Exploiting Mobility in Ad-Hoc Wireless Networks with Incentives

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Abstract

Nodes participating in a wireless ad-hoc network, where individual battery power and bandwidth are scarce resources, have a strong disincentive to cooperate and are likely to refuse to relay traffic for others. To address this problem, researchers have suggested incorporating explicit incentive mechanisms that promote cooperative behavior in the network. Although numerous analyses have shown that incentives can improve network performance, an important factor has been consistently neglected from such evaluations, namely, user mobility. By allowing nodes to move strategically in the network, the dynamics generated by an incentive mechanism can change significantly. In this paper, we consider node position as the strategy of a user that participates in an ad-hoc network where an incentive mechanism is strictly enforced. Using game theoretic models, we perform a qualitative study of the effect of different incentive mechanisms (e.g., reputation and payments) and show that if users are allowed to strategically choose their positions in the system, network performance degrades significantly. In particular, our results indicate that the best strategies for the users lead to topologies where the nodes are either tightly clustered together or spread into a chain.

Keywords: ad-hoc networks, incentive mechanisms, mobility

1 Introduction

The design of first generation peer-to-peer (P2P) applications assumed that peers would voluntarily cooperate by providing local resources to the system. However, measurement studies conducted on P2P file sharing applications have shown that the vast majority of the peers contribute very little to the system [18, 1].

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In the context of ad-hoc networks, peers are expected to comply with the routing protocol and forward traffic on behalf of other peers in the network. Such cooperative behavior is necessary to obtain good network performance. However, since local battery power and bandwidth are scarce resources, peers have an incentive not to cooperate and to refuse to relay traffic for others while expecting their own packets to be forwarded. Although free-riding has not been measured in the context of an ad-hoc network, a severe degradation in network throughput has been predicted even if only a small fraction of peers misbehave (e.g., systematically drop packets belonging to other peers) [13].

In recognition of this problem, researchers have recently proposed building incentive mechanisms into applications in order to promote cooperative behavior. In the context of ad-hoc networks, incentive mechanisms have been recognized as necessary to guarantee satisfactory network performance [13, 2, 4, 5, 22, 14, 7]. Most of the incentive mechanisms proposed have been modeled and evaluated to characterize the improvement in network performance. However, their analyses have all neglected an important aspect of the problem, namely, user mobility. In particular, user mobility has either not been considered at all, or assumed to be an exogenous process.

It is natural for users forming an ad-hoc network to be tempted to relocate if the benefits obtained are clearly perceivable. In fact, fully complying with an incentive mechanism may impose high costs or significant constraints to a node which may be location dependent. This was indeed observed by Crowcroft et al. as the throughput allowed by their incentive mechanism was heavily dependent on the geographical location of a node [7]. In their study, they observe that nodes located at the center of the network received much better service than nodes located at the edges. This illustrates clearly why nodes may wish to exploit mobility to improve their utility. However, if nodes strategically choose their location, then the dynamics induced by an incentive mechanism will change and this may affect overall performance.

In this paper, we specifically investigate the impact of strategic mobility on the performance of an ad-hoc network where peers are subject to incentives. We model the ad-hoc network using a game theoretic formulation where peers are strategic players that consider their positions in the network. The goal is to qualitatively evaluate the characteristic of network topologies that result from the equilibria of the game. Our conclusions are pessimistic: under different incentive mechanisms and different games, mobility leads to network topologies with poor performance. In particular, the best strategy for peers induce topologies where the nodes are either highly clustered together or stretched out in a chain. Given these results, we are tempted to say that "mobility decreases the capacity of ad-hoc wireless networks", as opposed to well-known results presented in [10], where selfishness and incentives are not considered.

The remainder of this paper is organized as follows. In Section 2 we present the related work on incentive mechanisms. Section 3 shows the impact of network position under different metrics and illustrates the benefits of mobility to a single peer. In Section 4 we develop a general game theoretic framework for studying the problem of mobility in incentive driven ad-hoc networks. In Section 5 we investigate two games that show the negative impact of mobility on the network. The behavior of greedy users is studied in Section 6 by means of simulation. Finally, Section 7 concludes the paper.

2 Related Work

The incentive mechanisms proposed in the literature for ad-hoc networks can be classified into two main categories: those based on reputation, and those based on pricing. In reputation mechanisms [2, 14, 17], peers keep track of the reputations of others (using local or distributed information) and use the reputation ratings to provide or decline service to a given peer. In pricing mechanisms [4, 5, 22, 7], peers exchange money (using real or virtual currency) for forwarding service: the peer forwarding a packet is remunerated, while the peer generating the packet is charged.

Modeling the performance of these incentive mechanisms in the context of ad-hoc networks has also received much attention from researchers. For example, reputation schemes have been studied using game theory in [15, 9, 19, 20]. The authors of [9, 19] suggest that cooperation can emerge as a Nash equilibrium even if peers make decisions based only on local observations. In [3] a Bayesian approach is proposed to represent and distribute reputation information.

Pricing schemes have been analyzed in [22, 7, 5]. In [22] the authors present a cheat-proof mechanism that is also robust in presence of collusion. An analytical framework based on optimal flow allocation is proposed in [7]. In [5] the authors show that network performance is improved under payments.

In all of the studies above, node mobility has either been neglected or considered to be exogenous. However, it has been observed that compliance cost and the performance of nodes may depend on network position. Moreover, some of the proposed incentive mechanisms are robust to various forms of cheating and collusion, such that it is advantageous for a node to comply with the incentive. For example, a node participating in an ad-hoc network coupled with a reputation mechanism has no option other than to forward other nodes' traffic in order to maintain a good reputation and receive service from others. However, to reduce the costs associated with forwarding (e.g., energy and bandwidth), the node can position itself in a location that minimizes the amount of relayed traffic. This would allow the node to enjoy the benefits of a high reputation at a much lower cost.

A similar argument can be established for the case of a pricing mechanism, where a node spends money to send data, but earns money from providing service to others. To reduce the cost of using the system, a peer can position itself in a location that balances the amount of relayed traffic and the average path length to its destinations (the cost to send a data packet increases with the number of hops). For the remainder of this paper, we assume that incentive mechanisms are strictly enforced and that a peer's only option to reduce the costs of complying with the mechanism is to reposition itself in the network.

