# Source Time Scale and Optimal Buffer/Bandwidth Trade-off for Regulated Traffic in an ATM Node

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**Abstract**

In this paper, we study the problem of resource allocation and control for an ATM node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability are considered. We investigate the relationship between source characteristics and the buffer/bandwidth trade-off under both services.

Our contributions are the following. For guaranteed lossless service, we find that the optimal resource allocation scheme suggests a time scale separation of sources sharing an ATM node with finite bandwidth and buffer space, with the optimal buffer/bandwidth trade-off is determined by the sources' time scale. For statistical service with a small loss probability, we present a new approach for estimating the loss probability in a shared buffer multiplexor with the so called "extremal" on-off, periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth trade-off is again determined by time scale separation.

Besides their obvious application to resource allocation and call admission control, our results have many other implications in network design and control such as network dimensioning and traffic shaping.

**Keywords:** *ATM, Call Admission Control, Network Dimensioning, Performance Bounds, Quality-of-Service Guarantees, Resource Allocation, Statistical Multiplexing*.

## 1 Introduction

Resource allocation is an extremely challenging and important problem in the design and control of high-speed networks such as ATM networks. The problem is particularly complicated by the need to support *Quality-of-Service* (QoS) guarantees for a variety of applications with very diverse traffic characteristics.

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In [EMW95], a new approach to resource allocation in an ATM node with fixed bandwidth and a finite, shared buffer is presented. In this approach, an ATM node is modeled by a shared buffer multiplexor with the so-called "extremal" on-off, periodic arrival processes, which account for the worst-case stochastic behavior (as proved for the bufferless multiplexor in [MM95, ZKST96]). The ingenuity of the approach is the reduction of the two-resource (*i.e.*, buffer and bandwidth) allocation problem to a single-resource allocation problem, *i.e.*, the known problem of estimating loss probability of a bufferless multiplexor. This reduction is made possible by introducing the concept of a virtual buffer/trunk system and establishing the "exchangeability" of buffer and bandwidth. Based on the analytical results, a qualitative theory is then described which provides many insights in call admission control.

Motivated and inspired by the work in [EMW95], we study source characteristics and their impact on buffer/bandwidth trade-off in the design and control of an ATM node. The starting point of our approach is the examination of optimal resource allocation schemes for guaranteed lossless service. We find that under such service, the optimal resource allocation scheme for an ATM node where each virtual circuit has its own allocated bandwidth and buffer space with no resource sharing (referred to as a lossless segregated system) is no different from that for an ATM node with all virtual circuits sharing the resources (*i.e.*, a lossless multiplexing system). Hence, in this case, there are no benefits in resource sharing, and the two systems are effectively equivalent. We also find that the optimal resource allocation scheme suggests an interesting separation of time scales among sources sharing an ATM node with finite bandwidth and buffer space, with the optimal buffer/bandwidth trade-off being determined by this time scale separation. Sources are classified as having either "fast" or "slow" time scales, reflecting the efficacy of either buffer sharing or bandwidth sharing among the sources.

For statistical service where a small loss probability is allowed, we derive a new approach to estimate the loss probability using our results for the optimal buffer/bandwidth trade-off obtained for lossless service. By giving a new interpretation to the virtual trunk/buffer systems introduced in [EMW95], we are able to transform the two-resource allocation problem into two independent single-resource allocation problems. The best buffer/bandwidth separation is explored by optimizing resource allocation along the optimal buffer/bandwidth trade-off curve. Through numerical examples, we demonstrate that source time scales also have a major impact on the optimal resource allocation under statistical service, and the optimal buffer/bandwidth trade-off is again reflected by the source time scale separation.

Our work differs from [EMW95] in several aspects. First, our perspectives on resource allocation and control problems are somewhat different. The authors in [EMW95] are primarily interested in call admission control. This is reflected in their fixing the system bandwidth $C$ and buffer space $B$. Resource allocation to each source is independent of the sources' characteristics. In our approach, we fix one resource (bandwidth $C$) and find the optimal allocation of the other resource (buffer space $B$). Furthermore, resource allocation is made according to the time scale separation of the system and the source' own time scale. Due to this difference in perspectives, we are able to study the role of source time scale and investigate optimal buffer/bandwidth trade-off for both lossless service and statistical service with a given loss probability. We are also able to explore the maximal benefits of both buffer sharing and bandwidth sharing. This is important, as the efficacy of buffer sharing and that of bandwidth sharing for sources with different time scales are quite different. Numerical examples indicate that our approach provides a better estimate of the system loss probability than [EMW95].

The remainder of this paper is organized as follows. We start with the optimal resource allocation problem for guaranteed lossless service in Section 2, and demonstrate the relationship between source time scale and optimal buffer/bandwidth trade-off. In Section 3, we study the optimal resource allocation problem for statistical service with small loss probability. We present a new approach to estimate the system loss probability by maximizing the efficacy of buffer and bandwidth sharing. In Section 4, numerical examples are presented to illustrate the relationship between source time scale and buffer/bandwidth trade-off. The effectiveness of our approach is demonstrated and comparison with the results in [EMW95] is also made. The paper is concluded in Section 5.

## 2 Guaranteed Lossless Service

The starting point of our study is the analysis of the optimal resource allocation scheme for guaranteed lossless service. Consider an ATM node with a total amount of bandwidth $C$ and buffer space $B$. Suppose there are $N$ virtual circuits sharing the node. Each virtual circuit is associated with a traffic source that is leaky bucket regulated [ATM, Tu86]. We consider the following two scenarios. In the first scenario, each virtual circuit is allocated a fixed portion of the total bandwidth and buffer space with no resource sharing among the virtual circuits. We call this system *lossless segregated system* (see Figure 1). In the second scenario, the resources are shared among the virtual circuits. We call such a system *lossless multiplexing system* (see Figure 2). We are interested in optimal resource allocation schemes that, for given bandwidth $C$, minimize the buffer requirement while ensuring that no virtual circuits ever incur losses in the above scenarios. Because of resource sharing, one may expect that the latter system requires less resources than the former for supporting lossless service. However, we show that for guaranteed lossless service, the optimal resource allocation schemes for both systems are the same. Hence in terms of resource requirements, the lossless multiplexing system is effectively equivalent to the lossless segregated system, and resource sharing in this case does not yield any saving in resources. Before we present the optimal resource allocation problems for the two systems,


Figure 1: Lossless Segregated System.

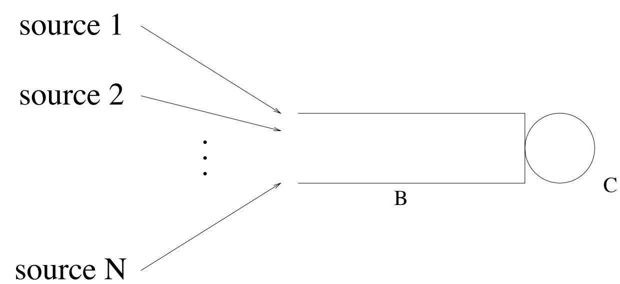



Figure 2: Lossless Multiplexing System.

we first describe the regulated traffic sources.

A leaky bucket regulator is characterized by three parameters: the token rate $\rho$, the token bucket size $\sigma$ and the peak rate $P$, where $P \geq \rho$. Let $A[\tau, \tau + t]$ denote the amount of traffic passing through the regulator in the time interval $[\tau, \tau + t]$. Then

$$A[\tau, \tau + t] \leq \mathcal{E}(t) := \min\{Pt, \sigma + \rho t\}, \ \tau \geq 0, \ t \geq 0 \tag{1}$$

where $\mathcal{E}(t)$ is called the minimum envelope process for the regulated source [Ch94]. It bounds the amount of traffic departing from the regulator during any time interval of length $t$.

Let $T_{on}$ denote the maximum length of a peak rate burst, *i.e.*,

$$T_{on} = \frac{\sigma}{P - \rho}. \tag{2}$$

A traffic source which generates traffic at peak rate $P$ for $T_{on}$ time and switches to rate $\rho$ for the rest of the time has a regulated traffic such that $A[0, t] = \mathcal{E}(t)$. We call such a traffic source *greedy*.

For the purpose of exposition, we assume that the $N$ traffic sources are classified into $J$ classes according to their regulator characterization, where all regulated sources in class $j$ have the same leaky bucket parameters $(\rho_j, \sigma_j, P_j)$, $1 \leq j \leq J$. There are $K_j$ Class $j$ sources, and $\sum_{j=1}^{J} K_j = N$. We assume that all classes have different peak rate burst length $T_{on}$. Without loss of generality, let $T_{on_1} > T_{on_2} > \cdots > T_{on_J}$. In order to have a stable system, we require that $\sum_{j=1}^{J} K_j \rho_j \leq C$. Also in order to avoid triviality, we assume that $C \leq \sum_{j=1}^{J} K_j P_j$.

### 2.1 Lossless Segregated System

We first consider the optimal resource allocation problem for the lossless segregated system (Figure 1). We fix the total bandwidth $C$ for the system, and consider allocation schemes that minimize the total buffer space required to ensure that no virtual circuits incur any losses.

For $j = 1, \ldots J$, suppose each source in class $j$ is allocated bandwidth $c_j$ and buffer space $b_j$. Let $C_j = K_j c_j$ and $B_j = K_j b_j$ denote, respectively, the total amount of bandwidth and buffer space allocated to class $j$ sources. The stability condition requires that $c_j \geq \rho_j$ for each $j$. Since the total bandwidth of the system is $C$, $\sum_{j=1}^{J} K_j c_j \leq C$. In order to ensure that no losses occur for any virtual circuit, the amount of buffer space $b_j$ allocated to a class $j$ source is determined by the maximum queue length for each segregated virtual circuit $j$, *i.e.*,

$$b_j \geq \sup_{t \geq 0} \{\mathcal{E}_j[t] - c_j t\} = \sigma_j - T_{on_j}(c_j - \rho_j). \tag{3}$$

The overall buffer space required to ensure that no virtual circuits encounter losses is thus $B_{seg} = \sum_{j=1}^{J} K_j b_j$. This determines the buffer requirement under the segregated allocation scheme.

