. A scheduling chip would also minimize latency because it can be tailored to the scheduling function and therefore be quite fast. However, the worst case latency must still be taken into account and this can be accomplished by computing a cutoff line – a time equivalent to worst case scheduling cost. Then the planning does not modify the application processor schedule prior to the cutoff line. This two part approach to dealing with scheduling overhead works and has been implemented in the Spring Kernel [Sta 91]. The technique is easily applied to the algorithm in this paper.

References

[Bar 91] S. Baruah, G. Koren, B. Mishra, A. Raghunathan, L. Rosier, and D. Shasha, "On-line Scheduling in the Presence of Overload," *Proc. of IEEE Symposium on Foundations of Computer Science*, San Juan, Puerto Rico, October 2-4, 1991.

[Bar 92] S. Baruah, G. Koren, D. Mao, B. Mishra, A. Raghunathan, L. Rosier, D. Shasha, and F. Wang, "On the Competitiveness of On-Line Real-Time Task Scheduling," *Proceedings of Real-Time Systems Symposium*, December 1991.

[Biy 88] S. Biyabani, J. Stankovic, and K. Ramamritham, "The Integration of Deadline and Criticalness in Hard Real-Time Scheduling," *Proceedings of the Real-Time Systems Symposium*, December 1988.

[Bla 76] J. Blazewicz, "Scheduling dependent tasks with different arrival times to meet deadlines" In E. Gelenbe, H. Beilner (eds), Modelling and Performance Evaluation of Computer Systems, Amsterdam, North-Holland, 1976 pp 57-65.

[Che 90] H. Chetto, M. Silly, and T. Bouchentouf, "Dynamic Scheduling of Real-Time Tasks under Precedence Constraints," *The Journal of Real-Time Systems*, 2, pp. 181-194, 1990.

[Der 74] M. Dertouzos, "Control Robotics: The Procedural Control of Physical Processes," *Proceedings of the IFIP Congress*, 1974.

[Geh 91] N. Gehani and K. Ramamritham, "Real-Time Concurrent C: A Language for Programming Dynamic Real-Time Systems," *Real-Time Systems*, 3, pp. 377-405, 1991.

[Har 91] J. R. Haritsa, M. Livny, and M. J. Carey, "Earliest Deadline Scheduling for Real-Time Database Systems," *Proceedings of Real-Time Systems Symposium*, December 1991.

[Jac 55] J. Jackson, "Scheduling a Production Line to Minimize Tardiness," Research Report 43, Management Science Research Project, University of California, Los Angeles, 1955.

[Jef 91] K. Jeffay, D. Stanat, and C. Martel, "On Non-Preemptive Scheduling of Periodic and Sporadic Tasks," *Proceedings of Real-Time Systems Symposium*, December 1991.

[Liu 73] C. L. Liu and J. Leyland, "Scheduling Algorithms for Multiprogramming in a Hard Real-Time Environment," *Journal of the ACM*, 20, pp. 46-61, 1973.

[Loc 86] C. D. Locke, "Best Effort Decision Making For Real-Time Scheduling," PhD Thesis, Computer Science Dept., CMU, 1986.

[Mok 83] A. K. Mok, "Fundamental Design Problems of Distributed Systems for the Hard Real Time Environment," PhD Dissertation, MIT, May 1983.

[Moo 68] J. Moore, "An n Job, One Machine Sequencing Algorithm for Minimizing the Number of Late Jobs," *Management Science*, Vol. 15, No. 1, pp. 102-109, September 1968.

[Ram 84] K. Ramamritham and J. Stankovic, "Dynamic Task Scheduling in Hard Real-Time Distributed Systems," *IEEE Software*, pp. 65-75, July 1984.

[Sch 92] K. Schwan and H. Zhou, "Dynamic Scheduling of Hard Real-Time Tasks and Real-Time Threads," *IEEE Transactions on Software Engineering*, Vol. 18, No. 8, pp. 736-748, August 1992.

[Sha 86] L. Sha, J. Lehoczky, and R. Rajkumar, "Solutions for Some Practical problems in Prioritized Preemptive Scheduling," *Proceedings of Real-Time Systems Symposium*, December 1986.

[Sha 90] L. Sha, R. Rajkumar, J. P. Lehoczky, "Priority Inheritance Protocols: An Approach to Real-Time Synchronization," *IEEE Transactions on Computers*, Vol. 20, No. 9, pp. 1175-1185, September 1990.

[She 90] C. Shen, K. Ramamritham, and J. Stankovic, "Resource Reclaiming in Real-Time," *Proceedings of Real-Time Systems Symposium*, December 1990.

[Sta 91] J. Stankovic and K. Ramamritham, "The Spring Kernel: A New Paradigm for Real-Time Systems," *IEEE Software*, Vol. 8, No. 3, pp. 62-72, May 1991.

[Tha 89] P. Thambidurai, and K. S. Trivedi, "Transient Overloads in Fault-Tolerant Real-Time Systems," *Proceedings of Real-Time Systems Symposium*, December 1989.

[Tok 87] H. Tokuda, J. Wendorf, and H. Wang, "Implementation of a Time-Driven Scheduler for Real-Time Operating Systems," *Proceedings of the Real-Time Systems Symposium*, December 1987.

[Tow 91] D. Towsley and S. Panwar, "Optimality of the Stochastic Earliest Deadline Policy for the G/M/c Queue Serving Customers with Deadlines," Univ. of Massachusetts Technical Report, September 1991.

[Zlo 91] G. Zlokapa, J. A. Stankovic, and K. Ramamritham, "Well-Timed Scheduling: A framework for Dynamic Real-Time Scheduling," Submitted to *IEEE Transactions on Parallel and Distributed Systems*, 1991.