The idea of using game theory to model non-cooperative interaction between selfish users that are faced with the task of constructing a network topology has been recently discussed in the literature. In [8, 6] the authors consider a game where nodes select neighbors and pay for the links they create. To characterize the outcome of the game they inspect the topologies induced by equilibrium strategies. Despite the similarities in the methodology, our approach is significantly different, as we consider extensive games (as opposed to one-shot games), different payoff functions, and the concept of subgame perfect Nash equilibria as the solution of the game.

3 Preliminaries

To illustrate the impact of the location of a peer on its cost to comply with the underlying incentive mechanism, we study the simple case in which N peers are uniformly distributed in a disc, and the traffic matrix is homogeneous, i.e. all peers communicate at the same rate λ packets/s with all other peers in the network. Without loss of generality, we consider the unit-area disc (of radius $R = 1/\sqrt{\pi}$).

We inspect two fundamental metrics used to determine the cost to peers that comply with the incentive mechanisms: (i) the amount of traffic relayed by a given peer on behalf of others (Section 3.1); (ii) the average hop count of the paths from a given peer to its destinations (Section 3.2).

3.1 Relayed traffic

The amount of traffic a node must forward on behalf of other nodes clearly depends on the position of the node within the network, as well as on the routing protocol. Intuitively, nodes at the edge of the network forward much less traffic than nodes located in the core of the network. To quantitatively demonstrate this fact, we consider a communication model in which a node in between a source-destination pair can be chosen as a relay if its distance from the straight line connecting the source to the destination is less than a constant B. Figure 1 illustrates a lightly shaded rectangle that comprises all nodes that could act as relays between source S and destination D. Note that not all of these nodes will actually forward the traffic between S and D. For a given node P within the rectangle, we consider all alternate nodes that could be used to relay traffic within the rectangle instead of P. This set is depicted in Figure 1 by a dark shaded rectangle and depends on the transmission range r. Among nodes in this smaller set, we assume that node P is equally likely to be selected to forward traffic.

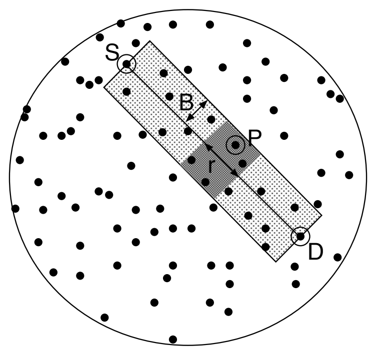

Figure 1: Peers that could act as relay between source S and destination D.

Due to the symmetry of our topology, the amount of traffic relayed by a node depends only on its distance x from the center of the disc, where $0 \leq x \leq R$. Consider a peer (node) P that is distance x from the origin. For each possible source, S, the set of destinations that could use P as a relay lie in a region of a sector starting from S whose sides pass a distance B from P, as illustrated in Figure 2. Using this set of destinations, we can compute the amount of traffic relayed by P on behalf of S.

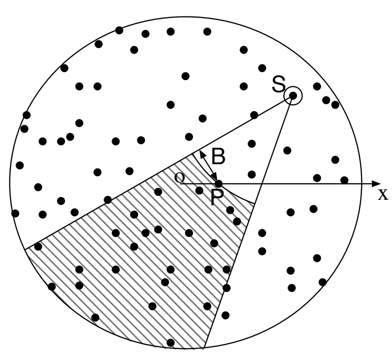

Figure 2: Destinations for which peer P may relay traffic for source S.

The total amount of traffic relayed by P is proportional to the number of nodes in the network N. We drop this dependency by introducing the *relayed traffic intensity* $\mathcal{R}(x)$, which is the average traffic forwarded by a node at distance x divided by the number of nodes in the network N. The quantity $\mathcal{R}(x)$ depends only on parameters R, B and r. Since a closed form solution for $\mathcal{R}(x)$ is difficult to obtain, we have evaluated $\mathcal{R}(x)$ using an approximate numerical method that discretizes the disc using a fine grid.

An illustrative set of results is shown in Figure 3 in the case of the unit area disc ($R \simeq 0.56$), $r = 0.01$ and different values of the band $B = 0.01, 0.1, 0.25$. An approximate formula valid for small values of r and B provides $\mathcal{R}(0) = 2\frac{R}{r}$. We observe that $\mathcal{R}(x)$ is a monotonically decreasing function of the distance. If we increase B, the traffic is spread more evenly in the network, yet nodes in the center have to relay more traffic than nodes near the edge.

Now consider the simple case where a single node can choose its position in the network. From our results, this node will position itself at the edge (center) of the network if it is simply trying to minimize (maximize) the amount of relayed traffic.

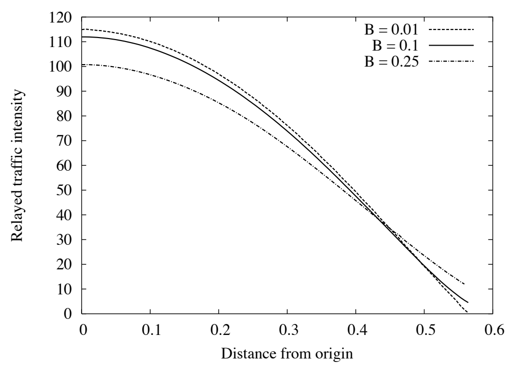

Figure 3: Relayed traffic intensity as a function of the distance from the center of a unit-area disc, using $r = 0.01$ and different values of band B.

3.2 Average path length

The other fundamental metric that comes into play in the case of payment-based incentive mechanisms is the average path length from a node to its destinations. This metric determines the cost of sending a packet through the network, which we assume to be proportional to the number of intermediate hops to reach a destination. Again, because of symmetry the average number of hops, $\mathcal{H}$, necessary to reach all possible destinations depends only on the distance x of the source S from the center of the disc, and can be expressed as

$$\mathcal{H}(x) = \int_D \left\lfloor \frac{\| D, S(x) \|}{r} \right\rfloor \mathrm{d}D \tag{1}$$

where $\|D, S(x)\|$ is the distance between $S(x)$ and a generic destination located at position D in the disc. Notice that in our model $\mathcal{H}(x)$ does not depend on the band B.

The above integral does not have a simple closed form solution, thus we resorted to a numerical method to evaluate $\mathcal{H}(x)$. The result is illustrated in Figure 4 as a function of the distance from the center, in the case of $r = 0.01$. An approximate formula valid for small values of r provides $\mathcal{H}(0) = \frac{2R}{3r}$. As expected, nodes near the center require fewer hops to reach their destinations than nodes close to the edges of the network, and $\mathcal{H}(x)$ is monotonically increasing.