Given the linearity of (3), the optimal buffer allocation problem can be formulated as the following Linear Programming (LP) problem:

**Problem** Minimize $B_{seg} = \sum_{j=1}^{J} K_j b_j = \sum_{j=1}^{J} K_j \sigma_j - \sum_{j=1}^{J} T_{on\,j}(C_j - K_j \rho_j)$

subject to:

$$\begin{aligned} b_j &= \sigma_j - T_{on_j}(c_j - \rho_j), \quad j = 1, \ldots, J \\ \sum_{j=1}^{J} C_j &= \sum_{j=1}^{J} K_j c_j \leq C, \\ \rho_j &\leq c_j \leq P_j, \quad j = 1, \ldots, J. \end{aligned}$$

The first term in the objective function denotes the buffer requirement if the total bandwidth is equal to the aggregate average rate of the sources, $\sum_{j=1}^{J} K_j \rho_j$, and the second term accounts for the buffer space reduction resulting from $C$ exceeding the aggregate average rate. By removing the first term (which is constant) from the objective function and reversing its sign, we can rewrite the optimization problem as follows[1]:

**Problem** Maximize $\sum_{j=1}^{J} T_{on\,j}(C_j - K_j \rho_j)$

subject to:

$$\begin{aligned} \sum_{j=1}^{J} C_j &\leq C, \\ K_j \rho_j \leq C_j &\leq K_j P_j, \quad j = 1, \ldots, J \end{aligned}$$

where note the new objective function is now to be maximized.

It is clear that the new objective function increases whenever bandwidth is taken from classes with smaller $T_{on}$ and is allocated to classes with larger $T_{on}$. As a consequence, the optimal allocation scheme consists of allocating

[1] We can also make the variable substitution $C_j' = C_j - K_j \rho_j$, $j = 1, \ldots, J$, in order to rewrite the LP problem in the standard form. But for the sake of clarity, we leave the problem in the present form.

peak rate to as many classes with large $T_{on}$ as possible without violating the constraint $\sum_{j=1}^{J} C_j \leq C$, while allocating only average rate to classes with small $T_{on}$. Formally, let $k$ be the smallest index such that

$$\sum_{j=1}^{k-1} K_j P_j + \sum_{j=k}^{J} K_j \rho_j \leq C < \sum_{j=1}^{k} k_j P_j + \sum_{j=k+1}^{J} K_j \rho_j. \tag{4}$$

Then the optimal resource allocation scheme that results in the minimum buffer requirement for the given bandwidth $C$ is as follows:

$$c_j = \begin{cases} P_j, & j = 1, \ldots, k-1, \\ \frac{C - \sum_{l \neq k} K_l c_l}{K_j}, & j = k, \\ \rho_j, & j = k+1, \ldots, J \end{cases} \tag{5}$$

and

$$b_j = \begin{cases} 0, & j = 1, \ldots, k-1, \\ \sigma_j - T_{on_j}(c_j - \rho_j), & j = k, \\ \sigma_j, & j = k+1, \ldots, J. \end{cases} \tag{6}$$

Note that $\sum_{j=1}^{J} C_j = \sum_{j=1}^{J} K_j c_j = C$, and the buffer requirement $B_{seg}$ has the following closed-form expression in terms of the regulated source parameters

$$B_{seg} = \sum_{j=k+1}^{N} K_j \sigma_j + [K_k \sigma_k - K_k T_{on_k}(c_k - \rho_k)]. \tag{7}$$

## 2.2 Lossless Multiplexing System

We now consider the optimal resource allocation problem for the lossless multiplexing system. Again we fix the total bandwidth of the system which is shared by all virtual circuits, and determine the minimal buffer space required to guarantee no losses. This shared buffer multiplexing system is equivalent to a shared single queue serviced by a server of capacity $C$. Given the regulated traffic sources defined earlier, the maximum queue length of the system is given by the following expression

$$Q_{max} \leq \sup_{t \geq 0} \left\{ \sum_{j=1}^{J} K_j \mathcal{E}_j(t) - Ct \right\} \tag{8}$$

where the equality is attained when all sources are greedy and start at the same time. Hence in order to ensure that no losses occur in the system, a minimum buffer size $B_{mux} = Q_{max}$ is required.

We proceed to derive a closed-form expression for $B_{mux}$ in terms of the parameters of the regulated sources. Note that since $\mathcal{E}_j(t)$ is piece-wise linear and concave for each $j$, so is $F(t) := \sum_{j=1}^{J} K_j \mathcal{E}_j(t) - Ct$. The maximum of $F(t)$ is attained at a point $t_{max}$ such that at $t_{max}$, the left derivative $\frac{d^- F}{dt}(t_{max}) \geq 0$ and the right derivative

$\frac{d^+F}{dt}(t_{max}) \leq 0$. By substituting the expression for $\mathcal{E}_j(t)$, we readily obtain that $t_{max} = T_{on_k}$, where $k$ is exactly as defined in (4). Specifically, $k$ is the smallest index such that

$$\sum_{j=1}^{k-1} K_j P_j + \sum_{j=k}^{J} K_j \rho_j \leq C < \sum_{j=1}^{k} K_j P_j + \sum_{j=k+1}^{J} K_j \rho_j. \tag{9}$$

This gives exactly the same index $k$ as in the lossless segregated system. In this case, however, $T_{on_k}$ has the following physical meaning: $T_{on_k}$ is the time the system reaches its maximum queue length when all sources are greedy.

From (8), we derive that the minimum buffer requirement is

$$B_{mux} = \sum_{j=k+1}^{J} K_j \sigma_j + [K_k \sigma_k - T_{on_k}(C - \sum_{j=1}^{k-1} K_j P_j + \sum_{j=k}^{J} K_j \rho_j)] \tag{10}$$

Comparing (7) and (10) we observe that $B_{seg} = B_{mux}$. Hence the minimum buffer requirement for the lossless multiplexing system is exactly the same as for the lossless segregated system. Thus we can define $B_{min} = B_{seg} = B_{mux}$ as the buffer requirement for lossless service.

From (9), we observe that if, for $1 \leq j \leq J$, we define $c_j$ and $b_j$ as in (5) and (6), then $\sum_{j=1}^{J} K_j c_j = C$ and $\sum_{j=1}^{J} K_j b_j = B_{min}$. This observation provides the following interesting interpretation as to how the system resources are optimally shared among the regulated sources in the lossless multiplexing system. Specifically, when resources are optimally allocated in the lossless multiplexing system, the virtual circuits behave *as if each of them was allocated fixed bandwidth* $c_j$ *and fixed buffer space* $b_j$, just as in the lossless segregated system. Hence the lossless multiplexing system can be effectively treated as if it were the lossless segregated system. This observation provides a motivation for the approach we take in Section 3 for studying resource allocation schemes under statistical multiplexing with small loss probability.

### 2.3 Source Time Scale and Optimal Buffer/Bandwidth Trade-off Curve

This far we have studied the resource allocation problem by fixing the bandwidth $C$. Now we consider the buffer/bandwidth trade-off for lossless service with a given set of regulated sources.

For any bandwidth $C$ such that $\sum_{j=1}^{J} K_j \rho_j \leq C \leq \sum_{j=1}^{J} K_j P_j$, the index $k$ defined in (9) is determined solely by the regulated source parameters and plays a key role in determining the minimal buffer requirement $B_{min}$ (see (7)). Hence in order to study the buffer/bandwidth trade-off, it suffices to study $k$ as a function of $C$. From (4), we see that $k$ is non-decreasing in $C$: $k = 1$ when $C = \sum_{j=1}^{J} K_j \rho_j$, and $k = J$ when $C = \sum_{j=1}^{J} K_j P_j$. As a consequence, from (5) and (6), we have that for class $j$ sources, $j = 1, \ldots, J$,

$$\frac{dc_j}{dC} \geq 0 \text{ and } \frac{db_j}{dC} \leq 0. \tag{11}$$

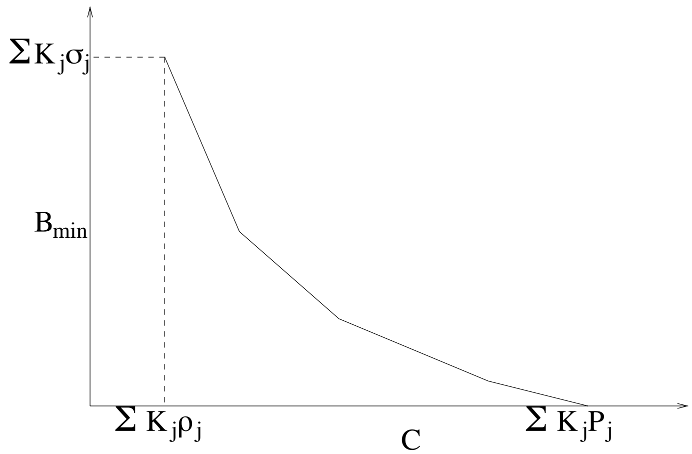



Figure 3: Optimal buffer/bandwidth trade-off curve.

That is, the allocated bandwidth $c_j$ to class $j$ sources is a non-decreasing function of $C$, whereas the buffer space $b_j$ allocated to these sources is a non-increasing function of $C$. Therefore, the buffer requirement $B_{min}$ is a decreasing function of $C$. Moreover, from (5) and (6), we obtain that

$$\frac{dB_{min}}{dC} = -T_{on_k}. \tag{12}$$

Thus, the buffer requirement $B_{min}$ is a piece-wise linear, decreasing convex function of $C$(see Figure 3). We call the curve $(C, B_{min})$ in Figure 3 the buffer/bandwidth trade-off curve for lossless service.

The optimal resource allocation scheme also suggests a taxonomy of the regulated sources according to their maximum peak rate burst length $T_{on_j}$, which we shall also refer to as the time scale of the regulated sources. Subsequently, we call the index $k$ the source *time scale index* with respect to $(C, B_{min})$. Sources in class $j$ are said to have either have "fast" time scale or "slow" time scale with respect to $(C, B_{min})$ according to whether $T_{on_j} < T_{on_k}$ or $T_{on_j} > T_{on_k}$. Under the optimal resource allocation scheme, we see that the most efficient way to accommodate "fast" time scale sources under lossless service is to allocate minimum amount of bandwidth (equal to their mean rates) and maximum buffer space (equal to their token bucket sizes), while the most efficient way to accommodate "slow" time scale sources is to allocate maximum bandwidth (equal to their peak rate) and zero buffer space. Sources in class $k$ have time scales in between, and accordingly, buffer/bandwidth trade-off for them are determined by the relation (12). Clearly, with larger $T_{on_k}$, increasing the bandwidth allocation to class $k$ sources will drastically reduce their buffer requirement. Thus, the optimal resource allocation scheme reveals a very interesting relationship between the source time scale and the buffer/bandwidth trade-off. In Section 4, we will present numerical examples to illustrate this relationship.

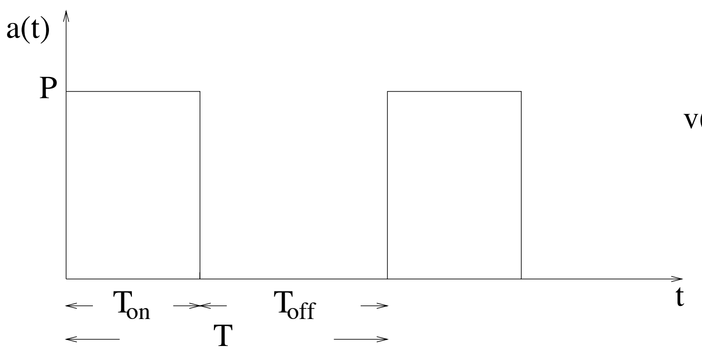



Figure 4: Extremal on-off, periodic curve.

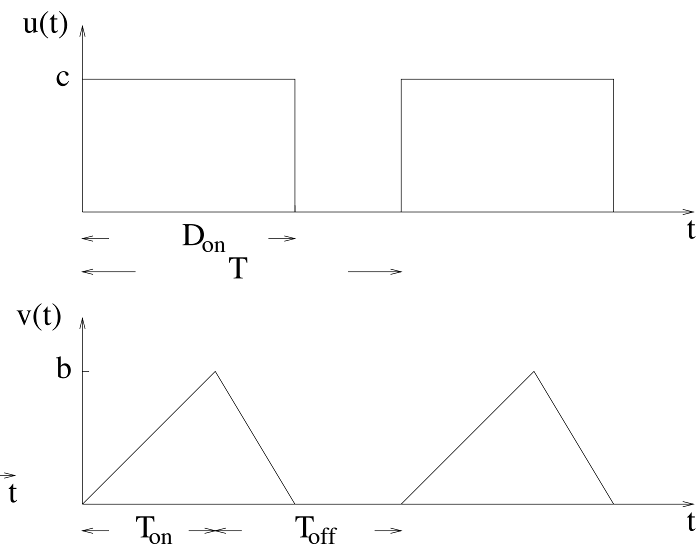



Figure 5: Buffer/bandwidth separation.

## 3 Statistical Service with Small Loss Probabilities

In the preceding section we established that under guaranteed lossless service, the optimal resource allocation is the same, regardless of whether resources are shared among sources. In this section, we study the benefits of resource sharing under statistical service where a small probability of loss, say, $10^{-9} \sim 10^{-7}$, is allowed, and investigate the buffer/bandwidth trade-off under such statistical service.

Once again, consider an ATM node with $N$ virtual circuits. Each virtual circuits is associated with a leaky bucket regulated traffic source. Suppose the node has a total amount of bandwidth $C$ and buffer space $B$, shared by all the sources. We assume that the system resources are not sufficient to provide guaranteed lossless service. In other words, the lossless service buffer requirement $B_{min}$ for the given value of bandwidth $C$ exceeds $B$. This implies that $(C, B)$ lies below the buffer/bandwidth trade-off curve in Figure 3. In this section, we explore the possibility of exploiting statistical multiplexing gains by considering statistical service with small loss probabilities.

Statistical multiplexing gains can be extracted by exploiting the bursty nature and statistical independence of traffic sources [EMW95]. To exploit the bursty nature of traffic sources regulated by leaky buckets with parameters $(\sigma, \rho, P)$, we follow [EMW95] and assume that the regulated sources after passing through leaky bucket regulators are extremal on-off, periodic processes (see Figure 4), where, when a source is active, it generates data at the peak rate $P$ until the depletion of its token bucket; it then stays inactive until the token bucket is completely filled again. Use of such processes are justified to a large extent by the work of [Do93, MM95, Wo94, YSS92, ZKST96]. In particular, such processes account for the *worst-case* statistical behavior in a bufferless multiplexor in the sense that they maximize the average loss rate [ZKST96] and the loss probability estimated by the Chernoff bound [MM95, ZKST96].

Let $a(t)$ denote an extremal on-off, periodic departure process from a leaky bucket regulator with parameters $(\rho, \sigma, P)$ as shown in Figure 4. Then the lengths of the on and off periods are $T_{on} = \frac{\sigma}{P-\rho}$ and $T_{off} = \frac{\sigma}{\rho}$. The source period is $T = T_{on} + T_{off} = P/\rho T_{on}$. Let $S$ denote the total amount of data generated during the on period. Then $S = PT_{on} = \sigma \frac{P}{P-\rho}$. $S$ is known as the source *burst-size*.

In order to model the statistical independence of traffic sources, we introduce indeterminate "phases" to the sources as in [EMW95]. Assume that traffic sources are grouped into $J$ classes, and for $1 \le j \le J$, there are $K_j$ sources in class $j$, $\sum_{j=1}^{J} K_j = N$. Each source $i$ of class $j$, $a_{ji}(t)$, is an extremal on-off, periodic process with leaky bucket parameters $(\rho_j, \sigma_j, P_j)$, but having an associated phase $\theta_{ji}$, *i.e.*, $a_{ji}(t) = a_j(t + \theta_{ji})$. For $1 \le j \le J$ and $1 \le i \le K_j$, the phases $\theta_{ji}$ are independent random variables uniformly distributed in the interval $[0, T_j]$.

In order to provide robust service, it is imperative to estimate the loss probability $P_{loss}$ at the ATM node due to buffer overflow. However, the loss probability of such a two-resource system with the given on-off sources is very difficult to compute directly. Several bounding and approximate approaches has been developed [KB92, NRST91, RV89, RV91], based on the so-called Benes approach [Be63]. A new approach is presented in [EMW95] by trading one resource for the other, using the notion of the virtual buffer/trunk.

Based on the work of [EMW95], in this section, we present a new approach to the problem of estimating system loss probability by transforming the two-resource problem into two independent single-resource problems which allows us to explore the optimal trade-off between buffer and bandwidth. This reduction is achieved via a virtual lossless segregate system functioning as a "resource separator", a new interpretation of the notion of the virtual buffer/trunk system introduced in [EMW95].

The rest of this section is organized as follows. In Section 3.1, we introduce the concept of virtual buffer/trunk system. In Section 3.2, we describe our approach for determining the optimal buffer/bandwidth separation and trade-off. The effectiveness of our approach is evaluated via numerical examples in section 4.

### 3.1 Virtual Buffer/Trunk System

Consider a virtual circuit with a single extremal on-off, periodic traffic source $a(t)$ characterized by parameters $(\rho, \sigma, P)$. Suppose it is allocated a trunk of bandwidth of $c$ where $\rho \le c \le P$. Then the maximum backlog at the virtual circuit is $b = \sigma - T_{on}(c - \rho) = T_{on}(P - c)$. Hence if the virtual circuit is allocated buffer space $b$, no losses will occur. Following [EMW95], we call such a virtual circuit a *virtual* buffer/trunk system.

Let $u(t)$ and $v(t)$ denote, respectively, the utilized bandwidth and the buffer content of the virtual circuit at time $t$. As shown in Figure 5, the two processes $u(t)$ and $v(t)$ are periodic with period $T$, the source period. The buffer fills at rate $P - c$ during a source's on period, reaches $b$ at the end of the source on period, and then, at the onset of an off period, depletes at rate $c$ until it becomes empty. The utilized bandwidth is $c$ whenever the buffer is not empty and 0 otherwise. Let $D_{on}$ denote the time in each cycle that the system is busy, *i.e.*, the buffer is not empty; $D_{on}$ exceeds the length of the source on period by the time required to deplete the buffer. Thus,

$$D_{on} = T_{on} + \frac{b}{c}. \tag{13}$$

We can view the virtual buffer/trunk system as a "resource separator" as it splits the traffic process $a(t)$ into two separate processes $u(t)$ and $v(t)$, representing, respectively, the bandwidth requirement and buffer requirement of the source at time $t$. Under this interpretation, by varying the trunk bandwidth $c$, the virtual buffer/trunk system can "regulate" the source's buffer and bandwidth requirements, thus providing an interesting buffer/bandwidth trade-off when allocating resources. When $c$ is increased, the source's bandwidth requirement during the system busy period is also increased. However, the system busy period is shortened and the source' buffer requirement is decreased. When $c$ is increased to the source's peak rate $P$, the system busy period $D_{on}$ equals the source on period $T_{on}$, during which the bandwidth requirement is $P$ while the buffer requirement is reduced to zero at all times. On the other hand, when $c$ is decreased, the reverse is true. In particular, when $c$ is decreased to the source's average rate $\rho$, the system is always busy and the buffer is never empty. Thus the source's bandwidth requirement is $\rho$ at all times, and the buffer requirement is uniformly distributed in $[0, \sigma]$.

The effect of varying $c$ on $u(t)$ and $v(t)$ can be concisely stated using the theory of stochastic orderings. The details are presented in Appendix A.

### 3.2 Estimating Loss Probability for Statistical Service

Recall that we are assuming that the system resources are not sufficient to support lossless service, i.e., $(C, B)$ lies below the buffer/bandwidth trade-off curve in Figure 7. In this section, we present a new approach for estimating the system loss probability in such cases. This approach exploits the buffer/bandwidth separation determined by a virtual buffer/trunk system and exploits the optimal buffer/bandwidth trade-off curve for lossless service.

Consider a lossless segregated system with a total amount of bandwidth $C_v$ and buffer space $B_v$ where $(C_v, B_v)$ lies on the buffer/bandwidth trade-off curve. Because the system resource pair $(B, C)$ lies below the buffer/bandwidth

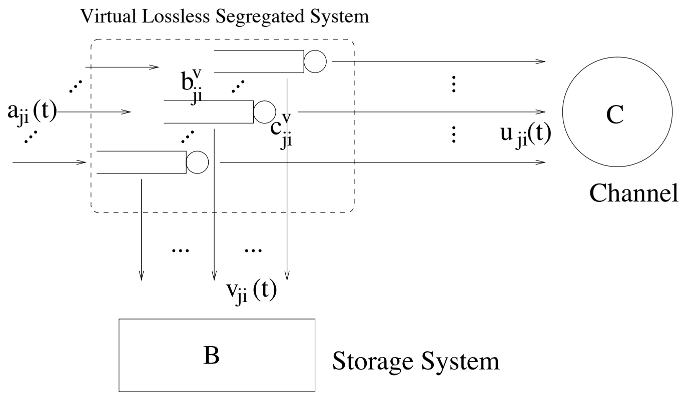



Figure 6: Buffer/bandwidth separation via virtual lossless segregated system.