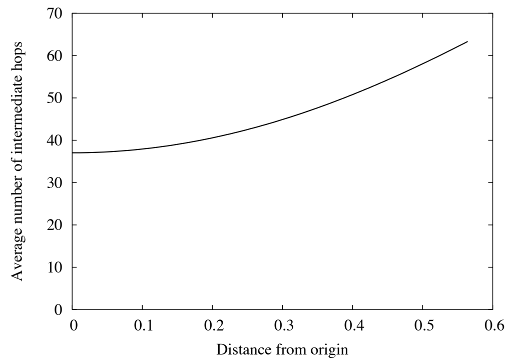

Figure 4: Average number of hops needed to send traffic to all destinations, as a function of the distance from the center of a unit-area disc in case of transmission range $r = 0.01$.

As a consequence, a node willing to just minimize (maximize) the average path length would position itself at the center (edge) of the network.

We observe that the two metrics $\mathcal{R}(x)$ and $\mathcal{H}(x)$ can be related by computing the total amount of traffic exchanged in the network. By equating the two measures of total traffic exchanged, we obtain the following relationship:

$$\int_0^R x\,\mathcal{R}(x)\,\mathrm{d}x = \int_0^R x\,\mathcal{H}(x)\,\mathrm{d}x \tag{2}$$

In a payment system in which each relayed packet costs one unit of money to the sender and provides one unit of money to the forwarding node, a node interested in accumulating money would choose the position that maximizes the difference $\mathcal{R}(x) - \mathcal{H}(x)$, which turns out to be at $x = 0$. Since $\mathcal{R}(0) > \mathcal{H}(0)$, a node could actually become rich going to the center of the network, at the expense of nodes located at the border, who have a negative balance.

However, our discussion so far describes only the behavior of a naive selfish user, who neglects the fact that other users also behave selfishly and will attempt to place themselves in optimal positions within the network.

4 Game Theoretic Model

Although in the previous section we have shown that physical location can impact the cost imposed by an incentive mechanism, it is essential to consider the scenario where all users in the network exploit mobility to reduce their costs. In this case, game theory offers a natural modeling framework to study the network formed by the strategic nodes. However, in order to use game theoretic models, the rules of the game played by the mobile users must be carefully defined. There are many different ways to establish a game for users to play, which can depend on issues such as the number of players, simultaneous or sequential movement, actions allowed to each player, initial state of the system, user's payoffs, etc. In what follows we introduce the common framework that will be used in the games described in Section 5.

Each node in the network is a player of the game. The set of movements available to each player corresponds to physical positions in the network. Players take turns in positioning themselves until the end of the game. Movements across the network occur instantaneously and have zero cost.[1] By assuming that at any time each node knows the location of all other nodes (a reasonable hypothesis if users can see each other), players will have perfect information. The games we consider can thus be modeled as finite, extensive form games with perfect information. We will use the solution concept of *subgame perfect equilibria* which is a refinement of Nash equilibria and, in the case of perfect information, coincides with the notion of sequential equilibria [16].

We choose to use the concept of subgame perfect equilibria since this eliminates Nash equilibria that are not meaningful, when players are assumed to be rational (i.e., utility maximizers). Consider the two player extensive form game described by the game tree illustrated in Figure 5. Although the strategy where player b plays b_1 no matter what player a does and player a plays a_2 is a Nash equilibrium, the threat posed by player b is not credible. In the sense that if player a plays a_1, then player's b best response will be b_1 and not b_2, since b will receive a higher payoff playing b_1.

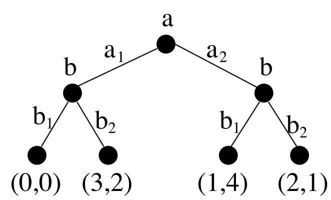

Figure 5: Example of a game tree of an extensive game with two players.

The set of of subgame perfect equilibria can be computed through a standard procedure known as *backward induction*. This algorithm operates recursively on the game tree starting from the leaves and moving towards the root. At each iteration, each parent node of a set of leaves chooses the leaf that maximizes its utility. The remaining leaves are pruned and the algorithm iterates. The game illustrated in Figure 5 has a single subgame perfect equilibrium that is given by player a playing a_1 and player b playing b_2 when player a plays a_1 and b_1 otherwise. An important existence result, known as Kuhn's theorem, ensures that every finite extensive game has a subgame perfect equilibrium [12]. In fact, if players are not indifferent among any two possible outcomes of the game, then there exists a unique subgame perfect equilibrium.

Let $p_i = (x_i, y_i) \in R^2$ be the physical position of player $i = 1, \dots, N$ in the plane at the end of the game. Then $\mathbf{p} = (p_1, \dots, p_N)$ is a possible outcome of the game that denotes the position of all players. Let $\mathbf{p}_{-i}$ be $\mathbf{p}$ with the i-th component removed.

We consider two fundamental metrics that impact the costs of a user complying with an incentive mechanism. Recall $\mathcal{R}$ and $\mathcal{H}$, and redefine them such that $\mathcal{R}(p_i, \mathbf{p}_{-i})$ and $\mathcal{H}(p_i, \mathbf{p}_{-i})$ are the amount of relayed traffic by node i and the average number of hops from i to all destinations, given that i is located at p_i and all other nodes are located at $\mathbf{p}_{-i}$.

We add an interference component, $\mathcal{I}$, to the cost of a node, that is related to the noise produced by other nodes' transmissions. Assuming that all transmissions occur at the same signal power, and that signal power

[1]The cost of moving can be interpreted as a fixed cost, since nodes will eventually reach an equilibrium. Costs associated with complying with the incentive mechanism are operational costs and last for the lifetime of the network.

decays with distance x as $1/x^a$, with $2 \leq a \leq 4$, the interference at node i can be expressed as

$$\mathcal{I}(p_i, \mathbf{p}_{-i}) = \sum_{j \neq i} \frac{N(j,i)}{\|p_i, p_j\|^a} \tag{3}$$

where $\|p_i, p_j\|$ is the distance between nodes j and i, and $N(j,i)$ is the average number of transmissions at j that interfere with the reception of a useful signal at i.

Finally, the payoff function to each user, which we assume to be identical for all users, is defined as

$$U(p_i, \mathbf{p}_{-i}) = \alpha\mathcal{R}(p_i, \mathbf{p}_{-i}) + \beta\mathcal{H}(p_i, \mathbf{p}_{-i}) + \gamma\mathcal{I}(p_i, \mathbf{p}_{-i}) \tag{4}$$

where the weights α, β and γ will be chosen to suit different systems and incentive mechanisms.