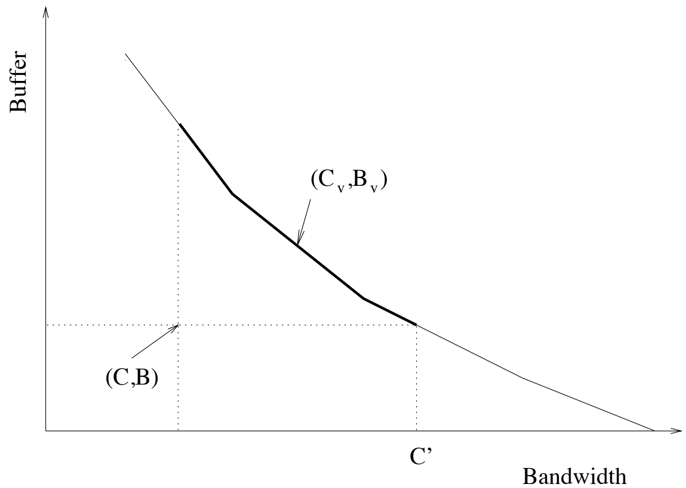



Figure 7: Relationship between $(C, B)$ and the buffer/bandwidth trade-off curve $(C_v, B_v)$.

trade-off curve, we must have either $C_v > C$ or $B_v > B$ or both. We call such a system a *virtual* lossless segregated system. In the virtual lossless segregated system, each source $i$ of class $j$, $a_{ji}(t)$, $1 \leq j \leq J, 1 \leq i \leq K_j$, has a trunk of fixed bandwidth $c^v_j$ and a buffer of fixed size $b^v_j$ such that $\sum_{j=1}^{J} K_j c^v_j = C_v$, $\sum_{j=1}^{J} K_j b^v_j = B_v$, and the resources $c^v_j$ and $b^v_j$ are allocated to each virtual buffer/trunk system according to the optimal resource allocation scheme described in Section 2.1. Hence no sources suffer any losses in the virtual buffer/trunk systems.

Let $u_{ji}(t)$ and $v_{ji}(t)$ denote the utilized bandwidth and the buffer contents of source $a_{ji}(t)$ in the virtual buffer/trunk system, where $u_{ji}(t)$ and $v_{ji}(t)$ are two periodic processes synchronized with source $a_{ji}(t)$ (*i.e.*, they all have the same phase $\theta_{ji}$). Interpreting the virtual buffer/trunk system as a "resource separator", then $u_{ji}(t)$ and $v_{ji}(t)$ represent, respectively, the bandwidth and buffer consumed by source $a_{ji}(t)$ at time $t$. Thus at any time $t$, the total bandwidth requirement of all sources is $U(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}(t)$, while the total buffer requirement of all sources is $V(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}(t)$. The virtual lossless segregated system separates the bandwidth and buffer requirements of the sources, thus enabling us to treat them separately.

By imagining that the traffic sources go through a virtual lossless segregated system that separates their bandwidth and buffer requirements, we reduce the difficult task of estimating the system loss probability in a buffered multiplexor with finite resources into that for two simpler systems: a trunk with bandwidth $C$ (but no buffer) and a storage system with buffer space $B$ (but no server)(see Figure 6 for an illustration). *At any time $t$, the sources demand a total amount of bandwidth $U(t)$ from the trunk and a total amount of buffer space $V(t)$ from the storage system. The sources will incur losses if either $U(t) > C$ or $V(t) > B$.* Therefore, we can use the probability that either event occurs as an upper bound on the loss probability $P_{loss}$ of the real system. (This buffer/bandwidth separation approach for estimating the system loss probability is justified and made rigorous in Appendix B.) By choosing different resource pairs $(C_v, B_v)$ along the buffer/bandwidth trade-off curve, the virtual lossless segregate system "regulates" the sources' bandwidth and buffer requirements, thus providing a trade-off between them. The rest of this section is devoted to the determination of the resource pair $(C_v, B_v)$ that optimizes the system loss probability estimate.

Let $u_{ji}$ and $v_{ji}$ be two random variables that represent respectively the instantaneous bandwidth requirement and buffer requirement of source $a_{ji}$ at a random time. Then $u_{ji}$ is a Bernoulli random variable taking value $c_j^v$ with probability $w_j = \frac{\rho_j}{c_j^v}$ and 0 with probability $1 - w_j$, and $v_{ji}$ has the distribution $Pr\{v_{ji} \le x\} = 1 - w_j + w_j \frac{x}{b_j^v}$, $0 \le x \le b_j^v$ (see Appendix A). Moreover, $u_{ji}, 1 \le j \le J, 1 \le i \le K_j$, are all independent, as are $v_{ji}, 1 \le j \le J$, $1 \le i \le K_j$.

Define $U = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}$ and $V = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}$. From the above discussion it follows that

$$P_{loss} \le \Pr\{V > B \text{ or } U > C\} \le \Pr\{V > B\} + \Pr\{U > C\}. \tag{14}$$

Since (14) is valid for any choice of a resource pair $(C_v, B_v)$ on the buffer/bandwidth trade-off curve, we have

$$P_{loss} \le \inf_{(C_v, B_v)} \{\Pr\{U > C\} + \Pr\{V > B\}\}. \tag{15}$$

The minimization problem on the right hand side of (15) reveals an interesting trade-off between buffer and bandwidth in the virtual buffer/trunk systems. Intuitively, the resource pair $(C_v^*, B_v^*)$ that minimizes the probability on the right hand side of (15) represents the "best" separation of bandwidth and buffer requirements of the resources. This separation is obtained by optimizing the resource allocation along the optimal buffer/bandwidth trade-off curve.

For any $C_v$, $\Pr\{U > C\}$ and $\Pr\{V > B\}$ can be estimated using the Chernoff bound. For $1 \le j \le J$, let $M_{u_j}(\theta) = E[e^{\theta u_{ji}}]$ and $M_{v_j}(\theta) = E[e^{\theta v_{ji}}]$ denote the moment generating functions of $u_{ji}$ and $v_{ji}$. Then $M_{u_j}(\theta) = E[e^{\theta u_{ji}}] = 1 - w_j + w_j e^{\theta c_j^v}$ and $M_{v_j}(\theta) = E[e^{\theta v_{ji}}] = 1 - w_j + \frac{1}{b_j^v \theta} w_j (e^{\theta c_j^v} - 1)$, where $(c_j^v, b_j^v)$ is the resource allocation to a source of class $j$ under the optimal resource allocation scheme with total bandwidth $C_v$. Define $\Lambda_{C_v}(\theta) = \sum_{j=1}^{J} \kappa_j \log M_{u_j}(\theta)$ and $\Lambda_{B_v}(\theta) = \sum_{j=1}^{J} \kappa_j \log M_{v_j}(\theta)$ where $\kappa_j = \frac{K_j}{N}$, $1 \le j \le J$, and $\sum_{j=1}^{J} \kappa_j = 1$. Then Chernoff bound [Bi86] yields

$$\Pr\{U > C\} = \Pr\left\{ \sum_{j=1}^{J} \sum_{i=1}^{K_J} u_{ji} > C \right\} \le e^{-N\Lambda_{C_v}^*(C/N)} \tag{16}$$

and

$$\Pr\{V > B\} = \Pr\left\{ \sum_{j=1}^{J} \sum_{i=1}^{K_J} v_{ji} > B \right\} \le e^{-N\Lambda_{B_v}^*(B/N)} \tag{17}$$

where

$$\Lambda^*_{C_v}(\alpha) = \sup_{\theta\geq 0}\{\theta\alpha - \Lambda_{C_v}(\theta)\} \text{ and } \Lambda^*_{B_v}(\alpha) = \sup_{\theta\geq 0}\{\theta\alpha - \Lambda_{B_v}(\theta)\}. \tag{18}$$

Hence,

$$P_{loss} \leq \inf_{(C_v,B_v)} \left\{ e^{-N\Lambda^*_{C_v}(C/N)} + e^{-N\Lambda^*_{B_v}(B/N)} \right\}. \tag{19}$$

The optimization problem can be greatly simplified by considering the asymptotic scaling $N \to \infty$ with $C/N = \overline{c}, B/N = \overline{b}$ and $K_j/N = \kappa_j, 1 \leq j \leq J$, held constant. In this case, we have that

$$\lim_{N\to\infty} \frac{1}{N} \log P_{loss} \leq - \sup_{(C_v,B_v)} \left\{ \min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\} \right\}. \tag{20}$$

Actually we can show (see Appendix C) that the choice of $(C_v, B_v)$ can be restricted to the segment of the buffer/bandwidth trade-off curve where $C_v \geq C$ and $B_v \geq B$ (the highlighted segment on the buffer/bandwidth trade-off curve in Figure 7). Let $\mathcal{C} = [C, C']$ be the corresponding range of $C_v$, *i.e.*, $C_v \in \mathcal{C}$ *iff* $B_v \geq B$ and $C_v \geq C$, where $C'$ is the bandwidth such that the minimum buffer requirement in the lossless segregated system is exactly $B$. Then we have,

$$\lim_{N\to\infty} \frac{1}{N} \log P_{loss} \leq - \sup_{C_v\in\mathcal{C}} \left\{ \min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\} \right\}. \tag{21}$$

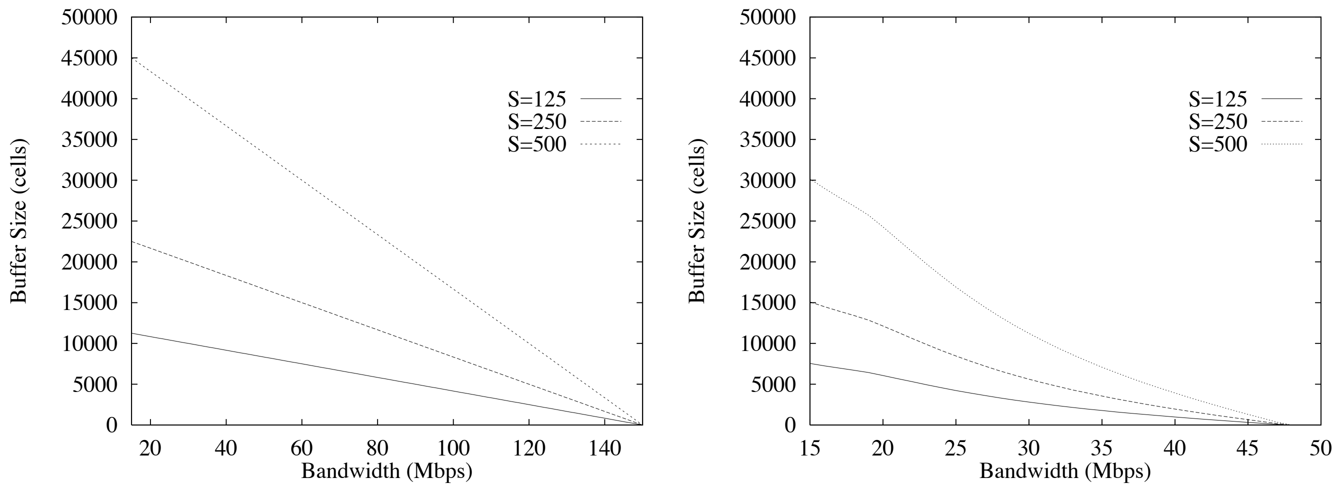
Since $\Lambda^*_{C_v}(\overline{c})$ is a decreasing function in $C_v$ while $\Lambda^*_{B_v}(\overline{b})$ is an increasing in $C_v$, the above optimization can be solved in a straightforward manner; details can be found in Appendix C. Let $C_v^*$ denote the solution to the optimization problem, then the loss probability $P_{loss}$ can be estimated by $e^{-N\Lambda^*_{C_v^*}(\overline{c})} + e^{-N\Lambda^*_{B_v^*}(\overline{b})}$. This estimate can be further refined by adding a prefactor that represents an asymptotic correction term for the Chernoff bound [BR60]. Therefore,