Using the above payoff function we can focus on the two classes of incentives previously mentioned: in the reputation mechanism, users minimize the amount of traffic relayed and are indifferent to the number of hops required to reach the destinations. In this system, $\alpha = -1$ and $\beta = 0$; in pricing mechanisms, users maximize their credit balance by minimizing the average number of hops to reach the destinations and by maximizing the amount of relayed traffic. In the simple case in which each hop requires one credit unit per packet, and each relayed packet provides one credit unit [4], we have $\alpha = 1$ and $\beta = -1$.

Interference is either neglected ($\gamma = 0$) or it is assigned a small, negative value ($|\gamma| \ll 1$ and $a = 2$). This latter case corresponds to a system where users regard interference as a second order cost – interference will only be considered if a user cannot improve further the utility determined by $\mathcal{R}$ and $\mathcal{H}$. In the games we study, interference does not play an important role; however, as we will see shortly, it helps to reduce the number of equilibria in the game simplifying the solution set.

We assume, for simplicity, that the traffic matrix is constant and homogeneous as in the previous section. Moreover, we also assume that the traffic pattern produced by the nodes is feasible under any topological formation of the network. We assume that all nodes have a fixed transmission range, and employ the same transmission power to communicate with other nodes. Nodes are not allowed to determine traffic routes, although they can influence them by selecting different positions on the network. The routing algorithm produces unique routes among any pair of nodes, based on minimum hop count. Finally, the incentive mechanisms enforce nodes to relay traffic for other nodes and to comply with the routing protocol.

5 Examples of Games

We present a case study of two different games. In the first one, which we call the *incremental topology game*, we assume that a given number of users N sequentially join an ad-hoc network, one after another. Each player makes a single move which corresponds to selecting its physical position when joining the network. There are two constraints when selecting a position: (i) nodes cannot occupy the same position; (ii) nodes must be connected to the current network. Connectivity is determined by a fixed transmission range. The very first node is arbitrarily placed at coordinates $(0, 0)$, and its move is not actually part of the game. The game ends after the arrival of the Nth user. This scenario is examined in Section 5.1.

In the second game, which we call the *mobility game*, all N nodes are initially placed in a particular network topology. Each player makes a single move that consists of changing their physical position in the network. Nodes play in a pre-determined order and their choice of position is subject to the same two constraints of the first game. This scenario is considered in Section 5.2.

To simplify the analysis, in both of these games we consider a discrete space, such that $p_i \in \mathbb{N}^2$. Thus, nodes have only a finite number of positions to choose from and the game is finite. We solve for the set of subgame perfect equilibria by applying backward induction to the game tree. Although in some cases the equilibrium is unique, in many other cases some nodes are indifferent with respect to a large number of positions in the network. This indifference leads to a large number of subgame perfect equilibria. Given the size of the game tree we consider, enumerating all such equilibria becomes computationally infeasible. Therefore, we have applied a randomization technique to explore the solution space: ties among the best possible actions for a player are broken randomly during the backward traversal of the game tree. Backward induction is then repeated several times using different seeds of the random number generator, in order to obtain (possibly) different solutions.

Note that each subgame perfect equilibrium will induce a network topology. The goal is to characterize these topologies to understand their performance. However, we first describe properties of different subgame perfect equilibria with respect to the induced topologies. We say that the solution of a game is *unique* when the game indeed has a single strategy that is subgame perfect, i.e, no ties are found during the backward induction. We say that the set of all subgame perfect equilibria of the game is *topology-equivalent* when for any two equilibria strategies s_1 and s_2 in this set, the Euclidean distance between any given pair of nodes in the topology induced by s_1 equals the Euclidean distance between that same pair of nodes in the topology induced by s_2. Notice that the topologies induced by the equilibria have exactly the same shape. We say that the set of all subgame perfect equilibria is *shape-equivalent* when the equilibria form topologies that have the same shape, but the position of nodes within the topologies can permute. Note that *topology-equivalent* implies *shape-equivalent* but not the converse. In all other cases we say that the game has *multiple* equilibria, i.e., different subgame perfect equilibria form topologies with different shapes.

Unfortunately, the complexity of the analysis grows exponentially with both the number of nodes and the connectivity radius, so that we are able to consider only a limited combination of parameters. However, all examined cases provide the same qualitative behavior, suggesting the emergence of well-defined strategies irrespective of the number of players.

5.1 Incremental topology game

5.1.1 Topologies on the line

In order to limit the complexity of the game, we first consider the simple case of a unidimensional grid. Node 1 is located at the origin, while all other nodes have to choose coordinates in $\mathbb{Z}$. In order to be connected, each node joining the network must be within distance K of a joined node.

The routing protocol produces minimum hop routes with ties resolved by choosing hops with longest physical distance towards the destination. For example, in the topology depicted in Figure 7, assuming $K = 5$, a message from node 5 to node 1 is routed through node 4. Note that routes are not necessarily symmetric.

In this scenario, the induced topology can be characterized by its spread along the line, which will be denoted by s. We observe that the minimum span of a network topology with N nodes is $N - 1$ ($s_{\min} =$

Table 1: Reputation system on the unidimensional grid with $K = 3$ ($\alpha = -1\,, \beta = 0\,, \gamma = 0$).

N	$\bar{s}$	$\bar{s^*}$	s^*_{std}	$s^*_{\min}$	$s^*_{\max}$	h	runs
3	3.81	3.40	1.12	2	6	1.70	10^4
5	7.18	5.55	0.95	4	10	1.39	10^4
7	10.35	7.02	1.04	6	11	1.17	10^3
9	13.40	8.87	0.97	8	12	1.11	10^2

$N - 1$), while the maximum span is $(N - 1)\,K$ ($s_{\max} = (N - 1)\,K$). Topologies with span $s_{\min}$ will be referred to as *minimal span topologies*.

Recall the **pricing scheme** introduced in Section 4, associated to a utility function (4) in which $\alpha = 1$ and $\beta = -1$. Although the game has many subgame perfect equilibria, we have found that, whenever $N > K$, the equilibria set is *shape-equivalent* with minimal span topologies ($s = N - 1$). This is somehow intuitive: there is no reason to leave "holes" in the network, as occupying a hole always provides a better utility than going to a border position. When $N \leq K$, there are *multiple* solutions, but all topologies correspond to cliques where all nodes are within transmission range of each other. Thus, the network span in this case is always less than or equal to K. Putting these results together, we have $s \leq \max(K, N - 1)$, which holds even if interference is added to the utility function of the users.

The **reputation scheme** introduced in Section 4, in which users want to just minimize the amount of relayed traffic, generates a more interesting game. In this case, getting close to other nodes, or even going to an empty slot in between other nodes may not necessarily be a good strategy, as the amount of relayed traffic typically increases in the middle of the network (see Section 3). However, we have observed that the formation of a cluster of nodes emerges as the best possible strategy for the users. In particular, the clustering increases with an increase in $N > K$.

To demonstrate our claim, we fix $K = 3$ and consider different values of N ranging from 5 to 9. Results are reported on Table 1. The column $\bar{s}$ indicates the average span of all possible topologies that can be built for a given pair (N, K). Columns labeled $\bar{s^*}$, s^*_{std}, $s^*_{\min}$ and $s^*_{\max}$ report the average, standard deviation, minimum and maximum span of topologies that correspond to the *multiple* subgame perfect equilibria found using the randomized search described at the beginning of this Section. The column labeled h reports the ratio between $\bar{s^*}$ and $s_{\min}$, the minimum achievable span.

The last column on the table reports the number of repetitions of the randomized search. Values of N larger than 9 were not considered due to excessive memory and CPU requirements. From the table we observe that the average span of equilibria topologies, $\bar{s^*}$, is always smaller than the average span over all possible topologies, $\bar{s}$, characterized by the same parameters. Notice that this average value is close to minimal span topology and approaches $s_{\min}$ as the number of nodes increases. In fact, the ratio h approaches 1 as N increases. Moreover, the minimal span topology is indeed the outcome of several subgame perfect equilibrium of the game, as shown by the $s^*_{\min}$ column on the table. If we add interference to the utility function of users, the minimum span topology is no longer a subgame perfect perfect equilibrium; however, this is an artifact due to the strategy of the last node that joins the network: knowing to be the last player of the game, it clearly prefers to position itself K hops away from the furthest node in order to reduce the interference received from the network. In fact, a typical outcome of the game is as depicted in Figure 6, in the case of $N = 9$, which shows that nodes tend to form a dense cluster even adding the interference term.

From Table 1 we also note that topologies with the maximum span do not appear to be equilibria (for $N > 3$). It can be shown that the the topology with span $s_{\max}$ is never the outcome of a subgame perfect equilibria for games with $N > 3$ and $K > 1$.

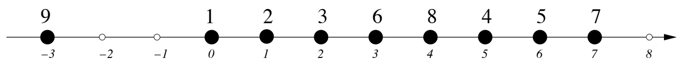

Figure 6: Example of equilibrium of the incremental game based on reputation and interference, with $N = 9$, $K = 3$.

It is, unfortunately, difficult to analyze games with K large and N sufficiently large to allow the strategies that lead to minimal span topologies to emerge as subgame perfect. In order to reduce the complexity of the game (and the number of symmetric solutions), we restrict the grid to the positive integers only, i.e. the position of the nodes are expressed by numbers in $\mathbb{N}$. Interestingly, the game based on the *reputation scheme*, with the addition of interference, now has a *unique* subgame perfect equilibrium. For example, the solution with $K = 5$, $N = 8$ is shown in Figure 7, which has a span $s = 12$ (notice that possible span values range from 7 to 35).

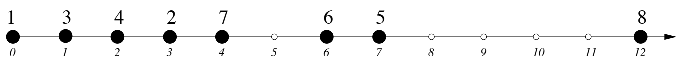

Figure 7: (a) Example of routing on the unidimensional grid: path from source node 5 to destination 1 is routed through node 4 ($K = 5$); (b) Unique solution of the incremental game based on reputation and interference, with $N = 8$, $K = 5$.

These and other experiments using different values of K and N reveal that in a subgame perfect equilibria, the majority of nodes are clustered around the origin. Since a network topology constrained to a line does not have good performance in general, the impact of having a clustered topology may not be significant. However, a topology placed on a two-dimensional space can have much better performance, and a clustering strategy could severely degrade its transport capacity [11]. In what follows, we study this case.

5.1.2 Topologies on the plane

We extend the incremental topology game to a two-dimensional grid in order to investigate if subgame perfect equilibria will also consist of clustered topologies. The routing protocol must be slightly modified with ties in the minimum hop count being resolved by choosing the shortest path length in terms of Euclidean distance. Further ties are broken deterministically[2].

We will consider two measures of compactness of a topology, as suggested in [21]. The *characteristic path length* l is defined as the average distance (minimum number of hops) between every pair of nodes in the topology. The *clustering coefficient* measures how closely nodes are clustered together in a topology. The clustering coefficient of a node is defined as $\frac{|E|}{|V|(|V|-1)/2}$, where V is the set of neighbors of the node according to the connectivity graph, E is the set of edges between nodes in V, and $|\cdot|$ denotes the cardinality of a set. The average clustering coefficient over all nodes is denoted the clustering coefficient, c, of the topology, which ranges from 0 (low clustered) to 1 (highly clustered).

Given a transmission range K, the number of nodes necessary to form an interesting game must be on the order of K^2. In fact, fewer nodes can choose random positions within transmission range of each

[2]Based on the difference between the indexes of the vertices modulus N. Although this is not a standard practice, it is a simple approach to solving the problem of route ties on the grid. Different approaches do not change significantly the numerical results presented in this section.

Table 2: Reputation system on a two-dimensional grid with $N = 9$, $K = 1$ ($\alpha = -1, \beta = 0$), with or without interference.

$\mathcal{I}$	$\bar{l}$	$\bar{l^*}$	l^*_{std}	l^*_{min}	l^*_{max}	runs
yes	2.58	2.34	0.12	2.17	2.55	10^2
no	2.58	2.33	0.13	2.11	2.72	10^2

other without having to forward any traffic. Unfortunately, the complexity of the solution on the plane soon becomes very prohibitive even for small values of K.

However the case $K = 1$, which is trivial on the line, is already interesting on the two-dimensional grid. In this case, while an arriving node necessarily has to choose a position that neighbors another node, there are many possible outcomes which will result in more or less clustered topologies.

With $K = 1$, we are able to study games comprising up to 9 nodes. In such topologies, the two extremes for the characteristic path length measure are represented by a chain of nodes where no node has more than two neighbors (referred to as *simple chain*), for which $l = 3.33$, and by a square cluster with edge 3, for which $l = 2$. Note that the clustering coefficient for any topology with $K = 1$ is always zero.