$$P_{loss} \approx \frac{1}{\theta^*_{C_v^*}\sqrt{2\pi\Lambda''_{C_v^*}(\theta^*_{C_v^*})}} e^{-N\Lambda^*_{C_v^*}(\overline{c})} + \frac{1}{\theta^*_{B_v^*}\sqrt{2\pi\Lambda''_{B_v^*}(\theta^*_{B_v^*})}} e^{-N\Lambda^*_{B_v^*}(\overline{b})} \tag{22}$$

where $\theta^*_{C_v^*}$ and $\theta^*_{B_v^*}$ are the solutions of the equations $\Lambda'_{C_v^*}(\theta) = C$ and $\Lambda'_{B_v^*}(\theta) = B$, respectively and where $\Lambda'(\theta)$ and $\Lambda''(\theta)$ denote the first and second derivatives of $\Lambda(\theta)$.

# 4 Numerical Examples

In this section, we present numerical examples to illustrate the results of the previous sections. Our focus is on the relationship between source time scale and the optimal buffer/bandwidth trade-off, and on the effect of this buffer/bandwidth trade-off on admissible regions under both deterministic lossless service and statistical service with small loss probabilities. In comparison to previous works that consider multiplexing of periodic on-off




(a) $L = 0$ (b) $L = 10^{-9}$

Figure 8: Buffer/bandwidth trade-off for a single class.

sources [EMW95, GBC95, KB92, NRST91, RV89, RV91], an important contribution of our present work lies in exploiting the different sources times scales existing in heterogeneous traffic sources. In particular, we show how the buffer/bandwidth trade-off is determined by the source time scale separation and that the boundary of the admissible region for heterogeneous sources can be severely non linear if the sources have very different time scales.

In the following examples, we describe regulated sources using the token rate $\rho$, the peak rate $P$, and the burst-size $S$ in place of the usual token bucket size $\sigma$. $\sigma$ can be obtained from $S$ from the identity $S = \sigma\frac{P}{P-\rho}$. The burst-size $S$ is preferred because it is related to the source time scale $T_{on}$ via $T_{on} = S/P$.

We begin by illustrating the relationship between source time scale and the optimal buffer/bandwidth trade-off. We first consider the case where there is a single class of sources. Under lossless multiplexing, for a multiplexor of bandwidth $C$ with $K$ homogeneous sources, from (10), we have that the minimal buffer requirement is given by

$$B_{min} = T_{on}(KP - C). \tag{23}$$

With $K$ fixed, the buffer requirement decreases linearly with the bandwidth and is proportional to the source time scale. At $C = KP$, it becomes zero. The source time scale also determines the rate of change in the buffer requirement, as $dB_{min}/dC = -T_{on}$. In Figure 8(a), the optimal buffer/bandwidth curve is plotted with $K = 100$ for three types of single-class sources with the same mean rate $\rho = 0.15$ Mbps and peak rate $P = 1.5$ Mbps, but three different burst-sizes $S = 125, 250, 500$, measured in cells (*i.e.*, 53 Bytes), yielding $T_{on} = 35, 70, 140$ ms., respectively. With $K = 100$, the aggregate average rate is 15 Mbps and the aggregate peak rate 150 Mbps. In Figure 8(b), we plot the bandwidth/buffer trade-off for statistical service with a loss probability of $L = 10^{-9}$, where

| class | $\rho$ | $P$ | $S$ | $T_{on}$ |
|---|---|---|---|---|
| 1 | 0.15 | 1.5 | 250 | 70 |
| 2 | 0.15 | 6 | 25 | 1.7 |

Table 1: Sources leaky bucket parameters.

for each given bandwidth value, the buffer requirement is computed as the minimum buffer size such that $P_{loss} \leq L$. In this case, the buffer requirement also decreases with the bandwidth, and is proportional to the source time scale. Note that the buffer requirement under statistical service is much less than under lossless service. For example, in this case, the buffer requirement drops to 0 as bandwidth approach 50 Mbps, as opposed to 150 Mbps in the case of lossless service. The presence of statistical multiplexing gains is clearly evident.

We now consider multiplexing two classes of sources. The parameters for the two classes are listed in Table 1. The parameters are chosen so that the two classes have drastically different $T_{on}$ ($T_{on_1} \gg T_{on_2}$), so as to highlight the effect of source time scale on the buffer/bandwidth trade-off curve when heterogeneous classes are multiplexed. We assume that $K_1 = K_2 = 50$. The aggregate average and peak rate of the two classes are 15 Mbps and 375 Mbps, respectively.

The optimal buffer/bandwidth trade-off curve for lossless service ($L = 0$) is plotted in Figure 9(a). We can distinguish two distinct components in the curve: the first one with a very steep slope, and the second one with a much lower slope. The knee point is at $C = 82.5$. This phenomenon can be explained by considering the structure of the optimal resource allocation scheme. With small values of bandwidth $C$, class 2 sources have a "fast" time scale (the time scale index $k$ defined in (9) is 1), and are thus allocated an amount of bandwidth equal to their mean rate $\rho_2$ and buffer space equal to their maximum token bucket size $\sigma_2$, while the remaining bandwidth and buffer space are allocated among class 1 sources, which have much larger time scale, $T_{on_1}$. As $C$ increases, the bandwidth allocated to class 1 sources increases, hence their buffer requirement decreases as a rate of $dB_{min}/dC = -T_{on_1}$. When $C$ reaches $82.5$ Mbps, class 1 sources are allocated peak rate (the time scale index, $k$, is 2), thus turning into "slow" time scale sources with no buffer requirements. As a consequence, as $C$ further increases, additional bandwidth is allocated only among class 2 sources, reducing their buffer requirements at a rate of $dB_{min}/dC = -T_{on_2}$.

In Figure 9, we also plot the buffer/bandwidth trade-off for statistical service with a loss probability of $10^{-9}$. As expected, both bandwidth and buffer requirements are reduced under statistical service, again providing evidence of statistical multiplexing gains. At $C = 85$ Mbps, the buffer requirement drops to $0$. For statistical service, the time scale separation of the two classes is easier to discern by examining the trade-off curve in log-scale (Figure 9(b)).

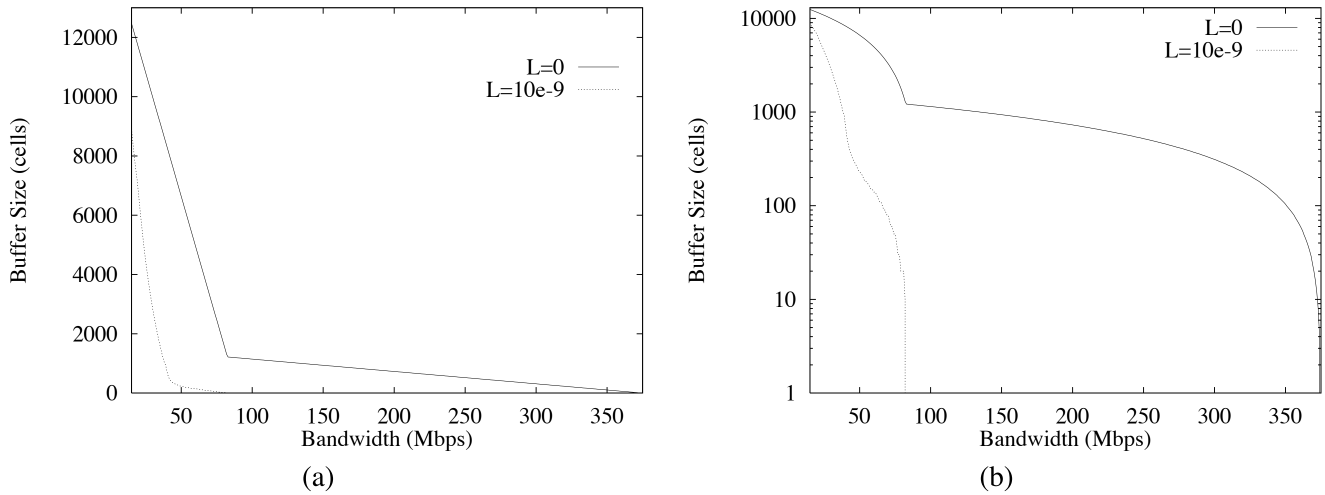



Figure 9: Buffer/bandwidth trade-off for two classes.

Observe that as the bandwidth $C$ approaches 50Mbps, the curve bends. As in the deterministic case, this phenomenon can be explained in terms of the structure of the optimal resource allocation for the virtual segregated system, which is determined by the optimization in (21). Due to statistical multiplexing gains, the time scale separation evident in the bandwidth/buffer trade-off curve occurs at a lower value of bandwidth.

Next, we study the impact of source characteristics on the system admissible region, defined as the number of sources that can be admitted without violating a given QoS requirement. We consider the same two classes of traffic discussed above. We fix the value of the system bandwidth $C$ at 45 Mbps and study the admissible region as a function of the system buffer size $B$. Because the two classes have the same average rate, the system utilization is maximized when the number of sources of both classes, $K_1 + K_2$ is maximized. Because $\rho/C = 1/300$, the utilization is $\frac{K_1+K_2}{3}\%$.