We have found that the equilibria of the game based on the **pricing scheme** yields the topology corresponding to the square cluster of edge 3. The equilibria set is *topology-equivalent* if the interference term is present in the utility function, and *shape-equivalent* if interference is not considered.

The game based on the **reputation scheme**, instead, has a more complex outcome, similar to what we observed on the line. Table 2 reports the results of our exploration of the set of subgame perfect equilibria for $N = 9, K = 1$. The first column indicates whether or not interference is present in the user utility function. Column $\bar{l}$ contains the average *characteristic path length* of all topologies that can be built when $N = 9$, $K = 1$. Columns labeled $\bar{l^*}$, l^*_{std}, l^*_{min} and l^*_{max} reports the average, standard deviation, minimum and maximum *characteristic path length* of topologies corresponding to the *multiple* subgame perfect equilibria.

We observe that $\bar{l^*}$ is smaller than the average over all possible topologies, about half way between $\bar{l}$ and the minimum achievable value of 2. However, both of the extreme cases mentioned above ($l = 2$ and $l = 3.33$) are not possible outcomes of any subgame perfect equilibria.

Since obtaining the equilibria for the incremental topology game on the plane with $K > 1$ requires a huge computational effort, we have constrained the action space by considering a grid only with positive coordinates. We study the case with $K = 2$, $N = 7$. In this constrained game, we have found that the equilibria set of the game based on the **pricing scheme** is *shape-equivalent*, irrespective of the presence of interference in the utility function. An example of solution is depicted in Figure 8(a). This topology is characterized by $l = 1.19$, which is the minimum characteristic path length achievable with $N = 7$ and $K = 2$.

The game based on the **reputation scheme** produces slightly different results depending on the presence of interference. Adding the interference component, the equilibria set is *topology-equivalent*, with $\bar{l^*} = 1.38$ and $\bar{c^*} = 0.77$. An example of an equilibrium solution is depicted in Figure 8(b). Removing this term, produces *multiple* equilibria characterized by the numbers reported in Table 3.

The first row refers to the *characteristic path length*, while the second to the *clustering coefficient*. The first column reports the average value of the above metrics over all possible topologies that can be generated with $N = 7$, $K = 2$. We observe that the average clustering coefficient of topologies corresponding to game

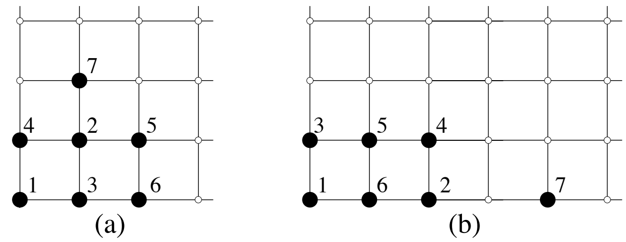

Figure 8: Topology resulting from the incremental game based on pricing (a) and on reputations with interference (b) using $N = 7$, $K = 2$.

Table 3: Reputation system on the positive two-dimensional grid with $N = 7$, $K = 2$ ($\alpha = -1$, $\beta = 0$), without interference.

metric	$\bar{l}$ / $\bar{c}$	$\bar{l^*}$ / $\bar{c^*}$	$l_{\rm std}$ / $c_{\rm std}$	$l_{\min}$ / $c_{\min}$	$l_{\max}$ / $c_{\max}$	runs
l	1.91	1.36	0.12	1.19	1.76	10^2
c	0.42	0.73	0.08	0.48	0.84	10^2

equilibria,$\bar{l^*}$, is large, while the characteristic path length is small (the minimum possible value of 1.19 is actually achieved).

All examples considered above suggest that node clustering emerges as the best strategy for all incremental topology games. Moreover, under the pricing mechanisms, the topologies associated with the set of subgame perfect equilibria are *shape-equivalent* and induce minimal span topologies. The topologies associated with the solutions generated by the reputation mechanism are not as regular, but consistently exhibits clusters. These observations do not hold well for an ad-hoc wireless network, as a clustered topology exhibits poor network capacity, especially when considering the two-dimensional space. In fact, if all nodes happen to be within transmission range of each other, only a single transmission would be possible at any given time, while it has been shown in [11] that much greater capacity can be achieved if nodes are spaced apart.

5.2 Mobility game

This game starts with nodes arranged in a given network topology, and proceeds with each node moving to (possibly) a different location in a pre-determined order, with the constraint of maintaining network connectivity. Note that the order of play, which is known to all players, may affect the outcome of the game. To emphasize the claim that network capacity decreases if users exploit mobility strategically under incentive mechanisms, the game will start with topologies that provide optimal connectivity and transport capacity. We will show that users can exploit mobility to reduce their costs of complying with the incentives.

We first consider the case of $N = 4$ nodes. The optimal topology at the start of the game is depicted in Figure 9(a). This topology is characterized by $l = 1.33$, $c = 0$, and allows two transmissions to occur simultaneously, significantly improving the overall capacity with respect to the case in which all nodes are within transmission range.

Under the **pricing scheme**, we have found that all subgame perfect equilibria yield topologies characterized by $l^* = 1$, $c^* = 1$, which means that all nodes end up within transmission range of each other. This occurs irrespective of the order of play. In particular, if the interference component is considered, the equilibria set is *shape-equivalent*, with an example topology shown in Figure 9(b).

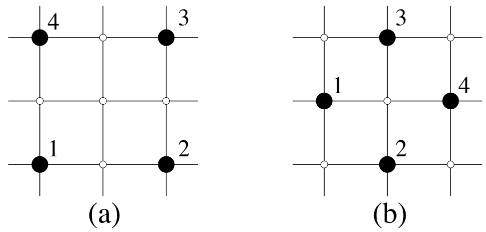

Figure 9: Initial topology of the mobility game with $N = 4$ (a) and unique solution of the game based on pricing and interference (b).

Table 4: Reputation system starting from the optimal topology of parameters $N = 4$, $K = 2$, with interference ($\alpha = -1\,, \beta = 0\,, \gamma = -0.001$).

metric	$\bar{l^*}$ $\bar{c^*}$	l_{std} c_{std}	$l_{\min}$ $c_{\min}$	$l_{\max}$ $c_{\max}$	runs
l	1.65	0.07	1.33	1.67	10^4
c	0.026	0.12	0	0.58	10^4

The game under the **reputation scheme** has *multiple* solutions, that differ in their topologies and depend on the order of play. To explore this game, we choose a random permutation of the order of play before the game starts, which is then known to all players. We consider only the case with interference, showing results in Table 4. It can be shown that the initial optimal topology is never induced by any subgame perfect equilibrium.