The admissible regions for lossless service with various values of buffer size are plotted in Figure 10. The key observation here is the non-linearity of the boundary of the admissible region for all values of the buffer size $B$ except for $B = 0$. As we will see, this non-linearity is caused by the different source time scales, $T_{on}$, of the sources. For $B = 0$, the boundary is linear with a slope given by the ratio of the source peak rates $dK_2/dK_1 = -P_1/P_2$. This is because in the bufferless case, sources are allocated peak rate. For a nonzero buffer size, we can identify two distinct segments of different slopes on the boundary of the admissible region. For relatively small numbers of class 1 sources, the boundary has the same slope as in the bufferless case, as shown in the figure. On the other hand,

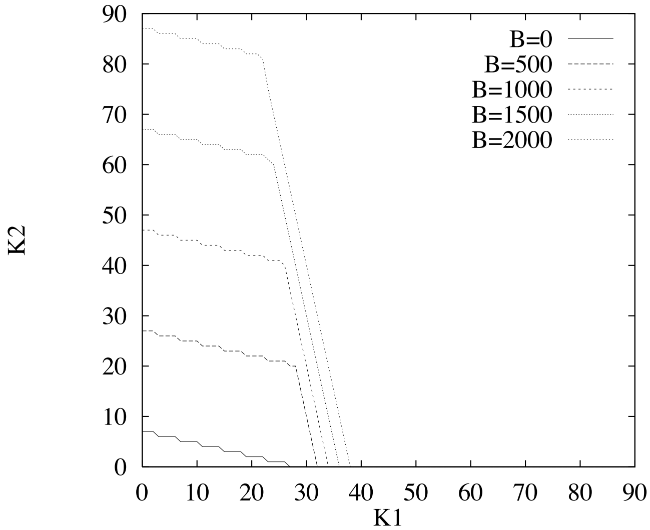



Figure 10: Admissible region for two heterogeneous classes: $T_{on_1} \gg T_{on_2}$.

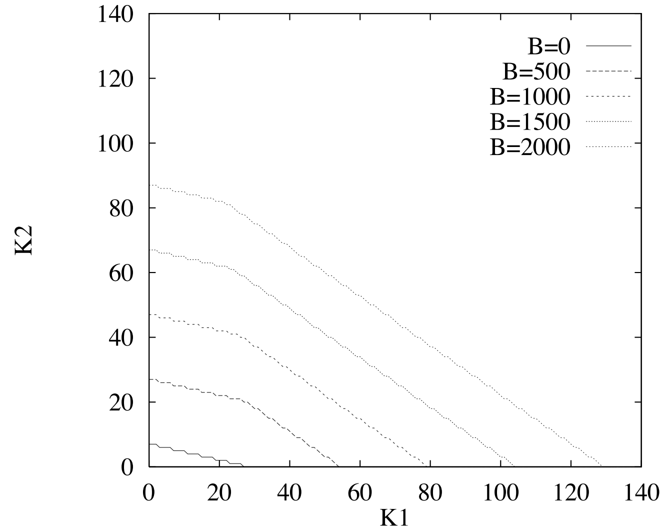



Figure 11: Admissible Region for two heterogeneous classes: $T_{on_1} \approx T_{on_2}$.

for relatively small numbers of class 2 sources, it is not difficult to show that the slope of the boundary is

$$\frac{dK_2}{dK_1} = -\frac{P_1}{\sigma_2/T_{on_1} + \rho_2} \leq -\frac{P_1}{\sigma_2/T_{on_2} + \rho_2} = -\frac{P_1}{P_2} \tag{24}$$

as $T_{on_1} \geq T_{on_2}$. As a result, the admissible region is convex.

From (24), we expect the boundary to be approximately linear when $T_{on_1} \approx T_{on_2}$, and linear when $T_{on_1} = T_{on_2}$. To illustrate this observation, we reduce the burst size $S$ of class 1 from 250 to 20, yielding a new source time scale $T_{on_1} \approx 5.8$, which is comparable to the time scale of class 2, $T_{on_2} \approx 3.5$. The resulting admissible region is plotted in Figure 11. We see that the boundary of the admissible region is now much closer to linear.

In Figures 12, 13 and 14, we plot the admissible regions for statistical service with various loss probabilities. The system bandwidth is again fixed at $C = 45$ Mbps, and three values of the system buffer size are considered: $B \in \{0, 1000, 10000\}$ cells. Note that statistical service provides considerable improvement in system utilization. It is also interesting to observe that for large buffer, a form of system saturation takes effect that limits statistical multiplexing gains. For example, for $B = 10000$ (Figure 14), because the most resource-efficient way to accommodate "fast" time scale is to allocate minimum bandwidth and maximum buffer space, when the input traffic is dominated by the "fast" time scale class 2 sources, it is possible to admit up to 300 sources, corresponding to $100\%$ utilization under lossless service. The same buffer size is less effective when input traffic is dominated by the "slow" time scale class 1 sources. In this case, because the most resource-efficient way to accommodate "slow" time scale is to allocate maximum bandwidth and no buffer space, no more than 70 sources can be admitted under lossless service

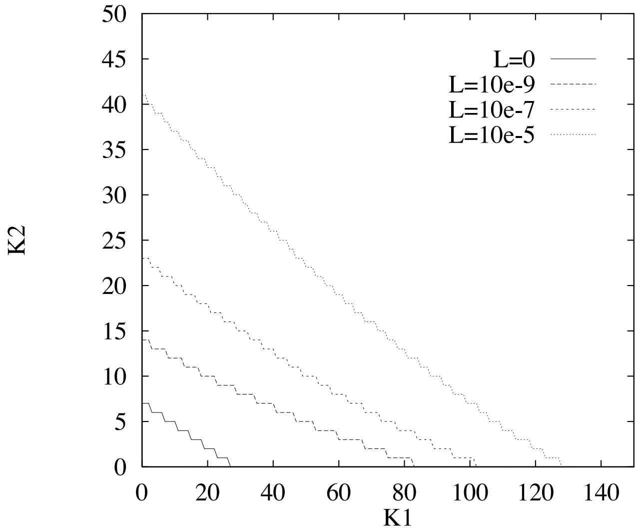



Figure 12: Admissible region: $B = 0$.

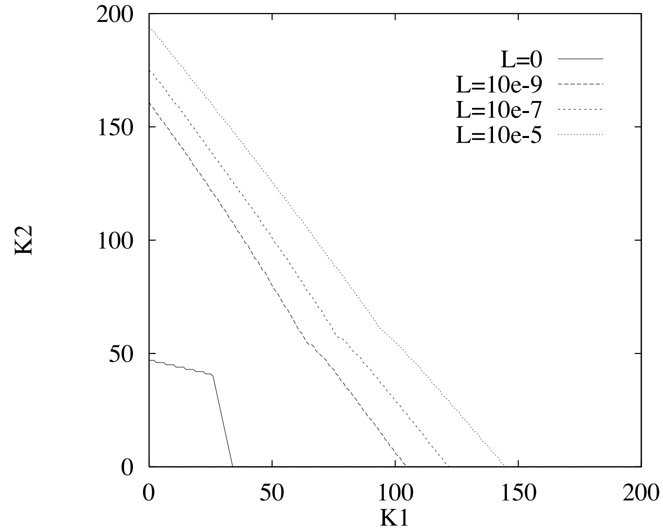



Figure 13: Admissible region: $B = 1000$.

(resulting in an utilization below $25\%$) and no more than approximately $210$ sources can be admitted under statistical service with a loss probability of $10^{-5}$ (resulting in an utilization of approximately $70\%$). For statistical service, the resulting admissible region is also clearly non-linear. We can distinguish two distinct components in the boundary: a linear region for $K_1 \leq 45$ and a concave region otherwise.

We now compare our approach with that in [EMW95]. In [EMW95] the system loss probability is estimated by reducing the two-resource allocation problem into a single resource allocation problem by fixing the buffer/bandwidth trade-off to the ratio $B/C$ regardless of sources characteristics. We extend their approach by transforming the two-resource allocation problem into two independent resource allocation problems. This allows us to explore the maximal benefits of both buffer sharing and bandwidth sharing. This is important, as the efficacy of buffer sharing and bandwidth sharing for sources with different time scales are quite different. As a consequence our loss probability estimate (21) that explores the "best" resource separation is expected to provide better results than the approach in [EMW95]. This has been confirmed by our numerical investigations. As an example, we compare our results in Figure 13 with the corresponding example of Figure 13 in [EMW95]. The parameters of the classes are swapped in their case, so our class 1 corresponds to class 2 in [EMW95] and vice versa. In Figure 15, we plot the admissible region obtained using both approaches, for loss probabilities of $10^{-9}$ and $10^{-5}$. The bandwidth and buffer size are fixed to $C = 45$ Mbps and $B = 1000$ cells, respectively. The admissible region computed by means of (21) is larger for all the loss probabilities considered.

Finally we evaluate the accuracy of our approach by comparison with simulation. In Table 2, we list the numbers

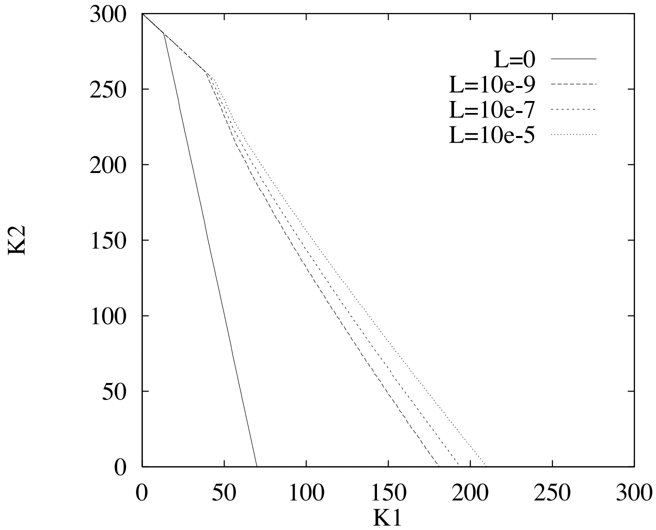



Figure 14: Admissible region: $B = 10000$.