From Table 4 we observe that in most of the cases the resulting topology is a simple chain of nodes. In fact, the average values of l and c are very close to the ones that characterize a line of 4 nodes separated by the transmission range (for which $l = 1.67$, $c = 0$). This particular behavior, which contradicts the clustering strategy observed in all games considered so far, can be explained as follows. Since we assume that nodes cannot break the network connectivity as a consequence of their movement, nodes find it advantageous to move to positions where they trap the nodes that would move next. For example, if the order of play is $\{4, 1, 2, 3\}$, an optimal strategy for node 4 is to move to the left of node 1. This prevents both nodes 1 and 2 (now in the middle of a simple chain) from moving away without disconnecting the network, while the the last node 3, who could move, now has no reason to do so. Thus, the resulting topology is a simple chain.

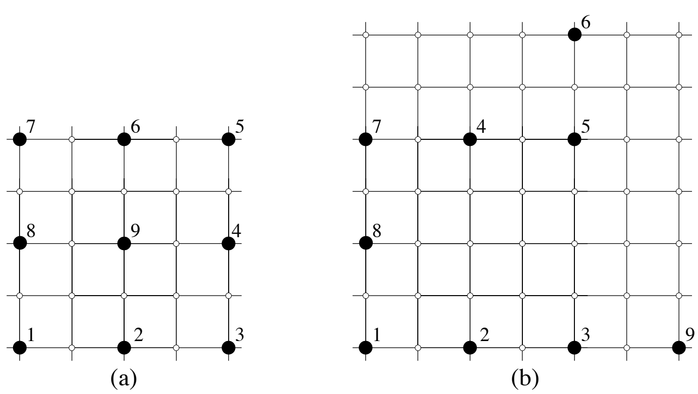

Figure 10: Initial topology of the mobility game with $N = 9$ (a) and unique solution of the game based on reputation and interference (b), when the order of play is $\{9, 6, 5, 4, 2, 8, 1, 3, 7\}$.

A similar outcome also emerges under the **reputation scheme** when we increase the number of nodes.

For instance, with $N = 9$ we have started the game from the regular grid depicted in Figure 10(a). If the order of play is $\{9, 6, 5, 4, 2, 8, 1, 3, 7\}$ (actually, any permutation starting with the sequence $\{9, 6, 5, 4\}$), the game based on reputation and interference has the unique solution depicted in Figure 10(b). This behavior can be explained with similar arguments to those used in the case of $N = 4$.

Several other experiments with different numbers of nodes, not reported here, confirm that the mobility game, under the **reputation scheme**, has subgame perfect equilibria that leads to simple chains, which have particularly poor network performance on the plane. We argue, however, that the a-priori knowledge of the order of play is fundamental to obtain unique solutions in the form of simple chains. As part of our future work, we plan to consider also the case in which the order of play is not known at the start of the game.

5.3 Mixing different populations of users

So far, we have assumed that all players are identical, strategically selecting their location in order to minimize monetary cost and/or battery power. However, in a diverse environment, users may have very different needs and constraints. In particular, some users may be unable or unwilling to change their position within the network, or simply be unaware of the fact that they could improve their utility by moving.

To illustrate the outcome in a diverse environment, we explore a simple scenario where network users are divided into two classes: strategic and indifferent. Strategic users are rational and behave as in all previous games considered, while indifferent users are static and do not move. We consider the *incremental topology game* introduced in 5.1 and investigate both the **pricing scheme** and **reputation scheme**. We will assume a fixed total number of users in the network, N, some of which are indifferent and the remaining strategic. Indifferent users are initially placed in the network in positions that maximize the network capacity. Figure 11 illustrates the position indifferent users would occupy (numbered accordingly) for the case where the network has a total of 6 users and $K = 2$. Strategic users then join the network one at a time as in the incremental topology game. For example, if we assume 2 indifferent users, then they would occupy positions 1 and 2 in Figure 11, and then 4 strategic users would then join the network. In the analysis that follows, we have neglected the interference term in the utility function of the strategic players.

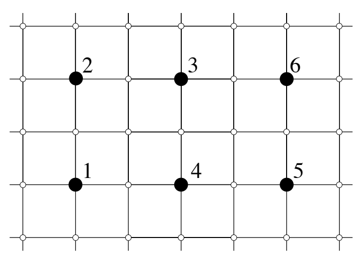

Figure 11: Fixed positions of the indifferent users, aiming at maximizing the network capacity

In Figure 12 we have plotted the average utility obtained by users in both classes, under the reputation and pricing schemes. Confidence intervals reflect the variability observed in different equilibria of the game played by strategic users. The main observation is that in both incentive schemes indifferent users get "exploited" by strategic users. The utility gap is maximum for small fractions of indifferent users under the reputation scheme, while in the pricing scheme the major differences are observed for large fractions of indifferent users. Notice that, in the pricing scheme, the sum of utilities of all users is always zero, since the total amount of money in the network is constant. However, strategic players always have (on average) positive credit balance, at the expense of indifferent users, who have a negative balance. Similar results were

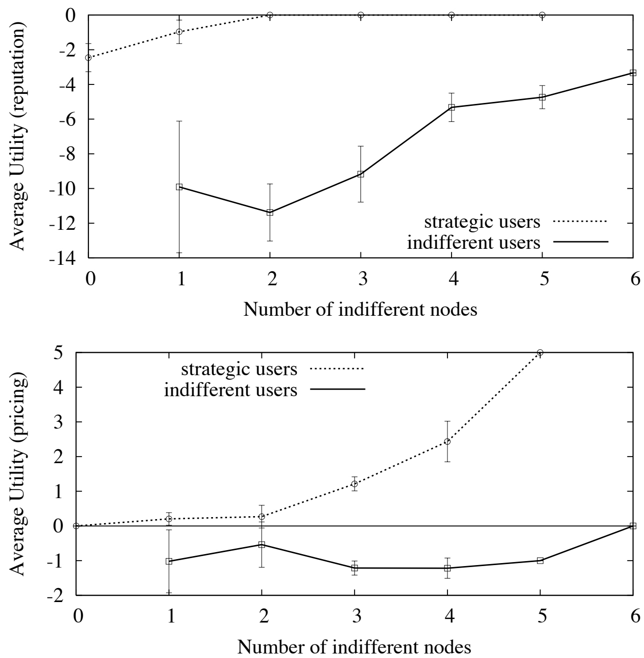

Figure 12: Average utility of strategic and indifferent users for the reputation scheme (top plot) and pricing scheme (bottom plot), while varying the number of indifferent users (lower values are better)

also obtained for other variations of this game.

We argue that strategic movement, besides decreasing the overall network capacity in general, can generate significant utility imbalances in the case of a mixed population of users. In particular, the fact that indifferent users may be exploited by users that move strategically poses a severe fairness problem to the deployment of ad-hoc networks.

6 Simulation Study of Greedy Users

In this section we examine more complex games that cannot be easily solved using a game theoretic approach. In particular, we consider the *infinite mobility game*. In this game, all N nodes in the network are initially placed randomly in a connected topology. At each step of the game, a node is selected randomly to move. Node positions are no longer constrained to a discrete grid and can be anywhere in the real plane. The game has an infinite number of steps.

Although this is a well-defined game, obtaining strong game theoretic solutions, such as the subgame perfect equilibria, seems infeasible. Thus, we resort to a simple heuristic in order to obtain an equilibrium. We consider a greedy strategy, where each node, when given the opportunity to move, selects a position randomly on the plane that provides a better payoff, given the current network topology. This strategy does not consider the future movement of other nodes and could represent the limited reasoning capability of the nodes.

To study this game under the greedy strategy, we construct a simple simulator to compute the equilibria topologies that might be reached. The simulation starts with an initial topology with nodes placed in a random connected network. At each step of the game, the next player to move is chosen uniformly at random among all players. The simulation terminates when a topology is formed and all nodes, after receiving an opportunity to move, do not find a better position in the network (after inspecting a very large number of positions).

Under the **reputation scheme** with interference, we have inspected games with up to 20 nodes. We have found that the equilibrium topology reached at the end of the simulation is always characterized by a stretched line. An example illustrating the initial random topology and the resulting topology for the case with $N = 10$, $K = 1$, is shown in Figure 13. This solution is similar to the simple chain solutions obtained in the mobility game described in the previous section.

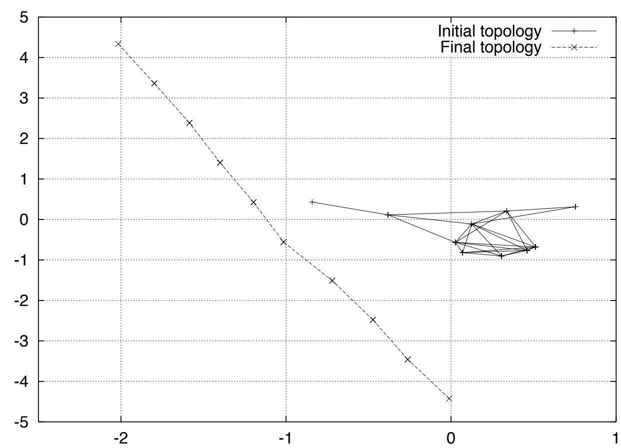

Figure 13: Example of initial and equilibrium topologies of the infinite mobility game under reputation scheme and interference, with $N = 10$, $K = 1$.

We have formally proved that this equilibrium, where nodes form a stretched straight line, always emerges independently of the number of players and initial topology, as the number of moves goes to infinity. Intuitively, since players are greedy and only consider the next best position, nodes routing traffic will move to the "edge" of the network provided they do not disconnect the network. Nodes not routing traffic will also move to the "edge" of the network to reduce interference. Eventually a simple chain will be reached. To minimize the interference, nodes in the simple chain will stretch out to form a straight line. It is clear that the equilibrium of this game has very poor network capacity as the node in the middle of the topology must forward traffic for $N^2/2$ source-destination pairs.

We have also investigated the **pricing scheme**, considering games with up to 20 nodes. In all cases, we have found that the equilibria of the game is a topology where nodes are tightly clustered together within transmission range of each other. Interestingly, if we add interference, nodes place themselves onto one or more "orbits" within the circle of diameter K. The choice of such position reflects the effort of the nodes to minimize interference, as this becomes the only non-zero term in their utility function. This solution is very similar to the ones observed in the games studied in the previous section. An example illustrating the outcome of the game with $N = 20, K = 1$ is illustrated in Figure 14.

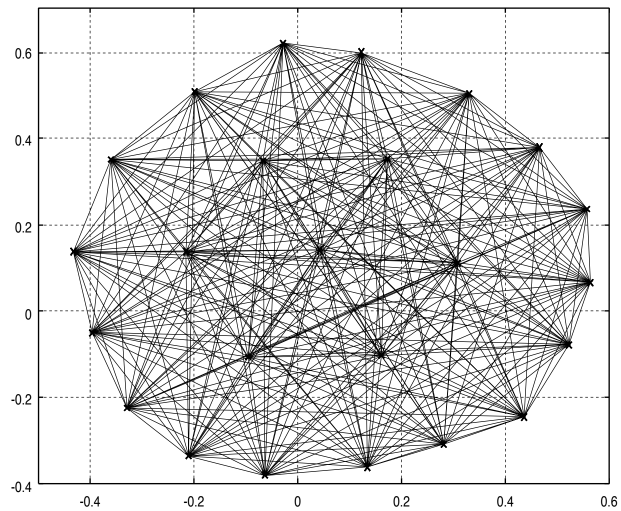

Figure 14: Example equilibrium topology of the infinite mobility game under pricing scheme and interference, with $N = 20$, $K = 1$.

7 Conclusions

In this paper we have shown that incentive mechanisms used to foster cooperation in ad-hoc networks can produce perverse side-effects if users decide their strategy based on factors that were not originally considered by the system designers.

In particular, we have considered strategic user mobility which so far has been neglected in the analysis of incentive mechanisms and can have dramatic consequences on the network performance. We have demonstrated this threat using simple game theoretical models having analytically tractable solutions. The degradation of network performance is essentially due to poor network topologies associated with game equilibria.

We have investigated both reputation and payment based incentive mechanisms, and different types of strategic games. Our results indicate that two forces are at play in determining the best strategies of the users: a collapsing force, which pushes nodes to cluster together as much as possible, and a scattering force, which eventually leads to the formation of simple chains of nodes. Although the outcomes of the games can differ, the effect on the network is negative for all considered cases, bringing to the conclusion that "mobility decreases the capacity of ad-hoc wireless networks".

So far, incentive mechanisms proposed for ad-hoc networks have mainly focused on promoting cooperation among the peers. Since requiring all peers to be just fully cooperative may not lead to optimal network performance when nodes are allowed to move strategically, an interesting question is whether incentives mechanisms can be designed to avoid users from forming topologies with poor network performance.

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