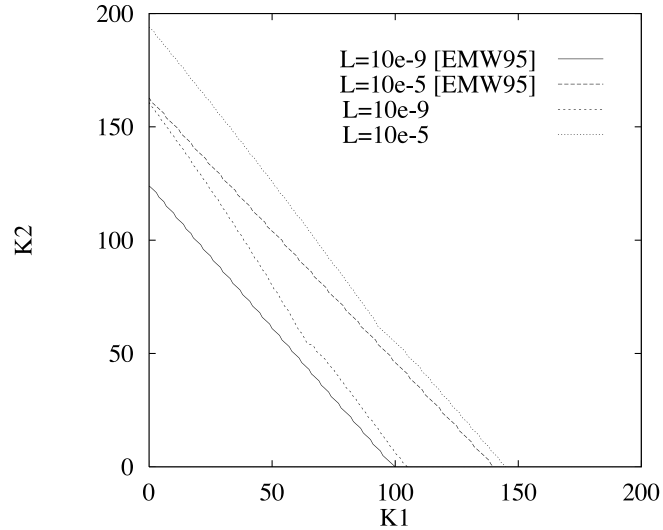



Figure 15: Comparison of our approach with [EMW95].

| Loss Probability | $2.2 \cdot 10^{-8}$ | $1.7 \cdot 10^{-7}$ | $1.3 \cdot 10^{-6}$ | $6 \cdot 10^{-6}$ |
|---|---|---|---|---|
| $K_1$ (simulation) | 150 | 160 | 170 | 180 |
| $K_1$ (analysis) | 116 | 123 | 134 | 142 |

Table 2: Comparison with simulation.

of class 1 admissible sources, $K_1$, obtained from simulation and our numerical approach for various loss probabilities, where $C = 45$Mbps and $B = 1000$ cells. The simulations results are taken from [EMW95]. As expected, our approach provides a conservative estimate of number of admissible sources when compared with simulation.

## 5 Conclusions

In this paper, we studied the problem of resource allocation and control for an ATM node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability were considered. We investigated the relationship between source characteristics and buffer/bandwidth trade-off under both services.

For guaranteed lossless service, we identified the optimal resource allocation scheme for an ATM node with finite bandwidth and buffer space, and found that the optimal resource allocation scheme suggests an interesting time scale separation of sources sharing the ATM node. With respect to this time scale separation, the time scale of a source can be defined and the optimal buffer/bandwidth trade-off determined by the sources' time scale. For statistical service with small loss probability, we presented a new approach for estimating loss probability in a shared buffer multiplexor with extremal on-off, periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth trade-off is again determined by the time scale separation and reflects the efficacy of buffer sharing and bandwidth sharing among sources with different time scales. Through numerical investigations, we illustrated the relationship of source time scale and the optimal buffer/bandwidth tradeoff and discussed the implications of our results in resource allocation and call admission control.

Our results have many other implications in network design and control such as network dimensioning and traffic shaping, in addition to resource allocation and call admission control in an ATM node. This will be the subject of future research.

# A Stochastic Orderings and Virtual Buffer/Trunk System

Let $X$ and $Y$ be two nonnegative real random variables with distributions $F_X$ and $F_Y$, respectively. We say that $X$ is smaller than $Y$ under *stochastic order* (resp., under *convex order*), denoted as $X \leq_{st} Y$ (resp. $X \leq_{cx} Y$), if for all increasing (resp. convex) functions $h$,

$$E[h(X)] \leq E[h(Y)] \tag{25}$$

provided that the expectations exist.

If $X \leq_{st} Y$, it is easy to show that $\Pr\{X > u\} \leq \Pr\{Y > u\}$ for any $u \in [0, \infty)$. In other words, $X$ is less likely than $Y$ to take on large values. Thus, if $X \leq_{st} Y$, we also say $X$ is stochastically smaller than $Y$.

If $X \leq_{cx} Y$, then it follows that $E[X] = E[Y]$ and $Var(X) \leq Var(Y)$. It can also be shown that that $X \leq_{cx} Y$ if and only if for any $x \geq 0$,

$$\int_x^\infty (1 - F_X(u))dy \leq \int_x^\infty (1 - F_Y(u))dy \tag{26}$$

provided that the integrals exist. Intuitively, $X \leq_{cx} Y$ means that $X$ is less variable than $Y$ in the sense that $Y$ gives more weight to the extreme values. Thus, if $X \leq_{cx} Y$, we also say that $X$ is *stochastically less variable* than $Y$.

Both $\leq_{st}$ and $\leq_{cx}$ are closed under convolution. Namely, for any two sets of independent random variables, $\{X_i, i = 1, \ldots, n\}$ and $\{Y_i, i = 1, \ldots, n\}$, if $X_i \leq_{st} (\leq_{cx}) Y, i = 1, \ldots, n$, then $\sum_{i=1}^n X_i \leq_{st} (\leq_{cx}) \sum_{i=1}^n Y_i$.

Now we apply these notions of stochastic orderings to the study of the virtual buffer/trunk system.

Let $v$ and $u$ denote, respectively, the instantaneous buffer content and utilized bandwidth at a random time. The instantaneous service rate $u$ is a Bernoulli random variable which takes value $c$ with probability $w = \frac{D_{on}}{T} = \frac{\rho}{c}$, and 0 with probability $1 - w$. $w$ represents the fraction of time the system is busy. Clearly, $E[u] = \rho$, thus the average (utilized) service rate equals to the average rate of the traffic source.

The instantaneous buffer content $v$ is a random variable that takes value in $[0, b]$ and has the following distribution

$$\Pr\{v \leq x\} = 1 - w + w\frac{x}{b},\ 0 \leq x \leq b, \tag{27}$$

*i.e.*, it takes value 0 with probability $1 - w$, and is otherwise uniformly distributed in the interval $(0, b]$. $E[v] = \frac{\rho T_{on}}{2c}(P - c)$.

It is easy to verify that the following stochastic ordering results hold for the instantaneous buffer content and utilized bandwidth. For any $c'$ and $c''$ such that $\rho \leq c' \leq c'' \leq P$, let $u_x$ and $v_x$, $x \in \{c', c''\}$, denote the instantaneous buffer content and utilized bandwidth for $c = x$, respectively. Then:

$$u_{c'} \quad \leq_{cx} \quad u_{c''} \tag{28}$$

$$v_{c'} \quad \geq_{st} \quad v_{c''}. \tag{29}$$

Hence, increasing $c$ results in the instantaneous utilized bandwidth $u$ becoming stochastically more variable. On the other hand, increasing $c$ results in stochastically smaller instantaneous buffer content. This means that not only the peak buffer usage decreases as a function of $c$, but also the probability of exceeding any buffer level monotonically decreases with $c$.

These results are useful in solving the optimization problem of Appendix C.

# B Upper Bounding the System Loss Probability

In this Appendix, we justify our approach to estimate the system loss probability.

The ATM node considered in Section 3 can be modeled by an infinite queue with a server of capacity $C$, the arrival process to the queue is the aggregation of the independent extremal on-off, periodic sources $g_i(t), 1 \leq j \leq J$ and $1 \leq i \leq K_j$. The system loss probability $P_{loss}$ due to buffer overflow in the ATM node with finite resources can then be upper bounded by the probability that $Pr\{Q > B\}$ where $Q$ denotes the stationary queue length of the infinite queue system.

Now consider a virtual lossless segregate system with a total amount of bandwidth $C_v$ and buffer space $B_v$ where $(C_v, B_v)$ lies on the buffer/bandwidth trade-off curve. In the virtual lossless segregate system, each source $i$ of class $j$, $a_{ji}(t), 1 \leq j \leq J, 1 \leq i \leq K_j$, has a virtual trunk of fixed bandwidth $c_j^v$ and a virtual buffer of fixed size $b_j^v$ such that $\sum_{j=1}^{J} K_j c_j^v = C_v$, $\sum_{j=1}^{J} K_j b_j^v = B_v$, and the resources $c_j^v$ and $b_j^v$ are allocated to each virtual buffer/trunk system according to the optimal resource allocation scheme described in Section 2.1. Let $u_{ji}(t)$ and $v_{ji}(t)$ denote the utilized bandwidth and the buffer content of source $a_{ji}(t)$ in the virtual buffer/trunk system. Thus, at any time $t$, the total utilized bandwidth is $U(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}(t)$, while the total buffer content is $V(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}(t)$. Now, let $v'_{ji}$ denote the derivative of $v_{ji}(t)$. $v'_{ji}(t)$ is zero when the buffer is empty; otherwise it takes values $P_j - c_j^v$ and $-c_j^v$ during the buffer filling and emptying phase, respectively. In other words, $v_{ji}(t) = \int_0^t v'_{ji}(s)ds$ for any $t \geq 0$. Given the definition of $u_{ji}(t)$ and $v'_{ji}(t)$, it is easy to verify that for any $t \geq 0$,

$$a_{ji}(t) = u_{ji}(t) + v'_{ji}(t). \tag{30}$$

Now for any time interval $[\tau, t]$, define $A[\tau, t] = \int_\tau^t \sum_{j=1}^{J} \sum_{i=1}^{K_j} a_{ji}(s)ds$, $U[\tau, t] = \int_\tau^t \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}(s)ds = \int_\tau^t U(s)ds$, and $V[\tau, t] = \int_\tau^t \sum_{j=1}^{J} \sum_{i=1}^{K_j} v'_{ji}(s)ds = V(t) - V(\tau)$. Then clearly, $A[\tau, t] = U[\tau, t] + V[\tau, t]$.

Let $Q(t)$ denote the queue length of the infinite queue system at time $t$. Then

$$Q(t) = \sup_{\tau \leq t} \{A[\tau, t] - C(t - \tau)\}. \tag{31}$$

Hence

$$\begin{aligned} Q(t) &= \sup_{\tau \leq t} \{U[\tau, t] + V[\tau, t] - C(t - \tau)\} \\ &\leq \sup_{\tau \leq t} \{U[\tau, t] - C(t - \tau)\} + \sup_{\tau \leq t} \{V[\tau, t] - 0 \cdot (t - \tau)\} \\ &= Q_C(t) + Q_B(t) \end{aligned} \tag{32}$$

where we define

$$Q_C(t) = \sup_{\tau \leq t} \{U[\tau, t] - C(t - \tau)\} \text{ and } Q_B(t) = \sup_{\tau \leq t} \{V(t) - V(\tau)\} = V(t).$$

The last inequality determines the "separation" between bandwidth and buffer requirements that enables us to treat them separately. The key observation now is that $Q_C(t)$ and $Q_B(t)$ can be regarded as the queue length processes of two well-defined systems. In particular, $Q_C(t)$ is the queue length of a system with a server of capacity $C$, where the arrival process is $U(t)$. $Q_B(t) = V(t)$ is the queue length (content) of a storage system where data is stored and retrieved according to the rate process $V'(t) = dV(t)/dt = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v'_{ji}(t)$. Observe that the latter system can be regarded as system with a server of zero capacity.

Let $Q_C$ and $Q_B$ denote, respectively, the stationary version of $Q_C(t)$ and $Q_B(t)$ (which exist because of the system stability condition). Inequality (32) implies

$$\begin{aligned} P_{loss} \leq \Pr\{Q > B\} &\leq \Pr\{Q_C + Q_B > B\} \\ &\leq \Pr\{Q_C > 0 \text{ or } Q_B > B\} \\ &\leq \Pr\{Q_C > 0\} + \Pr\{Q_B > B\}. \end{aligned} \tag{33}$$

For small loss probability and large $C$, the first term in (32) is well approximated (see [EM95, CIK91, SG94]) by the loss probability of a *bufferless* multiplexor with capacity $C$ and stationary arrival process $U$, *i.e.*, $\Pr\{U > C\}$. This, along with the fact that $Q_B = V$, yields the upper bound (14) stated in Section 3.2

$$P_{loss} \leq \Pr\{U > C\} + \Pr\{V > B\}. \tag{34}$$

# C Optimizing the Loss Probability Estimation

Here we determine the optimal allocation that minimize the loss probability below

$$\lim_{N\to\infty} \frac{1}{N} \log P_{loss} \leq - \sup_{C_v \in \mathcal{C}} \left\{ \min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\} \right\}. \tag{35}$$

To this end, we first establish that the two rate functions $\Lambda^*_{C_v}(\overline{c})$ and $\Lambda^*_{B_v}(\overline{b})$ are, respectively, decreasing and increasing function of $C_v$.

Consider the rate function $\Lambda^*_{C_v}(\overline{c})$ of $U = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}$. In order to show that it is a decreasing function of $C_v$, we need to show that for any $C_v, C'_v \in [\sum_{j=1}^{J} K_j \rho_j, \sum_{j=1}^{J} K_j P_j]$, if $C_v \leq C'_v$, then $\Lambda^*_{C_v}(\overline{c}) \geq \Lambda^*_{C'_v}(\overline{c})$. From (18), this is equivalent to

$$\sup_{\theta \geq 0} \{\theta \overline{c} - \Lambda_{C_v}(\theta)\} \geq \sup_{\theta \geq 0} \{\theta \overline{c} - \Lambda_{C'_v}(\theta)\}. \tag{36}$$

Clearly, (36) holds if $\Lambda_{C_v}(\theta) \leq \Lambda_{C'_v}(\theta)$ for all $\theta \geq 0$.

Recall that $\Lambda_c(\theta) = \sum_{j=1}^{J} \kappa_j \log M_{u_j}(\theta)$ and $M_{u_j}(\theta) = E[e^{\theta u_j}]$. Since $e^{\theta X}$ is a convex function of $X$, it suffices to show that each $u_{ji}$ is increasing in convex order as a function of $C_v$. From Appendix A, we know that $u_{ji}$ is increasing in convex order with increasing $c_j$. From (11), it follows that $u_{ji}$ is also increasing in convex order with increasing $C_v$. Thus $\Lambda_c(\theta)$ is an increasing of $C_v$. This establishes that $\Lambda^*_{C_v}(\overline{c})$ is a decreasing function of $C_v$.

Similarly, we can prove that the rate function $\Lambda^*_{B_v}(\overline{b})$ of $V = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}$ is an increasing function of $C_v$ by using the fact that each $v_{ji}$ is decreasing in the stochastic order with increasing $C_v$.

Given the monotonicity of $\Lambda^*_{C_v}(\overline{c})$ and $\Lambda^*_{B_v}(\overline{b})$, it is easy to realize that the supremum in (35) is attained at a $C_v$ where the two rate functions are equal (provided that they intersect), *i.e.*,

$$\Lambda^*_{C_v}(\overline{c}) = \Lambda^*_{B_v}(\overline{b}). \tag{37}$$

Intuitively this means that the optimal buffer/bandwidth trade-off is the one that makes probabilities of exceeding either resources equal.

However, it is possible that there is no $C_v$ such that (37) holds. This occurs when two functions do not intersect, thus one of the two functions always takes smaller values than the other. Intuitively, this corresponds to the case where, for any buffer/bandwidth trade-off considered, the probability of exceeding one resource is always larger than the probability of exceeding the other. For example, if $\Lambda^*_{C_v}(\overline{c}) > \Lambda^*_{B_v}(\overline{b}), C_v \in \mathcal{C}$, then, from the fact that $\Lambda^*_{B_v}(\overline{b})$ is increasing in $C_v$, we have that

$$\sup_{C_v \in \mathcal{C}} \left\{ \min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\} \right\} = \sup_{C_v \in \mathcal{C}} \left\{ \Lambda^*_{B_v}(\overline{b}) \right\} = \Lambda^*_{B}(\overline{b}) \tag{38}$$

where $B^*_v = B$ and $C^*_v = C'$.

Similarly, if $\Lambda^*_{C_v}(\overline{c}) < \Lambda^*_{B_v}(\overline{b}), C_v \in \mathcal{C}$, then, from the fact that $\Lambda^*_{B_v}(\overline{b})$ is decreasing in $C_v$,

$$\sup_{C_v \in \mathcal{C}} \left\{ \min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\} \right\} = \Lambda^*_{C}(\overline{c}) \tag{39}$$

where $C^*_v = C$.

Finally, we justify the restriction of the choice of $(C_v, B_v)$ to the segment of the buffer/bandwidth curve trade-off curve where $C_v \in \mathcal{C} = [C, C']$ ( highlighted segment in Figure 7). To this purpose, consider a pair $({C_v}', {B_v}')$ on the buffer/bandwidth trade-off curve outside the given segment, say with $C' < {C_v}' \leq \sum_{j=1}^{J} K_j P_j$. This implies that $B'_v < B$. Then, from the result for the lossless segregated system, we have that $\Pr\{V > B\} = 0$, i.e., $\Lambda^*_{{B_v}'}(\overline{b}) = \infty$ and $\min\{\Lambda^*_{{C_v}'}(\overline{c}), \Lambda^*_{{B_v}'}(\overline{b})\} = \Lambda^*_{{C_v}'}(\overline{c})$. Because $\Lambda^*_{C_v}(\overline{c})$ is decreasing in $C_v$, we can always improve the estimate by decreasing the value of ${C_v}'$. As a consequence, for any pair $({C_v}', {B_v}')$, with $C' < {C_v}' \leq \sum_{j=1}^{J} K_j P_j$, we get a better estimate by considering the extreme of the interval, namely the point $(C', B)$. Similar arguments apply for any pair $({C_v}', {B_v}')$, with $C' < {C_v}' \leq \sum_{j=1}^{J} K_j \rho_j \leq {C_v}' < C$, the details of which are omitted. This establishes the validity of the optimization problem (21) of Section 3.2.

## References


[ATM] ATM Forum, Technical Committee. Traffic Management Specification Version 4.0, atmf95-0013R6.

[Be63] V. Benes. *General Stochastic Processes in the Theory of Queues*. Reading, MA: Addison Wesley, 1963.

[Bi86] P. Billingsley. *Probability and Measures*, 2nd ed. New York: Wiley, 1986.

[BR60] R. Bahadur and R. Rao. On deviations of the sample mean. *Ann. Math. Statis.*, Vol. 31, pp. 1015-1027, 1960.

[Ch94] C. S. Chang. Stability, Queue Length and Delay of Deterministic and Stochastic Queueing Networks, *IEEE Transaction on Automatic Control*, Vol. 39, No. 5, pp. 913-931, May 1994.

[CIK91] E. Coffman, B. Igelnik and Y. Kogan. Controlled Stochastic Model of a Communication System with Multiple Sources, *IEEE Trans. on Information Theory*, Vol. 37, No. 5, pp. 1379-1397, May 1991.

[Do93] B. Doshi. Deterministic Rule Based Traffic Descriptors for Broadband ISDN: Worst Case Behavior and Connection Acceptance Control, *Proceedings of IEEE INFOCOM 93*, pp. 1759-1764, San Francisco, March 1993.

[EM95] A. Elwalid and D. Mitra. Analysis, Approximation and Admission Control of a Multi-Service Multiplexing System with Priorities, *Proceedings of IEEE INFOCOM 95*, pp. 463-472, Boston, April 1995.

[EMW95] A. Elwalid, D. Mitra and R.Wentworth. A New Approach for Allocating Buffer and Bandwidth to Heterogeneous, Regulated Traffic in an ATM Node, *IEEE Journal on Selected Area in Communications*, Vol. 13, No. 6, pp. 1115-1127, August 1995.

[GBC95] J. Garcia, J. Barcelo and O. Casals. An Exact Model for the Multiplexing of Worst Case Traffic Sources, *Data Communications and Their Perfomance*, S. Faida and R. Onvural, Eds.: Chapman & Hall, pp. 3-17, 1995.

[KB92] K. Kvols and S. Blaabjerg. Bounds and Approximations for the Periodic on/off Queue with Applications to ATM Traffic control, *Proceedings of IEEE INFOCOM 92*, pp. 487-494, Florence, May 1992.

[MM95] D. Mitra and J. Morrison. Multiple Time Scale Regulation and Worst Case Processes for ATM Network Control, *Proceedings of the 34th Conference on Decision and Control*, pp. 353-358, New Orleans, December 1995.

[NRST91] I. Norros, J. Roberts, A. Simonian and J. Virtamo. The Superposition of Variable Bit Rate Sources in an ATM Multiplexer, *IEEE Journal on Selected Area in Communications*, Vol. 9, No. 3, pp. 378-387, April 1991.

[RV89] J. Roberts and J. Virtamo. Evaluating Buffer Requirements in an ATM Multiplexer, *Proceedings of GLOBECOM 89*, pp. 1473-1477, 1989.

[RV91] J. Roberts and J. Virtamo. The Superposition of Periodic Cell Arrival Streams in an ATM Multiplexer, *IEEE Trans. on Communications*, Vol. 39, No. 2, pp. 298-303, February 1991.

[SG94] A. Simonian and J. Guibert. Large Deviations Approximation for Fluid Queues fed by a large number of on/off sources, *Proc. ITC-14*, J. Labetoulle and J. Roberts, Eds. New York: Elsevier, 1994, pp. 1013-1022.

[Tu86] J. Turner. New Directions in Communications (or Which Way to the Information Age?), *IEEE Comm. Mag.*,October 1986.

[Wo94] T. Worster. Modeling Deterministic Queues: the Leaky Bucket as an Arrival Process, *Proc. ITC-14*, J. Labetoulle and J. Roberts, Eds. New York: Elsevier, 1994, pp. 581-590.

[YSS92] N. Yamanaka, Y. Sato and K. Sato. Performance Limitations of Leaky Bucket Algorithm for Usage Parameter Control of Bandwidth Allocation Methods, *IEICE Trans. Commun.*, Vol. E75-B. No. 2, pp 82-86, 1992.

[ZKST96] Z. Zhang, J. Kurose, J. Salehi and D. Towsley. Smoothing, Statistical Multiplexing and Call Admission Control for Stored Video, *Technical Report* UM-CS-96-29, Computer Science Department, University of Massachusetts, Febraury 1996.